DEVELOPMENT OF AN N-WORKING STAGES LOT SIZE MODEL CONSIDERING SEVERAL PRODUCTIVE AND LOGISTICS ISSUES AND ITS RESOLUTION BY NONLINEAR PROGRAMMING

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Abstract **- Since the Economic Production Quantity (EPQ) model was proposed by Taft almost a century ago, different authors have developed new models to obtain optimal lot size values closer to real ones. Even so, there is still much research to do in this field, mainly in taking into account not only productive but logistics costs when determining the optimal lot size. To fill this gap, the model proposed in this work considers several logistics and productive issues, while goes further by incorporating the possibility of working in *n* stages, idea almost not taken into account in previous works. Even more, the possibility of working with constraints – a main characteristic of real productive processes – is also considered. Finally, and as an additional contribution, a nonlinear programming that solves the proposed model – and how to face such programming with MS Excel® – is also detailed.**

Keywords **–– Supply chain, logistics, lot size, lot-sizing problems, nonlinear programming.**

I. INTRODUCTION

According to several of the most important business journals (in works such as Zatzick *et al.*, 2009; and Kaplan and Porter, 2011) the global economic crisis has led managers worldwide to pursue cost reductions in order to keep companies alive. Even without such crisis, production processes optimization and cost reduction should always be in managers' agendas (Baykasoglu and Kaplanoglu, 2006; Devec and Herakovic, 2010).

There are different approaches when facing cost reductions, including: Reducing staff, outsource activities, and optimizing processes, among others. In this paper we focus on optimizing a key element of the production process: The lot size.

The importance of determining the optimal lot size was well established in the foundational works of Harris (1915) and Taft (1918); in these works the EOQ (Economic Order Quantity) and EPQ (Economic Production Quantity) models were introduced. The importance of the EOQ and EPQ models is such that, even nowadays, they are still studied and used in several organizations (Valencia *et al.*, 2014).

Due to the importance of establishing the quantity to produce, several modifications have been proposed to the seminal works of Harris (1915), Taft (1918), and the dynamic version of the model proposed by Wagner and Whitin (1958).

Most of the alternatives commented above have been focused on determining optimal lot sizes using only production costs in a unique working stage.

Literature reviews related with this topic are presented by Brahimi *et al.* (2006), Karimi *et al.* (2006), Yano and Lee (1995), Eftekharzadeh (1993), Maes and Wassenhove (1988).

From the analysis of works previously commented can be inferred that some of the most significant works where authors incorporate new costs and issues in lot size models can be found in Abolhasanpour *et al.* (2009), Kim *et al.* (2008), Chiu *et al.* (2009), Parveen and Rao (2009), Sicilia *et al.* (2013), Cheng and Ting (2010) and Yang *et al*. (2012).

Recently, some authors have not only incorporated new costs but have also started changing some assumptions about the seminal ones. Darwish (2008) replaces the continuous inventory issuing policy for a periodic one which is commonly used in production-shipment systems. Also, some costs which traditionally were supposed fixed are now considered variable (e. g. Darwish, 2008; Chiu *et al*., 2011).

Similarly, the idea that the optimal quantity to produce should be determined by considering not just production costs but supply chain (mainly logistics) costs has arisen from the analysis of different companies. Lee *et al.* (2003) come to this conclusion from the study of third-party logistics (3PL) and Chu and Chu (2007) come to the same conclusion by analyzing a refinery. The later being in concordance with Olivares *et al.* (2012) who established the importance of considering the logistics cost.

Some contributions to EPQ literature not only incorporate costs which were not previously considered but also deal with *n*-stage processes. Related to these contributions are the JELS (Joint Economic Lot Size) models. These models do not focus on optimizing a single element of the supply chain but the costs incurred in different echelons. A complete review related with JELS is presented by Glock (2012) in such work concludes that only two JELS models (Leung, 2010; Seliaman and Ahmad, 2009) truly considered an *n*-stage supply chain.

Other research line related with this paper can be found in the literature as: Multi-echelon serial systems, being some of the most representative works of such line (Clark and Scarf, 2006; Shang and Song, 2003; Chen and Zheng, 2000).

A main gap of the models previously discussed is that most of them do not include constraints, what motivates that such models can't be truly used in *reality*.

In this paper, we propose an optimal lot sizing model which not only considers the contributions of other authors but also incorporates new features to optimize a productive process followed by several companies in different sectors including: Textile, food and metallurgical (Valencia, 2015).

Other main contribution of this work is that even when the model was designed for manufacturing companies, it could also be used in a supply chain environments.

II. METHODS AND RESULTS

A. The productive process to be optimized

The production process that will be optimized, detailed in Fig. 1, is as follows: The raw material enters the process – enough to produce Q items – and it is completely transformed, then a quality inspection of all the products in the batch is performed, determining which ones are not satisfactory and which therefore should be reprocessed. This is done immediately. Once the process itself re-works the products, another inspection is carried out to determine which products are satisfactory and which are unusable (scrap). Satisfactory WIP (work in process) is transported for temporary storage and, in turn, the unusable products receive an appropriate disposal. WIP is then advanced to the following working stage, where the next operation is developed and the process is repeated, including transport to a temporary storage and the storage itself.

The process previously commented keeps going until the last stage of the operation is reached. In this case, finished products are transported to a warehouse. Finally, the stored products should be delivered in fixed amounts to the client.

Productive processes similar to the one commented above have been analyzed in previous works but this paper is the first one dealing with such process incorporating constraints and nonlinear programming.

B. The model developed

The model here proposed calculates the optimal lot size of the process model detailed in A. by incorporating several costs incurred.

Considered costs are motivated by the productive and logistics operations needed to make and deliver all the products required by the client. All the costs considered in the proposed model are shown in Table 1.

To develop the mathematical formulation of the lot size model proposed the variables and parameters shown in Table 2 and Table 3 were required.

Table 2. Variables used in the model.

Variable	Meaning	Units	Domain
Q	Total *pieces* to enter the first work-station.	Pieces	$\mathbb{N}$
A	Auxiliary variable 1. Number of pieces to work in station *i*	Pieces	$\mathbb{N}$
B	Auxiliary variable 2. Number of pieces to re-work in station *i*	Pieces	$\mathbb{N}$
C	Auxiliary variable 3. Number of pieces to scrap in station *i*	Pieces	$\mathbb{N}$
$\mathbf{C_A}$	Cost of satisfying the entire demand	$	$\mathbb{R}+$

Table 1. Costs considered in the model.

Categories	Fixed costs	Variable costs
Raw materials	–	1. Cost of purchasing raw material to make Q products.
Processing costs	1. Set up cost of each working stage.	1. Cost of processing per time unit.
Reprocessing costs	–	1. Cost of reprocessing per time unit.
Inspection costs	1. Cost of inspecting after processing in each working stage. 2. Cost of inspecting after re-processing in each working stage.	–
Cost of storing	–	1. Cost of storing before processing in each working stage per time unit. 2. Cost of storing before re-processing in each working stage per time unit. 3. Cost of storing after processing in each working stage per time unit. 4. Cost of storing after reprocessing in each working stage per time unit. 5. Cost of storing WIP in temporary warehouses inter working stages per time unit. 6. Cost of storing finished products in the warehouse per time unit.
Transport	1. Cost of trans-porting raw material to the first working stage. 2. Cost of trans-porting WIP to the next working stage or finished products to the ware-house. 3. Cost of trans-porting finished products to the client a predefined number of times.	1. Cost of transporting the total finished products to the client.
Maintenance	–	1. Cost of maintenance of each working stage due the processing operations. 2. Cost of maintenance of each working stage due the reprocessing operations.
Disposal	–	1. Cost of an adequate disposal of the scrap generated in each stage.

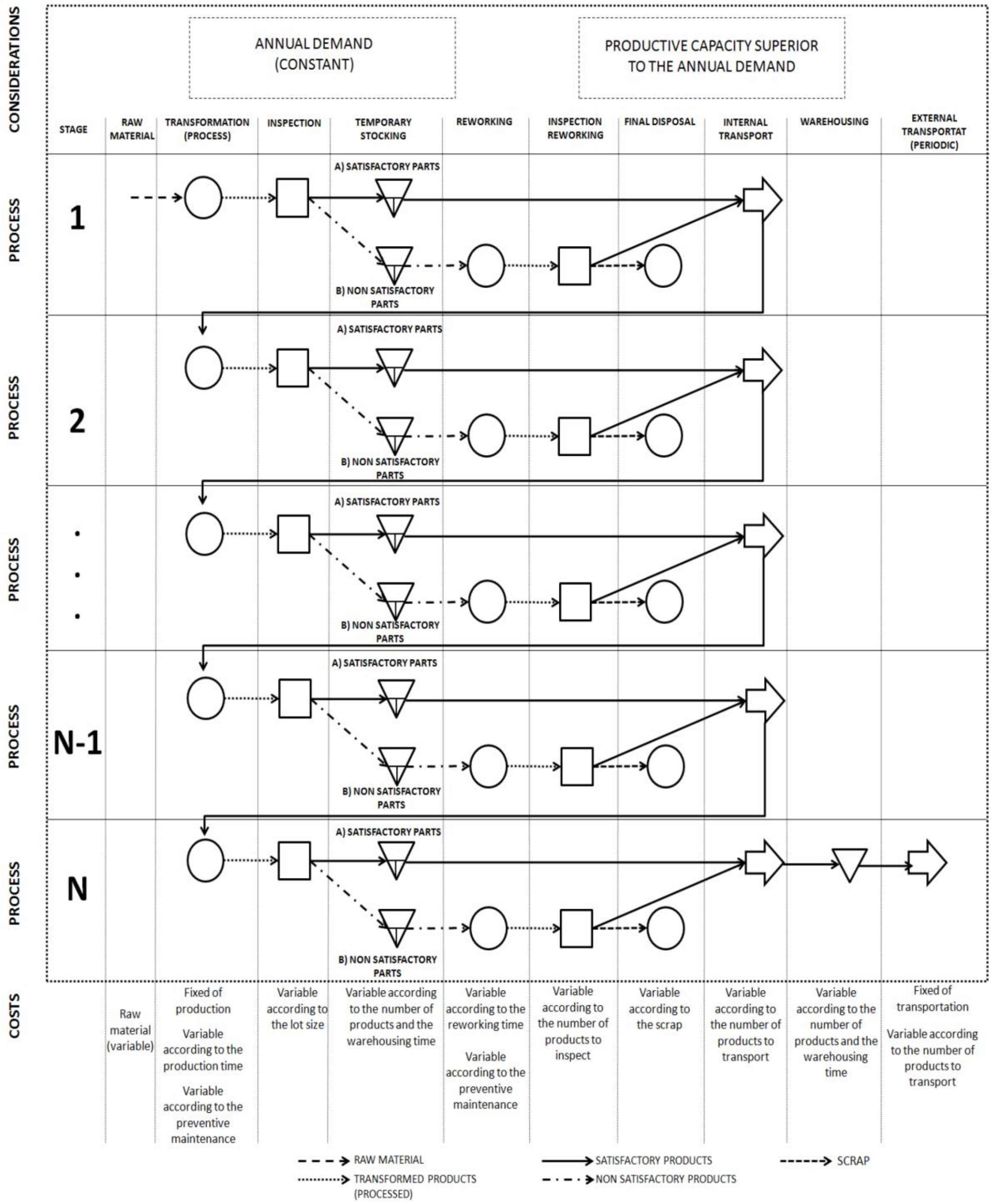

Figure 1. Productive Process

Considering the productive process previously commented and the parameters detailed in Table 2 the following nonlinear programming was developed in order to determine the optimal lot size.

Minimize

$$
\begin{aligned}
C_A(Q) = \frac{\lambda}{A_{n+1}} \Bigg\{ & C_y Q + \sum_{i=1}^{n} C_{ki} + \sum_{i=1}^{n} C_{p_i} A_i t_{p_i} + \sum_{i=1}^{n} C_{ri} B_i t_{r_i} \\
& + \sum_{i=1}^{n} C_{zi} C_i + C_a \left\lceil \frac{Q}{E} \right\rceil + \sum_{i=1}^{n} C_{b_i} \left\lceil \frac{A_{i+1}}{F_i} \right\rceil + C_c \left\lceil \frac{A_{n+1}}{G} \right\rceil + C_d A_{n+1} \\
& + \frac{1}{2} \sum_{i=1}^{n} C_{hi} A_i^2 t_{p_i} - \frac{1}{2} \sum_{i=1}^{n} C_{hi} A_i t_{p_i} + \frac{1}{2} \sum_{i=1}^{n} C_{gi} A_i^2 t_{p_i} \\
& - \frac{1}{2} \sum_{i=1}^{n} C_{gi} A_i t_{p_i} + \sum_{i=1}^{n} C_{gi} A_i B_i t_{r_i} - \sum_{i=1}^{n} C_{gi} B_i t_{r_i} \\
& + \sum_{i=2}^{n} C_{m\,i-1} A_i t_{m_{i-1}} + \left[\frac{C_u A_{n+1} (P\lambda)}{2N} \right] \Bigg[\frac{A_{n+1}}{\lambda} - \sum_{i=1}^{n} A_i t_{p_i} \\
& - \sum_{i=1}^{n} B_i t_{r_i} - \sum_{i=2}^{n} t_{m_{i-1}} - \sum_{i=1}^{n} t_{z_i} \Bigg] + \sum_{i=1}^{n} C_{v_i} + \sum_{i=1}^{n} C_{w_i} \\
& + \sum_{i=1}^{n} C_{z_i} A_i + \sum_{i=1}^{n} C_{z_i} B_i \Bigg\}
\end{aligned}
\qquad (1)
$$

where

Table 3. Parameters used in the model.

Parameter	Meaning	Units	Domain
n	Number of working stages.	Natural number	$\mathbb{N}$
λ	Expected annual demand.	Pieces per year	$\mathbb{N}$
θ	Percentage of parts becoming scrap after reprocessing at each station.	%	$\mathbb{R}^+$
E	Capacity of the transport which carries the raw material to the first stage.	Pieces	$\mathbb{N}$
F	Capacity of each transport which carry the WIP between stages.	Pieces	$\mathbb{N}$
G	Capacity of the transport which carries the finished products to the customer.	Pieces	$\mathbb{N}$
H	Capacity of each temporary ware-house.	Pieces	$\mathbb{N}$
I	Capacity of the final warehouse.	Pieces	$\mathbb{N}$
C_a	Fixed cost of transporting raw materials to the first workstation.	$	$\mathbb{R}^+$
C_b	Fixed cost of transporting the products to the next workstation (WIP), or final ware-house, either directly or through a tempo-rary warehouse.	$	$\mathbb{R}^+$
C_c	Fixed transport cost per shipping to the cus-tomer.	$	$\mathbb{R}^+$
C_d	Variable cost per shipping each product to the customer.	$	$\mathbb{R}^+$
C_g	Variable cost of storing each product per time unit, right after each production pro-cess.	$/year	$\mathbb{R}^+$
C_h	Variable cost of storing each piece, product or raw material item per unit of time, be-fore each production process.	$/year	$\mathbb{R}^+$
C_k	Set up costs per workstation.	$	$\mathbb{R}^+$
C_m	Variable cost of storing each unfinished product (WIP) per unit time in temporary warehouses.	$/year	$\mathbb{R}^+$
C_p	Variable processing cost per time unit.	$/year	$\mathbb{R}^+$
C_r	Variable rework cost per time unit.	$/year	$\mathbb{R}^+$
C_s	Variable cost of handling scrap.	$	$\mathbb{R}^+$
C_u	Variable cost of storing each product in the final warehouse, per time unit.	$/year	$\mathbb{R}^+$
C_v	Fixed cost of inspect after each processing.	$	$\mathbb{R}^+$
C_w	Fixed cost of inspect a batch after each re-processing.	$	$\mathbb{R}^+$
C_y	Variable cost of raw materials.	$/piece	$\mathbb{R}^+$
C_z	Variable cost of maintenance.	$/piece	$\mathbb{R}^+$
i	Index related with the each stage.	Natural number	$\mathbb{N}$
P	Customer replenishment period, as a frac-tion of a year.	Year	$\mathbb{Q}$
t_a	Available time of processing and re-processing of each stage.	year	$\mathbb{R}^+$
t_m	Storage time in each temporary ware-house.	$/year	$\mathbb{R}^+$
t_s	Processing time.	Year	$\mathbb{R}^+$
t_p	Re-processing time.	Year	$\mathbb{R}^+$
t_r	Transportation time of the work-stations to the next station or final warehouse, if there is a temporary warehouse between the sta-tions, should count time going to the ware-house and from there to the next station, not including storage time.	Year	$\mathbb{R}^+$
X	Percentage of defective parts produced at each station before reworking.	%	$\mathbb{R}^+$

$$A_i = \begin{cases} Q\prod_{i=1}^{n}\left(1-x_{i-1}\theta_{i-1}\right) & \text{for } \; i\in[1,n] \\ A_n\left(1-x_n\theta_n\right) & \text{for } \; i=1+n \end{cases} \tag{2}$$

$$B_i = A_i x_i \tag{3}$$

$$C_i = A_i\theta_i \tag{4}$$

Subject to:

$$Q \le E \tag{5}$$

$$A_{n+1} \le I \tag{6}$$

$$A_{i+1} \le H_i, \quad \text{for} \quad i\in[1,n-1] \tag{7}$$

$$A_{n+1} \ge P\lambda \tag{8}$$

$$t_{a_i} \ge A_i t_{p_i} + B_i t_{r_i} \tag{9}$$

The detail of each part of the Eq. (1) is showed in Fig. 2.

In the mathematical formulation showed above A_i is the total pieces (both finished and in process) that enters the stage i, B_i is the total pieces that must be reprocessed after passing the stage i, and C_i is the total pieces considered as scrap after passing the stage i.

The main idea of the proposed nonlinear programming is to determine the lot size (Q^*), which minimizes the total cost of the productive process ($), represented by C_A(Q) in the Eq. (1), while satisfying the entire constraints of the model (5-8).

As can be seen in Eq. (9), nonlinearities of (1) arrive from the fact that several terms of the model (fixed costs) must be multiplied by ($1/Q$) when substituting the value of the term (λ/A_{n+1}); such multiplications generate a rational equation that motivate to propose a nonlinear problem. Even more, an interesting research line to face the mathematical formulae was suggested by other authors; such line consist on solving the lot size problem with Mixed-Integer Linear Programming (MILP), that idea is beyond the scope of this paper so it remains as a future research line.

In the same model, constraint (5) avoids exceeding the capacity of the available transport to the first station. Transport constraints between stages are not supposed since it is allowed to do several trips between nodes (or use a homogeneous fleet), such cost it is considered in the transport fixed cost term ($C_{bi}\lceil A_{i+1}/F_i\rceil$).

$$\begin{aligned} C_A(Q) = \underbrace{\frac{\lambda}{A_{n+1}}}_{\text{Number of cycles}} \Bigg\{ & \underbrace{C_y Q}_{\text{Cost of the raw material}} + \sum_{i=1}^{n} \underbrace{C_{ki}}_{\text{Setup cost per stage}} + \underbrace{\sum_{i=1}^{n} C_{p_i} A_i t_{p_i} + \sum_{i=1}^{n} C_{ri} B_i t_{r_i}}_{\text{Process and reprocess cost per unit}} \\ & + \underbrace{\sum_{i=1}^{n} C_{zi} C_i}_{\text{Scrap handle cost}} + \underbrace{C_a\left\lceil \frac{Q}{E}\right\rceil}_{\text{Transport cost to } i=1} + \sum_{i=1}^{n} \underbrace{C_{b_i}\left\lceil \frac{A_{i+1}}{F_i}\right\rceil}_{\text{Transport cost to } i\neq 1} + \underbrace{C_c\left\lceil \frac{A_{n+1}}{G}\right\rceil}_{\text{Transport cost to client (fixed)}} + \underbrace{C_d A_{n+1}}_{\text{Transport cost to client (variable)}} \\ & + \underbrace{\frac{1}{2}\sum_{i=1}^{n} C_{hi} A_i^2 t_{p_i}}_{\text{Storing cost before processing}} - \underbrace{\frac{1}{2}\sum_{i=1}^{n} C_{hi} A_i t_{p_i}}_{\text{Storing cost after processing}} + \underbrace{\frac{1}{2}\sum_{i=1}^{n} C_{g_i} A_i^2 t_{p_i} - \frac{1}{2}\sum_{i=1}^{n} C_{g_i} A_i t_{p_i}}_{\text{Storing cost before re-processing}} \\ & \underbrace{\sum_{i=1}^{n} C_{g_i} A_i B_i t_{r_i} - \sum_{i=1}^{n} C_{g_i} B_i t_{r_i}}_{\text{Storing cost after re-processing}} + \underbrace{\sum_{i=2}^{n} C_{m i-1} A_i t_{m_{i-1}}}_{\text{Temporary storing cost between stages}} + \underbrace{\left[\frac{C_u A_{n+1}(P\lambda)}{2N}\right]}_{\text{Storing cost per time after processing}} \\ & \underbrace{\left[\frac{A_{n+1}}{\lambda} - \sum_{i=1}^{n} A_i t_{p_i} - \sum_{i=1}^{n} B_i t_{r_i} - \sum_{i=2}^{n} t_{m_{i-1}} - \sum_{i=1}^{n} t_{z_i}\right]}_{\text{Final products warehousing time}} \\ & + \underbrace{\sum_{i=1}^{n} C_{v_i} + \sum_{i=1}^{n} C_{w_i}}_{\text{Inspection cost (unitary, 1 \& 2)}} + \underbrace{\sum_{i=1}^{n} C_{z_i} A_i + \sum_{i=1}^{n} C_{z_i} B_i}_{\text{Inspection cost (unitary, 1 \& 2)}} \Bigg\} \end{aligned}$$

Figure 2. Detail of each element of Eq. (1).

Table 4. Input data of the numerical example.

Variable/ Parameter	Unique	Related with each stage 1	2
N	2	-	-
λ	100000	-	-
θ	-	2.00%	1.00%
E	5000	-	-
F	-	3000	3000
G	10000	-	-
H	-	20000	20000
I	43000	-	-
C_a	\$ 1000.00	-	-
C_b	-	\$ 10.00	\$ 9.00
C_c	\$ 50.00	-	-
C_d	\$ 1.00	-	-
C_g	-	\$ 1000.00	\$ 1200.00
C_h	-	\$ 1000.00	\$ 1200.00
C_k	-	\$ 1000.00	\$ 1200.00
C_m	\$ 1000.00	-	-
C_p	-	\$ 7000.00	\$ 12000.00
C_r	-	\$ 6000.00	\$ 11300.00
C_s	-	\$ 1.00	\$ 1.20
C_u	-	-	\$ 200.00
C_v	-	\$ 10.00	\$ 12.00
C_w	-	\$ 10.00	\$ 4.00
C_y	\$ 10.00	-	-
C_z	-	\$ 0.01	\$ 0.04
P	0.08	-	-
t_m	0.0001	-	-
t_s	-	0.0001	0.00032
t_p	-	0.00005	0.000012
t_r	-	0.0002	0.0002
X	-	\$ 1000.00	\$ 1200.00

Constrains (6) and (7) are related with the storing capacity in each stage and, finally, (8) guaranties that the amount of satisfactory products meets the demand of each period (i.e. Eq. (8) corresponds the production capacity constraint). Finally, demands are independent among different years as supposed by Clark *et al.* (2006).

An important and particularly not intuitive cost, considered in Eq. (1) is that of storing the finished products in the delivery periods. How this cost was calculated is discussed by Chiu *et al.* (2011).

Because nonlinear problems are not particularly easy to solve, is recommended to face the proposed model using specialized software, as MatLab®. However, if the number of stages is known and *sufficiently small*, the use of a spreadsheet, such as Microsoft Excel®, may be enough to solve the model. Evidence of this can be seen in Section C, where a particular case (n=2) is shown.

C. Numerical example

The proposed model has been tested in several situations – both real and theoretical – with very satisfactory results. In this paper a theoretical problem is presented.

The supposed situation corresponds to a company which productive process adjusts to the one already discussed. Also, the example variables are detailed in Table 4.

Because the process of the example only involves two working stages it is possible to solve it with a spread-sheet. To do so, the general nonlinear programming proposed can be re-written to the following one.

Minimize:

$$\begin{aligned} C_A(Q) = \frac{\lambda}{Q(1-x_1\theta_1)(1-x_2\theta_2)} \Big\{ & C_yQ + C_{k1} + C_{k2} + C_{p1}Qt_{p_1} + \\ & C_{p2}Q(1-x_1\theta_1)t_{p_2} + C_{r1}Qx_1t_{r_1} + C_{r2}Q(1-x_1\theta_1)x_2t_{r_2} + \\ & C_sQ(1-x_1\theta_1)x_2\theta_2 + C_a\left\lceil \frac{Q}{E} \right\rceil + C_{b_1}\left\lceil \frac{Q(1-x_1\theta_1)}{F_1} \right\rceil + \\ & C_{b_2}\left\lceil \frac{Q(1-x_1\theta_1)(1-x_2\theta_2)}{F_2} \right\rceil + C_c\left\lceil \frac{Q(1-x_1\theta_1)(1-x_2\theta_2)}{G} \right\rceil + \\ & C_dQ(1-x_1\theta_1)(1-x_2\theta_2) + \frac{1}{2}\Big[C_{h1}Q^2t_{p_1} + \\ & C_{h2}Q^2(1-x_1\theta_1)^2t_{p_2}\Big] + \frac{1}{2}\Big[C_{g1}Q^2t_{p_1} + C_{g2}Q^2(1-x_1\theta_1)^2t_{p_2}\Big] - \\ & \frac{1}{2}\Big[C_{g1}Qt_{p_1} + C_{g2}Q(1-x_1\theta_1)t_{p_2}\Big] + C_{g1}Q^2x_1t_{r_1} + \\ & C_{g2}Q^2(1-x_1\theta_1)^2x_2t_{r_2} - \Big[C_{g1}Qx_1t_{r_1} + C_{g2}Q(1-x_1\theta_1)x_2t_{r_2}\Big] + \\ & C_{m_1}Q(1-x_1\theta_1)t_{m_1} + \left[\frac{Cu\big[Q(1-x_1\theta_1)(1-x_2\theta_2) - P\lambda\big]}{2}\right] \\ & \Big[\frac{Q(1-x_1\theta_1)(1-x_2\theta_2)}{\lambda} - \Big[Qt_{p_1} + Q(1-x_1\theta_1)t_{p_2}\Big] - \\ & \Big[Qx_1t_{r_1} + Q(1-x_1\theta_1)x_2t_{r_2}\Big] - t_{m_1} - \Big[t_{s_1} + t_{s_2}\Big]\Big] + \\ & C_{v_1} + C_{v_2} + C_{w_1} + C_{w_2} + C_{z_1}Q + C_{z_2}Q(1-x_1\theta_1) + \\ & C_{z_1}Qx_1 + C_{z_2}Q(1-x_1\theta_1)x_2 \Big\} \end{aligned} \tag{10}$$

subject to:

$$Q \le E \tag{11}$$

$$A_2 \le H_1 \tag{12}$$

$$A_3 \le I \tag{13}$$

$$A_3 \ge P\lambda \tag{14}$$

As can be seen, some equations of the general model (2-4) have been incorporated in the one to minimize by doing the calculus detailed in formulas (15-21). This arrangement was possible because n is small enough. Otherwise, auxiliary variables are needed.

$$A_1 = Q \tag{15}$$

$$A_2 = Q(1-x_1\theta_1) \tag{16}$$

$$A_3 = Q(1-x_1\theta_1)(1-x_2\theta_2) \tag{17}$$

$$B_1 = Qx_1 \tag{18}$$

$$B_2 = Q(1-x_1\theta_1)x_2 \tag{19}$$

$$C_1 = Qx_1\theta_1 \tag{20}$$

$$C_2 = Q(1-x_1\theta_1)x_2\theta_2 \tag{21}$$

Considering formulae presented above and the information detailed in Table 4 the problem was transferred to a Microsoft Excel® spreadsheet, as can be seen in Fig. 3.

Finally, the Solver™ tool was used to solve the model; by doing so, and by considering all the constraints, the following optimal results were obtained: The optimal lot size (Q^*) is 8,008 and the total cost $C_A(Q)$ is \$70,643,757.

VARIABLE	STATION 1	STATION 2	WAREHOUSE	VARIABLE	STATION 1	STATION 2	WAREHOUSE
λ	100,000			C_p	$ 7,000.00	$ 12,000.00	
θ	2.00%	1.00%		C_r	$ 6,000.00	$ 11,300.00	
C_a	$ 1,000.00			C_s	$ 1.00	$ 1.20	
C_b	$ 10.00	$ 9.00		C_u		$ 200.00	
C_c	$ 50.00			C_v	$ 10.00	$ 12.00	
C_d			$ 1.00	C_w	$ 10.00	$ 4.00	
C_g	$1,000.000	$1,000.000		C_y	$ 10.00		
C_g	$1,000.000	$1,200.000		C_z	$ 0.01	$ 0.04	
C_h	$1,000.000	$1,200.000		N			6
C_k	$10,000.00	$ 7,500.00		μ_{tp}	0.00005	0.000012	
C_m	$1,000.000			μ_{tr}	0.0002	0.0002	

VARIABLE	STATION 1	STATION 2	WAREHOUSE
t_s	0.0001	0.00032	
t_m	0.0001		
x	5%	1%	
P			0.08

CAPACITIES	1	2	3
E	5,000.00		
F	3,000.00	3,000.00	
G			10,000.00
H	20,000.00	20,000.00	
I			43,000.00

Q^* =	
$C_A(Q)$=	

Figure 3. The numerical example.

III. CONCLUSIONS

Different approaches exist to pursue cost reductions, with the process optimization being one of the most recommended. In this paper, we focus on optimizing a key element of the production process: the lot size.

Lot size has been studied widely since the seminal work of Taft (1918) was published.

Several authors have proposed modifications to the work of Taft in order to obtain optimal lot size values closer to *real* ones. However, it has not been until nowadays that major modifications have been proposed, with some of the most important being those related with incorporating logistics activities.

Even considering recent efforts, there is still much research to do in this field, since several characteristics of *real* productive processes have not been taken into account. In this context, this work contributes to literature by proposing a lot size model that not only incorporates several productive and logistics issues but also considers the possibility of working in *n* stages.

An important difference between the proposed model and those previously proposed is that the one here commented incorporates constraints; by doing so, *real* productive processes can be optimized. To determine the optimal lot size it was necessary to develop a non-linear programming that is also showed.

In order to facilitate the application of the proposed model in multiple organizations, including SMEs, a numerical example solved with MS Excel® was also introduced.

Even when in this work several contributions to the literature where introduced, there are many research lines to continue this work. Some of the most interesting are the following: validate the proposed model in different companies, adapt the model proposed to other productive processes, and adapt heuristics and metaherustics methods – as those presented by Mayorano *et al.* (2013) and Schweickardt *et al.* (2011) – to solve the presented model when *n* is *too big*.

Moreover, a very interesting research line related with this paper was proposed by one of the blind reviewers who suggest to face the proposed model by using Mixed-Integer Linear Programming (MILP).

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