GYROTACTIC MICROORGANISMS FREE CONVECTION BOUNDARY LAYER FLOW ABOUT A VERTICAL TRUNCATED CONE IN NANOFLUID POROUS MEDIA

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***Abstract*— A non-similar boundary layer analysis is presented to examine the natural convection flow over a truncated cone with fixed apex half angle; pointing downwards in a nanofluid saturated porous medium contains gyrotactic microorganisms. A suitable coordinate transformation is performed, and the obtained non-similar equations are solved by fourth order Runge–Kutta method technique coupled with shooting scheme. The effects of the thermophoresis parameter, Brownian parameter, Lewis number, bioconvection Peclet number, bioconvection Lewis number, bioconvection Rayleigh number and buoyancy ratio on the reduced Nusselt, Sherwood and density of motile microorganisms numbers have been studied. A comprehensive numerical computation is carried out for various values of the parameters that describe the flow characteristics.**

***Keywords*— Bioconvection, truncated cone, gyrotactic microorganisms, nanofluid, boundary layer.**

I. INTRODUCTION

Heat transfer is an important process in Physics and Engineering, since the conventional heat transfer in fluids such as water, mineral oil and ethylene glycol are compared to those of most solids. Convective heat transfer can be enhanced passively by changing flow geometry, boundary conditions, or by enhancing thermal conductivity of the fluid.

Consequently improvements in heat transfer characteristics will improve the efficiency of many processes. A nanofluid is a new class of heat transfer fluid that contains a base fluid and solid nanoparticles of diameter 1–100 nm (Das *et al*., 2007; Anoop *et al*., 2009; Cheng, 2012; Abu-Nada, 2008; Kakać and Pramuanjaroenkij, 2009; Mahdy and Ahmed, 2012; Behseresht *et al*., 2014; Khanafer *et al*., 2003). The use of additives is a technique applied to enhance the heat transfer performance of base fluids. Nanofluids have been shown to increase the thermal conductivity and convective heat transfer performance of the base liquid. A comprehensive survey of convective transport was presented by Buongiorno (2006). The author discussed seven possible mechanisms associating convection of nanofluids through movement of nanoparticles in the base fluid. Among the investigated mechanisms, thermophoresis and Brownian diffusion were found important. Thermophoresis acts against temperature gradient, meaning that the particles tend to move from hot regions to cold ones. In addition, the Brownian motion tends to move the particles from high concentration areas to low concentration areas. Similarity solution to viscous flow and heat transfer of nanofluid over nonlinearly stretching sheet was investigated by Hamad and Ferdows (2012). Nield and Kuznetsov (2010) studied the Cheng–Minkowycz problem for natural convective boundary-layer flow in a porous medium saturated by a nanofluid considering the effects of Brownian motion and thermophoresis. Khan and Pop (2011) examined the free convection boundary layer flow past a horizontal flat plate embedded in a porous medium filled with a nanofluid.

Furthermore, Bioconvection has many applications in biological systems and Biotechnology. The term bioconvection refers to a macroscopic convection motion of fluid caused by the density gradient created by collective swimming of motile microorganisms (Becker *et al.*, 2004; Geng and Kuznetsov, 2004; Hillesdon and Pedley, 1996; Childress *et al*., 1975). These self-propelled motile microorganisms upgrade the density of the base fluid by swimming in a particular direction, thus causing bioconvection. On the other hand, the phenomenon of bioconvection in nanofluid convection is driven by the presence of denser microorganisms accumulating on the surface of lighter water. As the heavier microorganisms sink into the water, they are replenished by up swimming microorganisms, thus establishing bioconvection process within the system. The process is a meso-scale phenomenon in which the motion of motile microorganisms induces a macroscopic motion (convection). Unlike the motile microorganisms, the nanoparticles are not self-propelled. Thus, the motion of the motile microorganisms is independent of the motion of nanoparticles. Adding microorganisms to a nanofluid increases its stability as a suspension Kuznetsov (2011a), and could avoid nanoparticles from agglomerating and aggregating. Aziz *et al*. (2012) have numerically studied the free convection boundary layer flow past a horizontal flat plate in nanofluid containing gyrotactic microorganisms, and they found that the bioconvection parameters have strongly influenced the mass, heat, and motile microorganism transport rate. A detailed discussion (Khan *et al.*, 2013; 2014; Kuznetsov, 2011b; Mutuku and Makinde, 2014) of bioconvection in suspensions of oxytactic bacteria is made for the onset of bioconvection in a suspension of gyrotactic/oxytactic microorganisms in different cases. According to the investigations mentioned above, the main purpose of the present contribution is to study the problem of boundary layer natural convection flow due to gyrotactic microorganisms along a vertical truncated cone in a porous medium saturated by a nanofluid. Following Buongiorno (2006) model, the effects of Brownian motion and the thermophoresis are included for the nanofluid.

II. MATHEMATICAL FORMULATION

We consider the boundary layer flow of free convection due to gyrotactic microorganisms over a downward-pointing vertical truncated cone of half angle A immersed in a nanofluid saturated porous medium. The origin of the coordinate system is placed at the vertex of the full cone, with x being the coordinate along the surface of the cone measured from the origin and y being the coordinate perpendicular to the conical surface, as shown in Fig. 1. The surface of the cone is maintained at a constant temperature T_w, which is different from the porous medium temperature sufficiently far from the surface of the cone T_∞. The nanoparticle volume fraction and the density of motile microorganisms on the surface of the cone is C_w, N_w and the ambient value of the nanoparticle volume fraction and the density of motile microorganisms is denoted by C_∞, N_∞. The fluid properties are assumed to be constant except for density variations in the buoyancy force term. In addition, the nanoparticle suspension is assumed to be stable (there is no nanoparticle agglomeration). The presence of nanoparticles is assumed to have no effect on the direction in which microorganisms swim and on their swimming velocity. This is a reasonable assumption if the nanoparticle suspension is dilute (nanoparticle concentration is less than 1%). Bioconvection induced flow only takes place in a dilute suspension of nanoparticles; otherwise a large concentration of nanoparticles would result in a large suspension viscosity, which would suppress bioconvection. Assuming that the thermal and nanoparticle volume fraction boundary layers are sufficiently thin compared with the local radius, the governing equations for the conservation of total mass, momentum, energy, and nanoparticles within the boundary layer near the vertical truncated cone can be written in two-dimensional Cartesian coordinates as

$$\frac{\partial(ru)}{\partial x}+\frac{\partial(rv)}{\partial y}=0, \tag{1}$$

$$\frac{\partial P}{\partial y}=0, \tag{2}$$

$$\begin{aligned}\frac{\mu}{K}u=&-\frac{\partial P}{\partial x}+(1-C_\infty)g\beta\rho_{f\infty}\cos A(T-T_\infty)\\&-(\rho_p-\rho_{f\infty})g\cos A(C-C_\infty)\\&-(\rho_{m\infty}-\rho_\infty)g\gamma\cos A(N-N_\infty)\end{aligned}, \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\tau\left(D\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}+\frac{\overline{D}}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right), \tag{4}$$

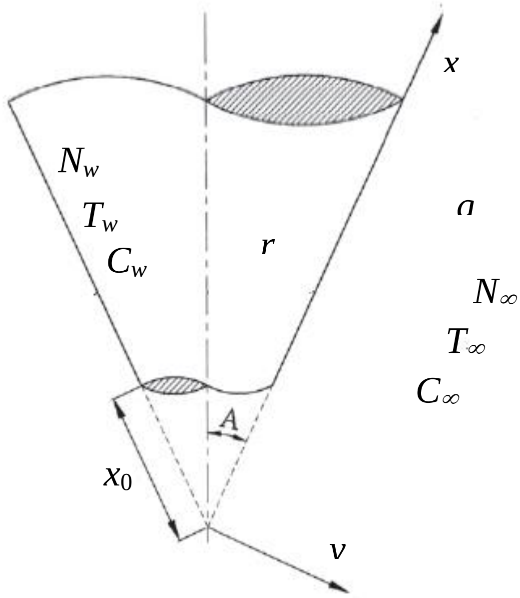

Fig. 1. Schematic view and the coordinate system.

$$\varepsilon^{-1}\left(u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}\right)=D\frac{\partial^2 C}{\partial y^2}+\frac{\overline{D}}{T_\infty}\frac{\partial^2 T}{\partial y^2}, \tag{5}$$

$$u\frac{\partial N}{\partial x}+v\frac{\partial N}{\partial y}+\frac{b\hat{W}}{\Delta C}\frac{\partial}{\partial y}\left(N\frac{\partial C}{\partial y}\right)=D^*\frac{\partial^2 N}{\partial y^2}. \tag{6}$$

In previous equations u and v are the velocity components along the x and y axes, μ is the dynamic viscosity of the fluid, α, β are the thermal diffusivity and volumetric expansion coefficient, ρ_f is the density of the base fluid, ρ_p is the density of nanoparticles, $\rho_{m\infty}$ is the microorganism density, γ is the average volume of microorganisms; $\hat{W}$ is the constant maximum cell swimming speed, K is the Darcy permeability of the porous medium, ε is the porosity; D, $\overline{D}$, D^* are the Brownian, thermophoretic diffusion and diffusivity of microorganisms coefficients, $\tau=\varepsilon(\rho c)_p/(\rho c)_f$ is the ratio of effective heat capacity of the nanoparticle material to the heat capacity of the fluid; $\Delta C=C_w-C_\infty$. In our problem, the dimensional boundary conditions are:

$$\begin{aligned}&v=0, T=T_w, C=C_w, N=N_w, y=0\\&u\to 0, T\to T_\infty, C\to C_\infty, N\to N_\infty, y\to\infty\end{aligned}. \tag{7}$$

Because the boundary layer thickness is small, the local radius to a point in the boundary layer r can be represented by the local radius of the truncated cone, $r = x\sin A$.

$$\begin{aligned}&\xi=\frac{\bar{x}}{x_0}=\frac{x-x_0}{x_0}, \eta=\frac{y}{\bar{x}}Ra^{1/2},\\&\psi=\alpha r Ra^{1/2}F(\xi,\eta), \theta(\xi,\eta)=\frac{T-T_\infty}{T_w-T_\infty},\\&\phi(\xi,\eta)=\frac{C-C_\infty}{C_w-C_\infty}, \chi(\xi,\eta)=\frac{N-N_\infty}{N_w-N_\infty},\\&Ra=\frac{(C-C_\infty)\rho_{f\infty}Kg\beta(T_w-T_\infty)\cos A}{\mu\alpha}\bar{x}\end{aligned}, \tag{8}$$

Upon using these variables and eliminating the pressure P from Eqs. (2) and (3) using cross-differentiation, the basic equations of the boundary layer for the problem under consideration can be written in non-dimensional form as

$$F'' - \theta' + N_r \phi' + R_b S' = 0, \quad (9)$$

$$\theta'' + \left(\frac{1}{2} + \frac{\xi}{\xi+1}\right) F\theta' + N_b \phi'\theta' + N_t \theta'^2 = \xi\left(F'\frac{\partial \theta}{\partial \xi} - \theta'\frac{\partial F}{\partial \xi}\right), \quad (10)$$

$$\phi'' + \left(\frac{1}{2} + \frac{\xi}{\xi+1}\right) L_e F\phi' + \frac{N_t}{N_b}\theta'' = \xi L_e\left(F'\frac{\partial \theta}{\partial \xi} - \theta'\frac{\partial F}{\partial \xi}\right), \quad (11)$$

$$\chi'' + \left(\frac{1}{2} + \frac{\xi}{\xi+1}\right) L_b F\chi' - Pe\left(\chi'\phi' + \phi''(\chi + \sigma)\right) = \xi L_b\left(F'\frac{\partial \chi}{\partial \xi} - \chi'\frac{\partial F}{\partial \xi}\right). \quad (12)$$

Subject to the dimensionless boundary conditions

$$F(\xi,0) = 0, \theta(\xi,0) = \phi(\xi,0) = \chi(\xi,0) = 1,$$
$$F'(\xi,\eta), \theta(\xi,\eta), \phi(\xi,\eta), \chi(\xi,\eta) \to 0, \eta \to \infty \quad (13)$$

The primes denote differentiation with respect to η. In addition, in Eqs. (8)–(12), Ra is the thermal Rayleigh number, R_b is the bioconvection Rayleigh number, N_r is the buoyancy ratio parameter, N_t is a modified diffusivity ratio parameter (somewhat similar to the Soret parameter that arises in cross-diffusion phenomena in solutions), N_b is the Brownian motion parameter, the parameter L_e is the traditional Lewis number (the ratio of the Schmidt number to the Prandtl number Pr), L_b is the bioconvection Lewis number, P_e is the bioconvection Péclet number, σ is the bioconvection constant. Furthermore, these dimensionless parameters are defined as

$$R_b = \frac{(\rho_{m\infty} - \rho_f)\gamma(N_w - N_\infty)}{(1 - C_\infty)\rho_{f\infty}\beta(T_w - T_\infty)}, L_b = \frac{\alpha}{\tilde{D}}, L_e = \frac{\alpha}{\varepsilon D},$$
$$N_r = \frac{(\rho_p - \rho_{f\infty})(C_w - C_\infty)}{(1 - C_\infty)\rho_{f\infty}\beta(T_w - T_\infty)}, N_b = \frac{\varepsilon(\rho c)_p D\Delta C}{(\rho c)_f \alpha},$$
$$N_t = \frac{\varepsilon(\rho c)_p \overline{D}(T_w - T_\infty)}{(\rho c)_f \alpha T_\infty}, P_e = \frac{b\hat{W}}{\tilde{D}}, \sigma = \frac{N_\infty}{N_w - N_\infty}.$$

The results of practical interest in many applications are the reduced Nusselt number Nu_x, the local Sherwood number Sh_x and the local density number of the motile microorganisms Nn_x which are defined as

$$Nu_x = \frac{xq_w}{k\Delta T}, Sh_x = \frac{xq_m}{D\Delta C}, Nn_x = \frac{xq_n}{D^*\Delta N} \quad (14)$$

where q_w, q_m and q_n are the wall heat, the wall mass and the wall motile microorganisms fluxes, respectively, and they are defined as follows

$$q_w = -k\frac{\partial T}{\partial y}, q_m = -D\frac{\partial C}{\partial y}, q_n = -D^*\frac{\partial N}{\partial y}, y = 0. \quad (15)$$

Using variables (8), (14) and (15), we obtain

Table 1. Comparison of values of $-\theta'(\xi,0)$ for various values of ξ with N_t=N_b=0 and N_r=0.

ξ	Cheng *et al.*, (1985)	Yih (1999)	Present
0	0.4437	0.4439	0.44391
0.5	0.5412	0.5285	0.54135
1	0.5991	0.5807	0.59913
2	0.6572	0.6373	0.65732
6	0.7219	0.7123	0.72199
10	0.7391	0.7330	0.73913
20	0.7532	0.7500	0.75325
40	0.7607	0.7592	0.76076
∞	0.7685	0.7686	0.76847

$$Nu_x = -Ra^{1/2}\theta'(\xi,0), \quad Sh_x = -Ra^{1/2}\phi'(\xi,0),$$
$$Nn_x = -Ra^{1/2}\chi'(\xi,0). \quad (16)$$

The reduced local Nusselt, Sherwood and local density of the motile microorganisms numbers can be written as (Mutuku and Makinde, 2014):

$$Nu_r = -Ra^{-1/2}Nu_x = -\theta'(\xi,0),$$
$$Sh_r = -Ra^{-1/2}Sh_x = -\phi'(\xi,0), \quad (17)$$
$$Nn_r = -Ra^{-1/2}Nn_x = -\chi'(\xi,0).$$

III. NUMERICAL METHOD

The boundary layer over the truncated cone, subjected to density of motile microorganisms, is described by the system of partial differential Eqs. (9)–(12), and its boundary conditions (13). The resulting system is numerically solved using fourth order Runge–Kutta method with symmetric estimation of $F'(\xi,0)$, $\theta'(\xi,0)$, $\phi'(\xi,0)$ and $\chi'(\xi,0)$ by the shooting technique. The basic step size used for the calculation is $\Delta\eta$=0.01. This value was arrived at after performing many numerical experiments to access grid independence. In the mentioned numerical method, an iteration process is employed and continued until the desired results are obtained within the following convergence criterion

$$\left|\left(f_{i+1}^{new} - f_i^{old}\right)\right| \le 10^{-6},$$

where f stands for F, θ, ϕ or χ and i refers to space coordinate, i.e. a maximum relative error of 10^{-6} is used as the stopping criteria for the iterations. An important criterion for the success of this numerical approach is to choose an appropriate finite value of η_∞. Thus, in order to estimating the realistic value of η_∞, the solution process has started with initial guess value of η_∞, and the system of Eqs. (9)–(12) is solved subject to the boundary conditions, Eqs. (13). The value of η_∞ is updated and the solution process is repeated until further changes (increment) in η_∞ did not change the values of results. In other words, the results are independent of the value of η_∞. The results show that the choice of η_∞=12 guarantees that all numerical solutions approach to their asymptotic values correctly. In addition, to assess the accuracy of the solution, the present results are compared with the results obtained by other researchers. Table 1 shows the numerical values of $-\theta'(\xi,0)$ for different values of ξ with N_r, N_t and N_b, the conditions for natural convection heat of a vertical truncated cone of Newtonian fluids in porous media with constant wall temperature. It is shown that the present results are in excellent agreement with the results reported by Cheng *et al*. (1985) and Yih (1999).

IV. RESULTS AND DISCUSSION

To obtain a clear insight of the behavior of rescaled velocity, temperature, nanoparticle volume fraction and rescaled motile microorganisms distributions, a comprehensive numerical computation is carried out using the method described in the previous section for various values of governing parameters. The parameters are used to be $1 \le L_e \le 10$, $0.1 \le N_t \le 1.2$, $0.6 \le N_b \le 1.1$, $0. \le \sigma \le 0.5$, $0.0 \le R_b \le 0.4$, $0 \le N_r \le 0.4$, $0.1 \le P_e \le 1.0$ and $2 \le L_b \le 8$.

Figure 2 illustrates the variation of the dimensionless velocity for different values of (a) buoyancy ratio N_r and (b) bioconvection Rayleigh number. In the absence of bioconvection parameter, the dimensionless velocity at the surface is found to be higher. It can be observed that the dimensionless velocity distribution decreases with an increase in the buoyancy ratio and bioconvection Rayleigh number. The thermophoresis parameter N_t can be described as the ratio of the nanoparticle diffusion, which is due to the thermophoresis effect, to the thermal diffusion in the nanofluid. According to Buongiorno (2006), the solid particles in the fluid experience a force in the direction opposite to the imposed temperature gradient. Therefore, the particles tend to move from hot to cold. The thermophoresis parameter is independent of the particle diameter in the case of very small particles.

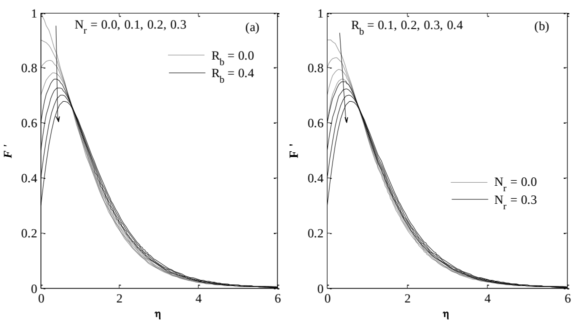

Fig. 2. Velocity distribution for various values of (a) N_r (b) R_b.

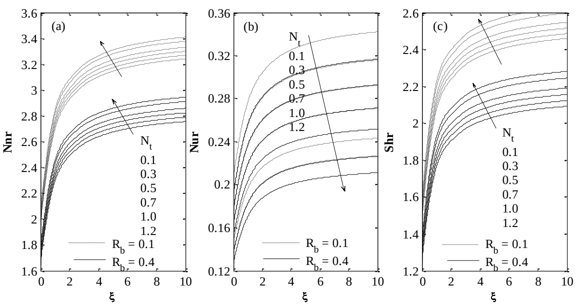

Fig. 3. Effect of the thermophoresis parameter on (a) Nnr (b) Nur and (c) Shr.

Figure 3 depicts the effect of thermophoresis parameter on the reduced (a) density number of the motile microorganisms, (b) Nusselt and (c) Sherwood numbers against ξ considering two cases namely, R_b=0.1 and R_b=0.4 . The figure reveals that increase in the thermophoresis parameter decreases the reduced Nusselt number while increases both of the reduced density number of the motile microorganisms and Sherwood number. This is because of the fact that the thermophoresis force, which tends to move particles from the hot zone to the cold zone, increases with the increase in N_t, which results in that the increase in the thermophoresis force increases the nanoparticle volume fraction. Furthermore, the Brownian motion parameter can be described as the ratio of the nanoparticle diffusion, which is due to the Brownian motion effect, to the thermal diffusion in the nanofluid.

Therefore, it is expected that the Brownian motion parameter increases with an increase in the difference between the nanoparticle volume fractions at the wall and ambient. Based on the Einstein-Stokes equation (Buongiorno, 2006), the Brownian motion is proportional to the inverse of the particle diameter. Hence, as the particle diameter decreases, the Brownian motion increases. On the other hand, the increase in the Brownian motion parameter decreases the reduced Nusselt number but increase both of the reduced density number of the motile microorganisms and reduced Sherwood number.

Again, the variation of reduced density of the motile microorganisms and Nusselt and Sherwood numbers against ξ for various values of buoyancy ratio N_r is illustrated in Fig. 5 (a,b,c). As it is observed, as N_r increases, both of reduced density of the motile microorganisms, Nusselt and Sherwood numbers decrease.

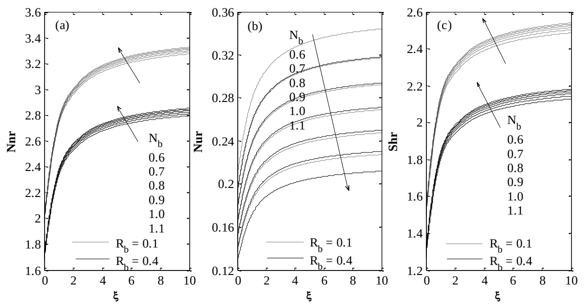

Fig. 4. Effect of the Brownian motion parameter on (a) Nnr (b) Nur and (c) Shr.

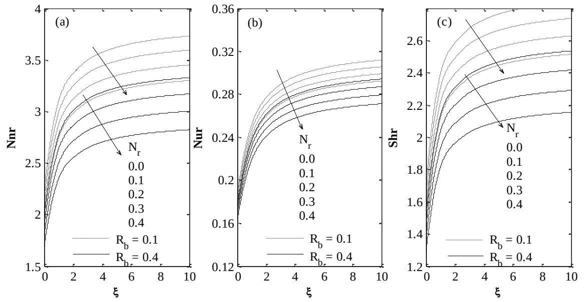

Fig. 5. Effect of the Buoyancy ratio parameter on (a) Nnr (b) Nur and (c) Shr.

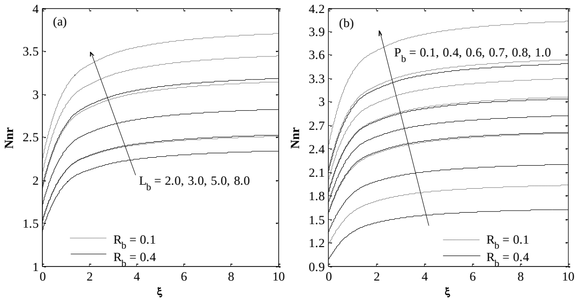

Fig. 6. Effect of bioconvection Lewis number and bioconvection Peclet number on Nnr.

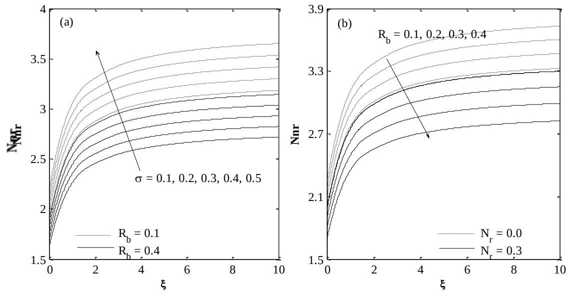

Fig. 7. Effect of bioconvection Lewis number and bioconvection Peclet number on Nnr.

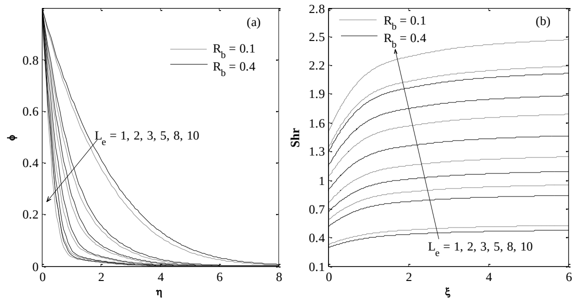

Fig. 8. Effect of Lewis number on (a) nanoparticle volume fraction (b) Shr.

Variations of reduced density number of the motile microorganisms against ξ for different values of bioconvection Lewisand bioconvection Peclet numbers in Fig. 6 (a, b). An increase in bioconvection Lewis number or Peclet number tends to increase reduced density number of the motile microorganisms. In addition as microorganisms concentration difference parameter increases, the reduced density number of the motile microorganisms increases, which is shown in Fig. 7(a), whereas the bioconvection Rayleigh number decreases reduced density number of the motile microorganisms Fig. 7 (b) . The increase in the Lewis number increases the reduced Sherwood number. The effect of Lewis number on rescaled nanoparticle volume fraction and reduced Sherwood number is shown in Fig. 8 (a, b). As it is seen that, the concentration boundary layer thickness depends upon Lewis numbers and it decreases with increasing Lewis numbers. In fact, Brownian motion coefficient decreases with increasing transverse distance and due to this reason the rescaled nanoparticle volume fraction decreases rapidly for large Lewis numbers. In addition, with an increase in Lewis number the reduced Sherwood number increases Fig. 8 (b).

V. CONCLUSIONS

Free convection boundary layer flow about a vertical permeable truncated cone embedded in a porous medium filled by a Newtonian nanofluid containing gyroytactic microorganisms is investigated numerically. The following important results are drawn from our contribution:

1. Both reduced density of motile microorganisms and Sherwood numbers increase with an increase in Brownian motion and thermophoresis but decrease as the buoyancy ratio parameter increases.
2. The rescaled velocity depends strongly on these bioconvection parameters.
3. The reduced density of motile microorganisms number decreases with an increase in bioconvection Lewis and Peclet numbers, whereas it decreases with increasing bioconvection Rayleigh number.

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