A DAMAGE PROGNOSIS METHODOLOGY FOR A SIMPLE CRACKED BEAM

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Abstract–– **Damage prognosis uses numerical and experimental responses to identify damage in structures or part of them, thus allowing the remaining structural life estimation at a high level of precision. Current methods focalize on crack identification; however, a complete methodology to estimate the remaining life of a cracked structure is less developed. A methodology is presented in this paper drawing on concepts such as wavelets transform, dynamic structures, and vibration signals for crack identification; and fracture mechanics and nonlinear optimization to obtain the remaining life. Finite element theory was applied to obtain its vibration modes. The crack was modeled as a flexural spring connected to the elements in the crack position and the crack identification was performed in the wavelet domain. Nonlinear optimization techniques and fracture mechanics concepts were used to estimate the remaining fatigue life. A numerical-experimental case study is solved to show the fundamentals of this methodology.**

Keywords–– **Damage prognosis, fatigue crack growth, wavelet transform, nonlinear optimization.**

I. INTRODUCTION

Failure in structures can cause severe financial losses and human injuries. In order to prevent the occurrence of failures, fracture mechanics provides tools to estimate the crack propagation rate and the critical crack size. After accidents related to fatigue crack growth, as the Aloha Airlines airplane in 1988 (NTBS, 1989), some criteria to verify whether the structure can keep working safely were established based on periodic non-destructive testing (NDT) measurements, loading knowledge, metallurgy, and failure prediction models.

More recently, damage prognosis looks to offer in-service tools to prevent failure and predict remaining life (Farrar *et al.*, 2005; Ling and Mahadevan, 2012; Zhang *et al.*, 2015). This new area requires the knowledge of a failure criterion together with the measurement of the state of structural health during operation. Furthermore, a precise knowledge of the system or structure is essential, whether by analytical or numerical equivalent mathematical models, *e.g.* finite element method.

Methods based on vibration measurements have received special attention for crack identification because of in-service variation of the dynamic characteristics like natural frequencies or vibration modes (Li *et al.*, 2005). Works introducing mathematical models to identify cracks in simple structures like beams have since followed (Khiem and Tram, 2014, Attar, 2012, Lu *et al.*, 2013).

Another concept recently introduced is the wavelet transform, which makes it possible to identify the effects caused by an incipient crack on the vibration mode. (Loutridis *et al.*, 2004, Srinivasarao *et al.*, 2010).

This work proposes a damage prognosis methodology that can be divided in two parts: the crack identification and the estimation of the remaining structural life. A numerical-experimental case study involving the identification of a simple cracked beam to illustrate the first part of the proposed methodology is presented. For this purpose, a combination of structure dynamics, wavelet transform, and non-linear optimization techniques was used. Once the crack was identified, a numerical simulation to determine the remaining life of the structure was performed.

II. PROPOSED DAMAGE PROGNOSIS METHO.DOLOGY

The scheme for the identification of a crack in a beam based on wavelet transform and nonlinear optimization techniques was first introduced conceptually by the authors (Presezniak *et al.*, 2007a,b) and is revised in this study, Fig. 1. Several aspects have to be considered in order to apply the proposed methodology: the mathematical/numerical model of the structure must be known, monitoring must be performed to determine the current structure state to update crack size or damage level, and failure criteria must be applied to estimate the remaining life. The critical crack size can be determined by means of load, geometry and mechanical properties by using fracture mechanics or limit load concepts. The remaining life can be estimated using a crack growth law like Paris law. The modeling of the structure can be performed by applying the finite element method (FEM) or even continuum mechanics solutions in the case of simple structures. The crack can be modeled, for instance, by a torsion spring or by using FEM. The modeling serves as the basis for obtaining numerical responses, and when compared to the experimental ones - provided by the periodic measurements - allowing the damage identification. The adjustment of curves to identify the crack is performed in a subspace of wavelet detail obtained from the wavelet transform using the scaling function.

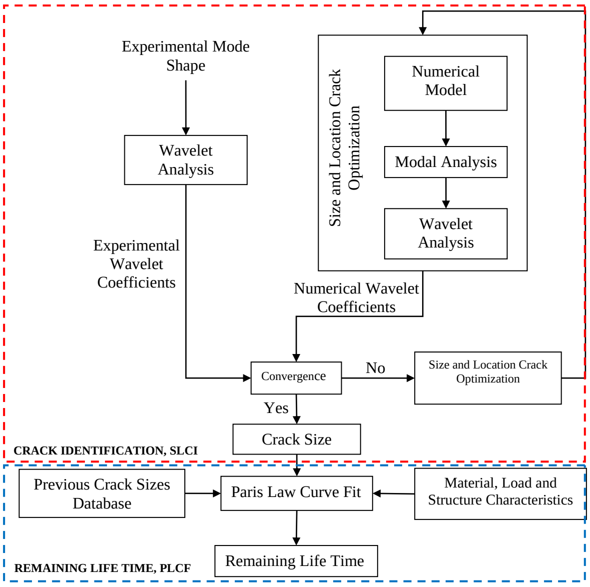

Figure 1: Flow diagram of the damage prognosis algorithm.

Table 1. Beam model dimensions.

Situation	Case	Node position [mm]	Crack size [mm]
	1		3
1	2	160	5
	3		10
	4	50	
2	5	250	5
	6	450	

The second part of the methodology uses fracture mechanics and non-linear optimization techniques to estimate the remaining fatigue life. It requires to know loads, geometry and material properties (Paris law parameters, yield stress, fracture toughness).

It also allows to know the minimum number of identified successive crack lengths necessary to have a good remaining life estimation.

III. NUMERICAL-EXPERIMENTAL CASE STUDY

In order to validate the methodology outlined above, a numerical-experimental example was introduced.

The beam under consideration was made of steel with dimensions 510mm length, 25mm height and 13mm width. The parameters *a* and L_t are the crack size and crack location, respectively. Table 1 shows the sizes and locations for the cracks analyzed.

A. Numerical Models

Numerical Model of the Beam

In order to allow the wavelet transform to identify the discontinuity caused by the crack, a FEM model of a simple cracked structure was implemented, Fig. 2. This way, the crack was identified through the stiffness variation during the optimization procedure.

The implemented cracked beam model used the two most common beam elements: Timoshenko and Euler-Bernoulli. The Timoshenko beam elements were used in non-adjacent-to-the crack elements due to the robustness of beam. On the other hand, the Euler-Bernoulli beam model was used in the elements adjacent to the crack.

Figure 2: Beam model.

According to Dimarogonas (1996), by using fracture mechanics relations between the strain energy release rate or the stress intensity factor and the Castigliano theorem, the local flexibility for a plane strain condition in the cracked region can be obtained as follows:

$$c=\frac{6\pi h}{bEI}F\left(\frac{a}{h}\right) \tag{1}$$

where *a* is the crack size, *b* is the beam height, *h* is the width of the rectangular cross-section, *E* is the Young modulus and *I* is the second order area moment. The function *F* is given by

$$F\left(\frac{a}{h}\right)=1.86\left(\frac{a}{h}\right)^2-3.95\left(\frac{a}{h}\right)^3+16.37\left(\frac{a}{h}\right)^4+37.22\left(\frac{a}{h}\right)^5+76.81\left(\frac{a}{h}\right)^6+126.9\left(\frac{a}{h}\right)^7+172.5\left(\frac{a}{h}\right)^8-144\left(\frac{a}{h}\right)^9+66.6\left(\frac{a}{h}\right)^{10} \tag{2}$$

and the torsional spring stiffness is:

$$K_t=\frac{1}{c} \tag{3}$$

Some considerations must be made in the global mass and stiffness matrix assembly due to the applied crack model. So, the nodes *n* and *n+1* have equal displacements although being able to rotate differently (Fig. 2). Thus, element stiffness and mass matrices are given by:

$$[K_t]=\begin{array}{c}\begin{matrix} v_1 & \theta_1 & v_2 & \theta_2 & \theta_3 & v_4 & \theta_4 \end{matrix}\\ \begin{bmatrix} \ddots & & & & & & & \\ & k_{11}^n & k_{12}^n & k_{12}^n & k_{14}^n & 0 & 0 & 0 \\ & k_{21}^n & k_{22}^n & k_{23}^n & k_{24}^n & 0 & 0 & 0 \\ & k_{31}^n & k_{32}^n & k_{33}^n+k_{11}^{n+1} & k_{34}^n & k_{12}^{n+1} & k_{13}^{n+1} & k_{14}^{n+1} \\ & k_{41}^n & k_{42}^n & k_{43}^n & k_{44}^n+K_t & -K_t & 0 & 0 \\ & 0 & 0 & k_{21}^{n+1} & -K_t & k_{22}^{n+1}+K_t & k_{23}^{n+1} & k_{24}^{n+1} \\ & 0 & 0 & k_{31}^{n+1} & 0 & k_{32}^{n+1} & k_{33}^{n+1} & k_{34}^{n+1} \\ & 0 & 0 & k_{41}^{n+1} & 0 & k_{42}^{n+1} & k_{43}^{n+1} & k_{44}^{n+1} \\ & & & & & & & \ddots \end{bmatrix}\end{array} \tag{4}$$

and

$$[M_t]=\begin{array}{c}\begin{matrix} v_1 & \theta_1 & v_2 & \theta_2 & \theta_3 & v_4 & \theta_4 \end{matrix}\\ \begin{bmatrix} \ddots & & & & & & & \\ & m_{11}^n & m_{12}^n & m_{12}^n & m_{14}^n & 0 & 0 & 0 \\ & m_{21}^n & m_{22}^n & m_{23}^n & m_{24}^n & 0 & 0 & 0 \\ & m_{31}^n & m_{32}^n & m_{33}^n+m_{11}^{n+1} & m_{34}^n & m_{12}^{n+1} & m_{13}^{n+1} & m_{14}^{n+1} \\ & m_{41}^n & m_{42}^n & m_{43}^n & m_{44}^n & 0 & 0 & 0 \\ & 0 & 0 & m_{21}^{n+1} & 0 & m_{22}^{n+1} & m_{23}^{n+1} & m_{24}^{n+1} \\ & 0 & 0 & m_{31}^{n+1} & 0 & m_{32}^{n+1} & m_{33}^{n+1} & m_{34}^{n+1} \\ & 0 & 0 & m_{41}^{n+1} & 0 & m_{42}^{n+1} & m_{43}^{n+1} & m_{44}^{n+1} \\ & & & & & & & \ddots \end{bmatrix}\end{array} \tag{5}$$

where the values of the matrix elements are given by the Euler-Bernoulli model. The Timoshenko model was used for the other elements of the global matrix.

Modal Parameters

In the absence of damping, or considering proportional damping model, the vibration modes are calculated solving the following generalized eigenvalues problem:

$$\lambda_j M\phi_j=K\phi_j \tag{6}$$

where M and K are the global mass and stiffness matrices of the cracked beam, λ_j is jth eingenvalue, with j=1 to n degrees do freedom, and ϕ_j is the jth eigenvector associated to the jth eingenvalue. According to Ewins (2000),

$$\lambda_j = \Omega_j^2 \tag{7}$$

where Ω_j is the j^{th} undamped natural frequency. In a matrix representation, the spectral and modal matrices are defined as (Ewins, 2000):

$$\Lambda = diag\left(\Omega_j^2\right) \quad \text{and} \quad \Phi = \left[\phi_1, \phi_2, \phi_3, ..., \phi_n\right] \tag{8}$$

where the size of these matrices is $n \times n$.

Wavelet Analysis

The wavelet transform presents some interesting properties when applied to complex functions. In certain cases, the Fourier transform needs many coefficients whereas the wavelet transform can make the same representation with a reduced number of coefficients (Barbosa, 2001). The most important characteristic of the wavelet system is that the functions are discrete in time and frequency domains, which means that the wavelet functions are defined in a restricted time interval.

A wavelet system consists of a scale function, ϕ, and the wavelet function, ψ, as continuous or discrete functions, which characterize both types of transforms.

The generation of the scale functions of the Daubechies wavelet system is bounded by certain restrictions to define them in the integer values of the interval where the function is defined (Daubechies, 1988).

These relations present some coefficients, called filter coefficients, defined by h_k and calculated in order to maintain the following relation true (Stark, 2005):

$$\phi(x) = \sum_{k=0}^{2N-1} 2^{j/2} h_k \phi(2^j x - k) \tag{9}$$

where N means the Daubechies wavelet family (1 to 9), j is *a* scaling parameter function, and k is a translation parameter function. The restrictions used to calculate the filter coefficient values can be found in Daubechies (1988).

Some filter coefficients, g_k, are also defined by the wavelet functions using the following relation

$$\psi(x) = \sum_{k=0}^{2N-1} g_k \phi(2x - k) \tag{10}$$

and the coefficients h_k and g_k are related by

$$g_k = (-1)^k h_{1-k} \tag{11}$$

The discrete wavelet transform can be improved as a signal processing analysis by using a filter bank.

Its coefficients are given by the h_k and g_k values, calculated from the scale and wavelet function definitions (Burrus *et al.*, 1997). Schematically, this analysis can be performed as represented in Fig. 3.

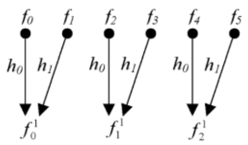

Figure 3: Signal filtering using scale function filters coefficients.

Fracture Mechanics

Considering a linear elastic fracture mechanics model and Mode I loading, the critical crack size can be estimated as a function of the material fracture toughness, K_{IC}, the stress state, σ, and the geometry, Y, as can be seen in Perez Ipiña (2004).

On the other hand, fatigue crack growth rate can be estimated applying the Paris law:

$$\frac{da}{dn} = C \Delta K^m \tag{12}$$

where da/dn is the crack growth rate, C and m are material characteristics, $\Delta K = \Delta\sigma(\pi a)^{1/2} Y$ is the variation of the driving force that depends on load variation ($\Delta\sigma$), crack length (a) and geometry (Y). Equation 1 can be integrated as follows:

$$N = \int_{a_i}^{a_f} \frac{da}{C(\Delta K)^m} \tag{13}$$

where a_i is the actual crack size, and a_f is the critical crack size. Equation (13) allows determining the number of cycles up to the structural failure, i.e., the remaining structural life (Larrainzar *et al.*, 2010). The inputs for the driving force can be obtained by an adequate instrumentation and stress analysis.

B. Experiments

Pre-cracked beams with different sizes and positions of the cracks were tested in order to validate the proposed methodology for crack identification in simple beams. A free-free beam was chosen both for simplicity and model precision. To achieve this condition, the beam was supported in a flexible polyurethane foam, in which case the rigid body natural frequency was disregarded.

The steel cracked beam was ink-marked in each node position corresponding to the FEM model (50 elements of 10 mm in length each). Then, the frequency response (inertance) measurements were obtained by attaching the accelerometer on a node of the beam and exciting in all the others, one by one, as shown in Fig. 4. Thus, an inertance row matrix was obtained.

The excitation was performed through a PCB 086C04 piezoelectric hammer, the response was measured with a PCBS 352C68 accelerometer and a HP 3560 Fourier analyzer was used to process the data.

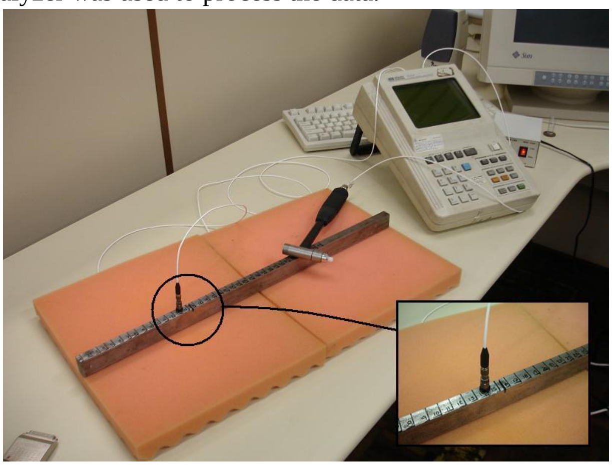

Figure 4: Experimental Setup.

A modal analysis was carried out with the FRFs obtained experimentally obtaining the first natural frequencies and vibration modes characteristics of the beam with different cracks (position and size of the crack). ICATS software was used for modal analysis. The first mode was used for crack identification.

The identification methodology consisted in comparing and adjusting the experimentally obtained mode and its corresponding numerical one. This adjustment was performed in the wavelet domain for both modes. In this area, due to irregularities from the measurement of FRF, the modal analysis resulted contaminated by noise, then the coefficients of detail wavelet were filtered to allow the crack identification.

A crack that grows by fatigue in a real structure follows the Paris law. In cases of constant known load, the estimation of the remaining life is straightforward once the crack was identified: the crossing between a critical crack length and the crack length curve estimated from Paris law. For situations where the load is variable and not known, it is possible to estimate the remaining life by applying an optimization procedure.

C. Optimizations Procedures

Two optimization procedures were proposed to perform this methodology. The numerical codes, named Size and Location Crack Identification (SLCI) and Paris Law Curve Fit (PLCF), as shown in Fig. 2.

Size and Location Crack Identification Code (SLCI)

In this part of the optimization codes, a structural modal analysis was performed to identify the position and the equivalent size of the crack.

After that, and using the wavelet transformed details, a least squares adjustment was conducted in wavelet domain between the identified mode and its numerical equivalent one. Then, size and location of the crack were identified.

The nonlinear optimization function, fminsearch of Matlab, based on Nelder-Mead or flexible polyhedron method, was used. In the particular case of the crack, the objective function is a multi-objective function considering the location and the size of the crack, simultaneously. The error functions used to obtain the objective function, required to identify position and size of the crack, are defined by:

$$error_1 = \left| Pos[\max(experimen)] - Pos[\max(numer.)] \right| \tag{14}$$

and

$$error_2 = \max\Big[\big| \left|\max(experimen)\right| - \left|\max(numer.)\right| \big| \Big] \tag{15}$$

where $Pos[\max(.)]$ represents the position of the maximum value of the Wavelet coefficient; and max[max(.)] corresponds to the Wavelet coefficient maximum value.

Thus, the objective function is defined as:

$$f_{obj}(x_a) = w_1 * error_1 + w_2 * error_2 \tag{16}$$

where w_k, with k=1 to 2, are weight parameters used to weigh up both errors and $x^T=\{x_{crack}\ a_{crack}\}$, referred to as 'design vector', whose components represent the position and size of the crack, respectively.

Paris Law Curve Fit Code (PLCF)

When both crack size and location are identified, PLCF can start. Crack size evolution as a function of the number of cycles can be obtained by incrementally solving Eq. (2), for example, as:

$$a_{i+dn}(N) = \left[\frac{1}{\dfrac{1}{a_i^{\left(\frac{1}{2}m-1\right)}} - \pi^{1,5} N(m-2) C \Delta\sigma^m Y^m} \right]^{\left(\frac{2}{m-2}\right)} \tag{17}$$

where $a_{i+dn}(N)$ is the crack length estimation for N cycles after the actual size, a_i, previously identified. Equation (17) allows identifying the structure remaining life when $a(N)$ equals the critical or tolerable crack size (a_c). An optimization process, using a certain number of crack sizes previously identified, allows a better prediction because there are unknown variables and/or uncertainties in others, like loads, material constants, and measurement errors. To this end, nonlinear optimization techniques were applied.

A design vector is defined as:

$$x_P^T = \{m\ C\ Y\} \tag{18}$$

Thus, the objective function is defined as:

$$f_{obj}(x) = \left\| \{a_{theoret.} - a_{experimen.}\} \right\|_2 \tag{19}$$

where the error between theoretical and experimental crack sizes is a vector composed by the number of previously identified crack size, and $\| \|_2$ is the Euclidian norm operator.

After the curve fitting, the optimal design parameters allow the identification of the crack growth model, and thus the remaining life of the structure.

IV. RESULTS AND DISCUSIONS

A. Crack Identification

The FEM results of the beam configurations, given in Table 1, were obtained from the mass and stiffness matrices presented in Section **III.A**. The solutions for the eigenvalue problem for uncracked and cracked beams (situation 1 of case 3 in Table 1) can be seen in Fig. 5. It is not possible to appreciate in Fig. 5 the influence of the crack on the first and second modes. However, when the wavelet transform was applied, this change in curvature became clear, as shown in Figs. 6 and 7 for situations 1 and 2, presented in Table 1, respectively.

The optimization algorithm to perform the crack identification, presented in **III.C**, was applied and its results are shown in Table 2. In order to compare these results with those obtained with a simple vibrations analysis, Table 3 shows the natural frequencies of Situation 1 for different crack lengths.

Figure 8 shows the first mode of this free-free boundary condition beam.

Imperfections in the setting of measuring sensors, can cause unwanted noise in the identified mode. In fact, since the points in the curve are not at the exact measurement locations, some high frequency noise was obtained when the wavelet transform was applied. Thus, a higher level resolution must be applied to filter this high level noise. Figure 9 shows that a peak in the wavelet transform coefficients appears where the crack is located (160mm).

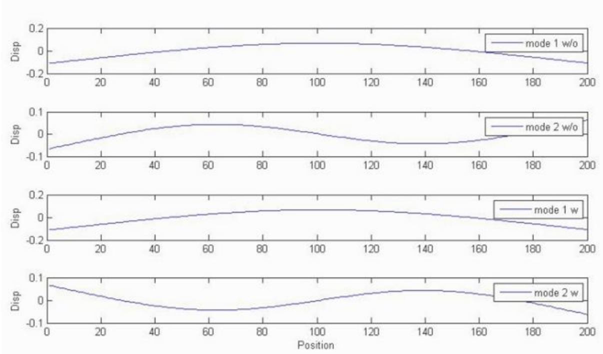

Figure 5: First and second modes.

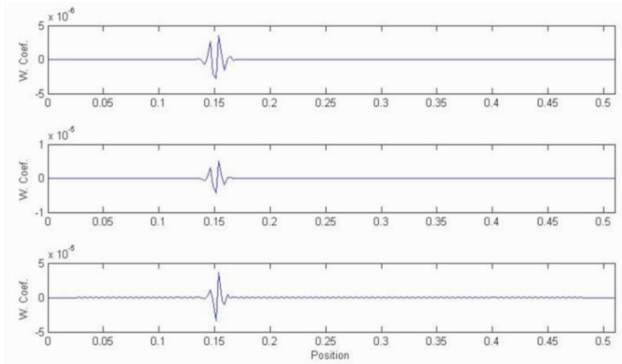

Figure 6: Wavelet details coefficients for situation 1.

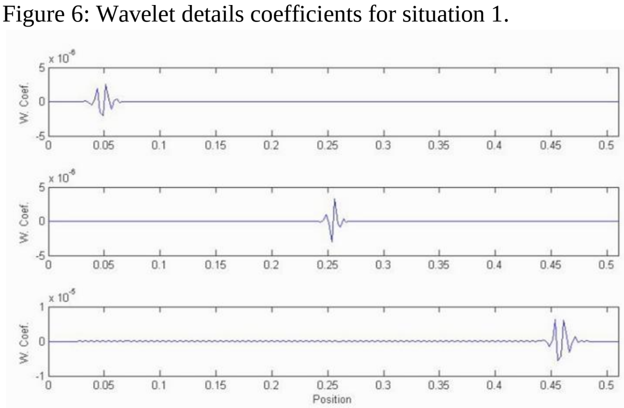

Figure 7: Wavelet details coefficients for situation 2.

Table 2. Actual and estimated location and size of cracks.

Situation	Case	Position [mm]		Size [mm]	
		Actual	Identified	Actual	Identified
1	1			3	2.46
	2	160	165	5	4.78
	3			10	10.00
2	4	50	51.0		
	5	250	249.9	5	4,60
	6	450	453.9		

Table 3. Natural frequencies for different crack lengths.

Crack length [mm]	Natural frequency [Hz]
0	504.4
1	504.4
5	504.2
10	501.3
15	475.0

The identification procedure was applied to the filtered wavelet coefficients resulting in an identified crack size of 14.66mm on a crack location of 163mm.

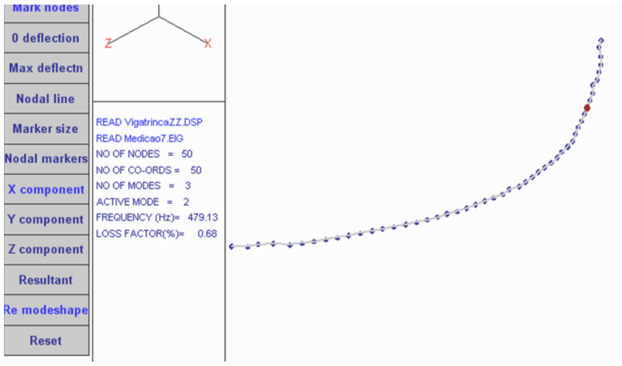

Figure 8: First Mode shape of the cracked beam.

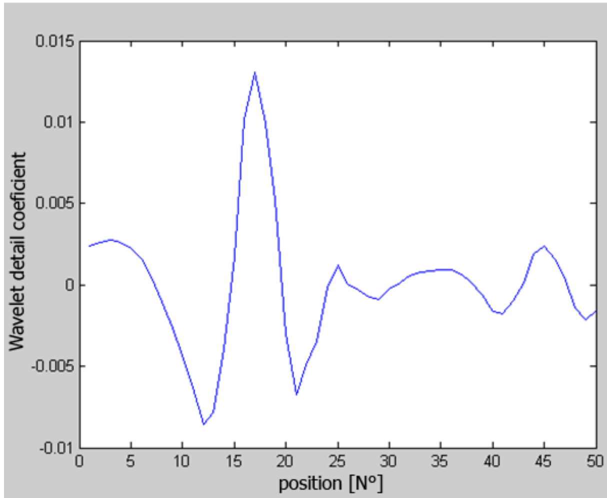

Figure 9: Wavelet detail coefficients to condition 1.

B. Paris Law Curve Fit Code (PLCF)

The Paris Law Curve Fit code (PLCF) was applied in order to have an estimation of the remaining life. This step aims also to establish the number of the identified crack lengths necessary to estimate, with a given precision, the remaining life of the structures.

Situation 1, case 3 (original crack size: 10mm) was analyzed in this example. The Paris Law parameters used in the simulation were C=7×10^{-9}, m=3.5 and Y=2. The final crack length was a_f =13mm.

Constant load condition, $\Delta\sigma$=17 MPa, was considered and results obtained by means of Eq. (17) are described by the blue curve in Fig. 10. Table 4 shows those points taken from the simulation and considered “actually measured by the identification technique” ones that were used in the optimization PLCF process. These points would be obtained through experimental identification technics in real situations.

Then, the Paris Law Curve Fit code (PLCF) was applied by means of Eq. (19) in order to obtain the “actual” values of crack length, simulate its growth and have an estimation of the remaining life.

Three estimation curves are shown in Fig. 10 (red curves) against the “true or actual one” (blue curve). They were obtained considering different quantities of “actually measured" points (1 to 3; 1 to 4 and 1 to 5 points). The real parameters versus the 5 points adjusted curve are shown in the detail. The "actual" and the estimated number of cycles to attain the limit crack size correspond to the intersection of each curve with the green horizontal line in Fig. 10 and are shown in Table 5. The precision of the optimization process depends on the number of points used and gave a very good approximation with only 5 "actual" points. It can be argued that the measured points in monitored structures will be a lot larger than five, but this rapid convergence might be very important in situations where the in service loads can vary (for example in-flight conditions).

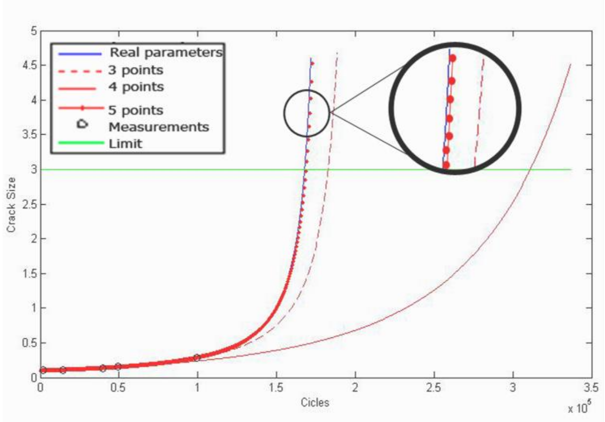

Figure 10: Tendencies in crack growth curves adjusted by PLCF using 3, 4 and 5 points.

Table 4. Number of cycles – crack size points used for the optimization PLCF process.

Point	Number of cycles	Simulated crack size
1	2000	10.1023
2	14500	10.1123
3	39500	10.1387
4	49500	10.1533
5	99500	10.2887

Table 5. Identified remaining life parameters

Points*	Paris' law			Number of Cycles		
	m	C	Y	Actual	Adjust	Error [%]
3	2.385	7.7e-9	2.685		3.11e5	85,41
4	3.294	5.4e-9	2.271	168e3	1.83e5	8,93
5	3.473	6.1e-9	2.103		1.69e5	0,59

* Quantity of points corresponding to successive crack sizes at the given numbers of cycles.

V. CONCLUSIONS

A methodology for damage prognosis analysis was introduced and applied to a simple cracked beam. The crack was modeled as a torsional spring and a combined Timoshenko and Euler-Bernoulli finite element model was employed for the simple cracked structure.

The wavelet transform was reviewed and a methodology to detect the presence of the crack was presented. This procedure was able to identify small discontinuities from a signal and was verified in the numerical-experimental example of a free-free beam.

The estimation of remaining life presented very good results for 5 or more "measured points." This methodology proved adequate for the analyzed case study, although further research is needed to be more widely applied in the important area of damage prognosis.

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