A NEW CLASS OF ALMOST UNBIASED ESTIMATOR

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***Abstract*— In this paper, we consider a new class of estimator in linear regression model, namely almost unbiased principal component two-parameter estimator. Under the mean square error matrix criterion, the superiority of the almost unbiased principal component two-parameter estimator relative to the ordinary least squares estimator and almost unbiased two-parameter estimator is discussed. In addition, we give a numerical example and a simulation study to study the performance of the new estimator.**

***Keywords*— ordinary least squares estimator, almost unbiased estimator, linear regression model, mean squared error matrix, mean squared error.**

I. INTRODUCTION

It is well known that the ordinary least squares (OLS) estimator the best linear unbiased estimator in a linear regression model. However, when there exists multicollinearity in the model, the OLS estimator may be unstable and produce a large mean squared error (MSE). To overcome this problem, statisticians have proposed many biased estimators, such as ridge estimator (Hoerl *et al.*, 1970), Liu estimator (Zhang, 2006), principal component regression (PCR) estimator (Massy, 1965), r-k class estimator (Baye *et al.*, 1984), r-d class estimator (Kaçıranlar *et al.*, 2007), two-parameter estimator (Yang *et al.*, 2010) and principal component two-parameter estimator (Chang, 2012). All these biased estimations are aimed at reducing the MSE by sacrificing the unbiasedness of the estimator.

Although biased estimator shows good performance in the presence of multicollinearity, its estimate deviates from the true value of the parameter. Therefore, some statisticians have begun to pay attention to reducing the bias of biased estimators while keeping the MSE of the estimator small, thus introducing almost unbiased estimators. Kadiyala (1984) first proposed a class of almost unbiased estimator. Since then, statisticians have proposed a series of almost unbiased estimators based on biased estimators. For example, Singh (1986) proposed an almost unbiased ridge (AUR) estimator based on the ridge estimator, and Akdeniz and Kaçıranlar (1995) proposed an almost unbiased Liu (AUL) estimator. Chang (2014) proposed two almost unbiased two-parameter (AUTP) estimators and discussed the properties of two AUTP estimator in the sense of MSE, respectively. The results show that under MSE criteria, AUTP estimator is better than the OLS estimator and the corresponding biased estimator is improved. Li and Yang (2014) propose the almost unbiased ridge principal component estimator (AURPC) and the almost unbiased Liu principal component (AULPC) estimator by combining PCR with the AUR estimator and AUL estimator (Martínez *et al.*, 2017).

In this paper, we propose an almost unbiased principal component two-parameter estimator based on the AUTP and PCR estimator. Then, we will discuss the superiority of the new estimator with OLS and AUTP under the mean squared error matrix (MSEM). Finally, a numerical example and a simulation study are given (Ahmadizadeh *et al.*, 2017).

II. THE PROPOSED ESTIMATOR

A general linear regression model can be expressed as seen in Equation 1:

$$y = X\beta + \varepsilon \qquad (1)$$

In the formula above, y is an $n\times1$ vector of observed values on response variable, X is an $n\times p$ full column rank matrix of n observations on p explanatory variables, β is a $p\times1$ vector of unknown parameters, ε is an $n\times1$ vector of disturbance assumed to be distributed with mean vector **0** and variance covariance matrix $\sigma^2 I$ and I is an identity matrix of order $n\times n$.

Let Φ be the orthogonal matrix such that $\Phi^T X^T X\Phi = \Lambda = diag(\lambda_1, \lambda_2, \cdots, \lambda_p)$, where $\lambda_1 \ge \lambda_2 \ge \cdots \ge \lambda_p$ denote the ordered eigenvalues of $X^T X$. Then model (1) can be written in canonical form as follows in Eq. 2:

$$y = Z\alpha + \varepsilon \qquad (2)$$

where $\alpha = \Phi^T\beta$, $Z = X\Phi$.

According to the model (2), the AUR estimator proposed by Singh (1986), the AUL estimator proposed by Akdeniz and Kaçıranlar (1995), the AUTP proposed by Chang (2014), the AURPC and AULPC estimator proposed by Li and Yang (2014) can be written as can be seen respectively in Eqs. From 3 to 7:

$$\hat{\alpha}_{AUR} = (I - k^2(\Lambda + kI)^{-2})\hat{\alpha}_{OLS} \qquad (3)$$

$$\hat{\alpha}_{AUL} = (I - (1-d)^2(\Lambda + I)^{-2})\hat{\alpha}_{OLS} \qquad (4)$$

$\hat{\alpha}_{AUTP} = \{2I - (\Lambda + I)^{-1}(\Lambda + dI)(\Lambda + kI)^{-1}\Lambda\}\ (\Lambda+I)^{-}$

$$^{1}(\Lambda+dI)(\Lambda+kI)^{-1}\Lambda\hat{\alpha}_{OLS} \quad (5)$$

$$\hat{\alpha}_{AURPC} = \Phi_r\Phi_r^T(I - k^2(\Lambda+kI)^{-2})\hat{\alpha}_{OLS} \quad (6)$$

$$\hat{\alpha}_{AULPC} = \Phi_r\Phi_r^T(I - (1-d)^2(\Lambda+I)^{-2})\hat{\alpha}_{OLS} \quad (7)$$

Motivated by Combining Li and Yang, combine the idea of the PCR estimator and the AUTP estimator, we propose a new class estimator called almost unbiased two parameter principal component (AUTPC) estimator as follows:

$$\hat{\alpha}_{AUTPC} = \Phi_r\Phi_r^T\hat{\alpha}_{AUTP}$$
$$=\Phi_r\Phi_r^T\left\{2I-(\Lambda+I)^{-1}(\Lambda+dI)(\Lambda+kI)^{-1}\Lambda\right\}$$
$$(\Lambda+I)^{-1}(\Lambda+dI)(\Lambda+kI)^{-1}\Lambda\hat{\alpha}_{OLS} \quad (8)$$

According the definition of AUTPC, we can get following results:

If $d=1$, then $\hat{\alpha}_{AUTPC} = \hat{\alpha}_{AURPC}$.

If $k=0$, then $\hat{\alpha}_{AUTPC} = \hat{\alpha}_{AULPC}$.

If $r=p$, then $\hat{\alpha}_{AUTPC} = \hat{\alpha}_{AUTP}$.

If $r=p$ and $d=1$, then $\hat{\alpha}_{AUTPC} = \hat{\alpha}_{AUR}$.

If $r=p$ and $k=0$, then $\hat{\alpha}_{AUTPC} = \hat{\alpha}_{AUL}$.

So, the AUTPC estimator can be seen as a generalization of the AURPC, AULPC, AUTP, AUR and AUL estimators.

Let $G=(\Lambda+I)^{-1}(\Lambda+dI)(\Lambda+kI)^{-1}\Lambda$, then the $\hat{\alpha}_{AUTPC}$ can be expressed as:

$$\hat{\alpha}_{AUTPC} = \Phi_r\Phi_r^T(2I-G)G\hat{\alpha}_{OLS} \quad (9)$$

Furthermore, we can obtain the bias, covariance matrix and MSEM of AUTPC as follows,

$Bias\ (\hat{\alpha}_{AUTPC}) = E\ (\hat{\alpha}_{AUTPC}) - \alpha =$

$$(\Phi_r\,\Phi^T_r\,(2I-G)\,G - I)\,\alpha \quad (10)$$

$$Cov(\hat{\alpha}_{AUTPC}) = \sigma^2\Phi_r\,\Phi^T_r\,(2I-G)\,G\,\Lambda^{-1}G^T\,\Phi_r\Phi^T_r \quad (11)$$

$MSEM\ (\hat{\alpha}_{AUTPC}) = \sigma^2\Phi_r\,\Phi^T_r\,(2I-G)\,G\Lambda^{-1}G^T(2I-G)$
$\Phi_r\,\Phi^T_r\ + (\Phi_r\,\Phi^T_r\,(2I-G)\,G-I)\,\alpha\alpha^T$
$\Phi_r\,\Phi^T_r\,(2I-G)\,(G-I)^2 \quad (12)$

According to Xu *et al.*, (2011), equations (10) and (12) can rewritten as follows:

$$Bias\ (\hat{\alpha}_{AUPCTP}) = [-\Phi_r\Phi^T_r(G-I)^2 - \Phi_{p-r}\Phi^T_{p-r}]\ \alpha \quad (13)$$

$MSEM\ (\hat{\alpha}_{AUPCTP}) = \sigma^2\Phi_r\,\Phi^T_r\,(2I-G)\,G\Lambda^{-1}G^T\,(2I-G)^T\,\Phi_r\,\Phi^T_r + [-\Phi_r\,\Phi^T_r\,(G-I)^2 - \Phi_{p-r}\,\Phi^T_{p-r}]\,\alpha\alpha^T\,[-\Phi_r\,\Phi^T_r\,(G-I)^2 - \Phi_{p-r}\,\Phi^T_{p-r}]^T \quad (14)$

III.PROPERTIES OF THE NEW ESTIMATOR

For convenience of following discussion, a definition, some lemmas and notations are listed as it can be seen in the next paragraphs.

Let the case of a matrix A, where $A>0$.

It means that A is a non-negative matrix, where A^+ denotes the Moore-Penrose inverse of A, $R(A)$ and $N(A)$ stands for the column space and null space, respectively.

Definition 1: Let $\hat{\alpha}_1$ and $\hat{\alpha}_2$ are two estimators of α, if $MSEM(\hat{\alpha}_2) - MSEM(\hat{\alpha}_1) \geq 0$, then $\hat{\alpha}_1$ is said to be superior to $\hat{\alpha}_2$ in the MSEM sense.

Lemma 1 (Farebrother, 1976): Suppose A is a non-negative definite and a is a compatible column vector. Then $A - aa^T \geq 0$ if and only if $a^T A^{-1} a \leq 1$.

Lemma 2 (Baksalary *et al.*, 1991): Let $C_{n\times p}$ be the set of $n\times p$ complex matrices, $H_{n\times n}$ be the subset of $C_{n\times p}$ consisting of Hermitian matrices. For given $L\in C_{n\times p}$, L^* and $\kappa(L)$ stand for the conjugate transpose and the set of all generalized inverse of L, respectively.

Let $D\in H_{n\times n}$, $D^-\in\kappa(D)$, a_1 and $a_2\in C_{n\times 1}$ be linearly independent, $f_{ij} = a_i^* D^- a_j$, $i,j=1,2$ and if $a_1\notin R(D)$, let

$$s = [\frac{a_2^*(I-DD^-)^*(I-DD^-)a_2]}{[a_1^*(I-DD^-)^*)(I-DD^-)a_1]}.$$

Then, $D + a_1a_1^* - a_2a_2^* \geq 0$ if and only if one of the following sets of conditions holds:

(a) $D\geq 0$, $a_i\in R(D)$, $i=1,2$, $(f_{11}+1)(f_{22}-1)\leq |f_{12}|^2$;

(b) $D\geq 0$, $a_1\notin R(D)$, $a_2\in R(D\vdots a_1)$, $(a_2 - sa_1)^* D^-(a_2 - sa_1)\leq 1 - |s|^2$;

(c) $D = U\Delta U^* - \lambda vv^*$, $a_i\in R(D)$, $i=1,2$, $v^*a_1\neq 0$, $f_{11}+1\leq 0$, $f_{22}-1\leq 0$, $(f_{11}+1)\times(f_{22}-1)\geq |f_{12}|^2$, where $(U\vdots v)$ is a sub unitary matrix (U possibly absent), Δ a positive-definite diagonal matrix (being U present) and λ has a positive scalar value. Further, the conditions (a), (b) and (c) are all independent of the choice of $D^-\in\kappa(D)$.

A. Comparing between estimators: the AUTPC and the OLS

Theorem 1:

Suppose $\Phi_r^T A\Phi_{p-r} = 0$ and $\Phi_{p-r}^T A\Phi_{p-r}$ is invertible, then the new estimator $\hat{\alpha}_{AUTPC}$ is superior to the $\hat{\alpha}_{OLS}$ under the MSEM criterion if and only if:

$b^T\,[\sigma^2\,(\Lambda_r^{-1} - \Phi_r\,\Phi_r\,(2I-G_r)\,G_r\Lambda^{-1}\,G^T_r\,(2I-G_r)^T\,\Phi_r\,\Phi^T_r)]\ b < 1 \quad (15)$

where $b=\left[-\Phi_r\Phi_r^T(G-I)^2-\Phi_{p-r}\Phi_{p-r}^T\right]\alpha$.

Proof. Where it is easy to compute that:

$$MSEM(\hat{\alpha}_{OLS})-MSEM(\hat{\alpha}_{AUTPC})=D-bb^T \quad (16)$$

$$D=\sigma^2\Phi_r[\Lambda_r^{-1}-(2I-G_r)G_r\Lambda_r^{-1}G_r^T(2I-G_r)^T]\Phi_r^T + \sigma^2\Phi_{p-r}\Lambda_{p-r}^{-1}\Phi_{p-r}^T$$

and $b=\left[-\Phi_r\Phi_r^T(G-I)^2-\Phi_{p-r}\Phi_{p-r}^T\right]\alpha$.

Then, $D>0$ if and only if:

$$\Lambda_r^{-1}-(2I-G_r)G_r\Lambda_r^{-1}G_r^T(2I-G_r)^T>0.$$

$$\Lambda_r^{-1}-(2I-G_r)G_r\Lambda_r^{-1}G_r^T(2I-G_r)^T=$$

$$diag\left\{\frac{1}{\lambda_i}-\frac{\left[\lambda_i^2+\lambda_i(2k+2-d)+2k\right]^2(\lambda_i+d)^2\lambda_i}{(\lambda_i+1)^4(\lambda_i+k)^4}\right\}_{i=1}^r$$

So, $D>0$, if and only if:

$$(\lambda_i+1)^4(\lambda_i+k)^4-\left[\lambda_i^2+\lambda_i(2k+2-d)+2k\right]^2(\lambda_i+d)^2\lambda_i^2>0$$

$$(\lambda_i+1)^2(\lambda_i+k)^2-\left[\lambda_i^2+\lambda_i(2k+2-d)+2k\right](\lambda_i+d)\lambda_i>0$$

Since $k>0$ and $0<d<1$, we have:

$$(\lambda_i+1)^2(\lambda_i+k)^2-\left[\lambda_i^2+\lambda_i(2k+2-d)+2k\right](\lambda_i+d)\lambda_i=[\lambda_i(k+1-d)+k]^2>0$$

Thus, we have the next expression:

$$\Lambda_r^{-1}-(2I-G_r)G_r\Lambda_r^{-1}G_r^T(2I-G_r)^T>0.$$

According to the Lemma 1, we get that: $D-bb^T>0$. So, the Proof is completed.

B. Comparing between estimators: the AUTPC and the AUTP

Firstly, by simple computation we get the variance matrix, bias and MSEM of the AUTP estimator as follows:

$$Cov(\hat{\alpha}_{AUTP})=(2I-G)G\alpha \quad (17)$$

$$Bias(\hat{\alpha}_{AUTP})=-(G-I)^2\alpha \quad (18)$$

$$MSEMS(\hat{\alpha}_{AUTP})=(2I-G)G\alpha \quad (19)$$

Theorem 2:

Suppose $\Phi_r^TA\Phi_{p-r}=0$ and $\Phi_{p-r}^TA\Phi_{p-r}$ is invertible, then the new estimator $\hat{\alpha}_{AUPCTP}$ is superior to the $\hat{\alpha}_{AUTP}$ under the MSEM criterion if and only if $\alpha\in N(F)$, where

$$F=\sigma^{-1}(\Phi_{p-r}^T\Lambda^{-1}\Phi_{p-r})^{-\frac{1}{2}}\Phi_{p-r}^T.$$

Proof. It is easy to get that:

$$b_1=-(G-I)^2\alpha$$

$$b_2=\left[-\Phi_r\Phi_r^T(G-I)^2-\Phi_{p-r}\Phi_{p-r}^T\right]\alpha$$

$$D_1=Cov(\hat{\alpha}_{AUTP})-Cov(\hat{\alpha}_{AUPCTP})=\sigma^2\Phi_{p-r}(2I-G_r)G_r\Phi_{p-r}^T\Lambda^{-1}\Phi_{p-r}G_r(2I-G_r)\Phi_{p-r}^T$$

The Moore-Penrose inverse D_1^+ of D_1 is:

$$D_1^+=\sigma^{-2}\Phi_{p-r}(2I-G_r)^{-1}G_r^{-1}(\Phi_{p-r}^T\Lambda^{-1}\Phi_{p-r})^{-1}G_r^{-1}(2I-G_r)^{-1}\Phi_{p-r}^T$$

$$D_1D_1^+=\Phi_{p-r}\Phi_{p-r}^T=I-\Phi_r\Phi_r^T$$

Since $D_1D_1^+b_1\neq b_1$, $b_2-b_1=D_1\gamma_1$,

so $b_1\notin D_1$, and $b_2\in R(D_1 \vdots b_1)$, where

$$\gamma_1=-\sigma^2\Phi_{p-r}(2I-G_r)^{-1}G_r^{-1}(\Phi_{p-r}^T\Lambda^{-1}\Phi_{p-r})^{-1}\Phi_{p-r}^T\alpha$$

$$X=\begin{pmatrix}1.9&2.2&1.9&3.7\\1.8&2.2&2.0&3.8\\1.8&2.4&2.1&3.6\\1.8&2.4&2.2&3.8\\2.0&2.5&2.3&3.8\\2.1&2.6&2.4&3.7\\2.1&2.6&2.6&3.8\\2.2&2.6&2.6&4.0\\2.3&2.8&2.8&3.7\\2.3&2.7&2.8&3.8\end{pmatrix}\quad y=\begin{pmatrix}2.3\\2.2\\2.2\\2.3\\2.4\\2.5\\2.6\\2.6\\2.7\\2.7\end{pmatrix}$$

according to the condition of part (b), in Lemma 2 that the $\hat{\alpha}_{AUTPC}$ is superior $\hat{\alpha}_{AUTP}$ in the MSEM criterion, if and only if $(b_2-b_1)^TD_1^-(b_2-b_1)=\gamma_1^TD_1\gamma_1\leq 0$.

Observing that:

$$\gamma_1^TD_1\gamma_1=\sigma^{-2}\alpha^T\Phi_{p-r}(\Phi_{p-r}^T\Lambda^{-1}\Phi_{p-r})^{-1}\Phi_{p-r}^T\alpha=\alpha^TF^TF\alpha\geq 0$$

where $F=\sigma^{-1}(\Phi_{p-r}^T\Lambda^{-1}\Phi_{p-r})^{-\frac{1}{2}}\Phi_{p-r}^T$, the Proof is completed.

C. Numerical examples

In order to illustrate the behavior of the new estimator, we are to conduct a real data study which is the total National Research and Development Expenditures as a percentage of Gross National Product by country: 1972-1986. It can be seen from Table 1 that the estimated MSE value of the AUTPC estimator is always less than the estimated MSE value of the OLS estimator and the AUTP estimator in this study. The results are consistent with Theorem 1 and Theorem 2.,

It is easy to get the eigenvalues of X^TX are 302.9626, 0.7283, 0.0446, 0.0345 and the condition number of X^TX is 8781.525.

Since the condition number is greater than 100, which express the multicollinearity existence in this data. And we can get the OLS estimator of β and σ^2 are $\hat{\beta}_{OLS}=(0.6455,0.0896,0.1436,0.1526)^T$ and $\hat{\sigma}_{OLS}^2=0.0015$.

In this example, we choose $r=3$, and set $d=0.9$. When the value of k changes from 0.001 to

0.009 with step length of 0.001, we can get the MSE value of OLS estimator, AUTP estimator and AUTPC estimator (see Table1).

IV. MONTE CARLO SIMULATION

To further study the performance of the new estimator, we are to perform a Monte Carlo study. According to [11], explanatory variables are generated by:

$$x_{ij} = (1-\rho^2)^{1/2} z_{ij} + \rho z_{i,p}, \quad i = 1,\cdots,n, \quad j = 1,\cdots,p$$

Where z_{ij} is an independent standard normal pseudo-random number, ρ is a specified constant, and ρ^2 gives the correlation which between any two explanatory variables.

The responses variable come from the next considerations:

$$y_i = \beta_1 x_{i1} + \beta_2 x_{i2} + \cdots + \beta_p x_{ip} + \varepsilon_i, \quad i = 1,\ldots,n$$

So, we have that ε_i is a normal pseudo-random number with mean zero and variance 1. In this simulation, we have chosen $p = 5$, $\rho = 0.8, 0.9, 0.99$, $n = 50$, and we have chosen the normalized eigenvector corresponding to the largest eigenvalue of $X^T X$ as β.

The simulation is repeated 2000 times by generating a new ε_i. The MSE value of estimators is calculated as follows:

$$MSE(\hat{\beta}) = \frac{1}{2000} \sum_{m=1}^{2000} tr(MSEM(\hat{\beta}_{(m)}))$$

Where $\hat{\beta}_{(m)}$ is the estimated value of the m th repeated experiment of β. The results are summarized in Tables 2, 3, and 4.

From Tables 2, 3 and 4, we can find that the MSE value of AUTPC estimator is smaller than the MSE value of the OLS estimator and the AUTP estimator. It means that the AUTPC estimator outperforms the OLS estimator and the AUTP estimator, which is consistent with the theorem exposed in Section 3. Moreover, we can see that the MSE value of estimators increase as the level of multicollinearity increase.

Table 1. Estimated mean squared error with $d = 0.9$

k	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
$\hat{\beta}_{OLS}$	0.0808	0.0808	0.0808	0.0808	0.0808	0.0808	0.0808	0.0808	0.0808
$\hat{\beta}_{AUTP}$	0.0786	0.0777	0.0768	0.0759	0.0749	0.0738	0.0728	0.0718	0.0708
$\hat{\beta}_{AUTPC}$	0.0619	0.0616	0.0613	0.0610	0.0607	0.0604	0.0601	0.0598	0.0595

Table 2. Estimated MSE with $\rho = 0.8$ and $d = 0.9$

k	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
$\hat{\beta}_{OLS}$	0.3037	0.3037	0.3037	0.3037	0.3037	0.3037	0.3037	0.3037	0.3037
$\hat{\beta}_{AUTP}$	0.3037	0.3036	0.3036	0.3036	0.3036	0.3036	0.3035	0.3035	0.3035
$\hat{\beta}_{AUTPC}$	0.1861	0.1861	0.1860	0.1860	0.1860	0.1860	0.1860	0.1860	0.1860

Table 3. Estimated MSE with $\rho = 0.9$ and $d = 0.9$

k	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
$\hat{\beta}_{OLS}$	0.6312	0.6312	0.6312	0.6312	0.6312	0.6312	0.6312	0.6312	0.6312
$\hat{\beta}_{AUTP}$	0.6307	0.6306	0.6305	0.6304	0.6302	0.6301	0.6299	0.6298	0.6296
$\hat{\beta}_{AUTPC}$	0.3306	0.3306	0.3305	0.3305	0.3304	0.3304	0.3303	0.3302	0.3302

Table 4. Estimated MSE with $\rho = 0.99$ and $d = 0.9$

k	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
$\hat{\beta}_{OLS}$	7.8419	7.8419	7.8419	7.8419	7.8419	7.8419	7.8419	7.8419	7.8419
$\hat{\beta}_{AUTP}$	7.7010	7.6212	7.5304	7.4315	7.3266	7.2177	7.1062	6.9931	6.8796
$\hat{\beta}_{AUTPC}$	4.1967	4.1679	4.1353	4.0997	4.0618	4.0220	3.9808	3.9387	3.8959

V. CONCLUSIONS

In this paper, we propose a new class of almost unbiased estimator in the linear regression model. The conditions under which the new estimator is better than the OLS estimator and the AUTP estimator are given in the MSEM sense. Furthermore, we give a numerical example and a simulation study to study the performance of the new estimator. The study results show that the new estimator is a reasonable alternative estimator to the OLS estimator and the AUTP estimator.

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