SENSITIVITY ANALYSIS OF VIBRATING MOTION OF NON-UNIFORM AFM PIEZOELECTRIC MICROCANTILEVER

R. GHADERI and M. H. KORAYEM

Department of Mechanical and Aerospace Engineering, Science and Research Branch, Islamic Azad University, Tehran, Iran. Email: Hkorayem@iust.ac.ir

Abstract**—Piezoelectric MCs (MCs) are a special type of MCs. Having self-actuating and self-sensing abilities, they can be used as micro-robots in AFM, sensors and actuators. This paper analyzes sensitivity of vibrating motion of a piezoelectric MC with the presence of geometrical discontinuities. As resonance amplitude and natural frequency are of paramount importance in vibrating motions and they are considered in most engineering applications such as AFM and MEMS, sensitivity analysis of these two parameters is conducted. Vibrating analysis is performed based on the non-uniform beam model and the Euler-Bernoulli theory. The Sobol method is used to conduct sensitivity analysis of MC's vibrating motion into the geometrical dimensions of layers and tip in order to specify the sensitive and insensitive parameters and their effects on vibrating motion of piezoelectric cantilever. The simulation results show that to achieve actuation ability, it is better to select a piezoelectric layer, which is thin, wide and long, and a tip, which is thin and short. The results also show that the piezoelectric layer has a different effect on natural frequency, as L_p/L becomes 0.66; natural frequency reaches its maximum amount, while the effect of other parameters on frequency is either absolutely ascending or absolutely descending.**

Keywords**— Sensitivity analysis, AFM, Piezoelectric MC, Resonance Amplitude, Natural Frequency**

I. INTRODUCTION

Due to high flexibility, sensitivity to atomic and molecular forces and very high response time, MCs (MCs) are extensively used in atomic force microscope (AFM) and MEMS switches (Lee *et al*., 2007; Chu *et al*., 2007) in different environments (air, liquid). Among MCs, piezoelectric MCs, with self-actuating and self-sensing abilities and higher imaging speed, can be used as the new generation of MCs in imaging AFM (Rogers *et al*., 2004; Adams *et al*., 2005; Bashash *et al*., 2010) bio mass sensing (Mahmoodi *et al*., 2008) and MEMS switches (Lee *et al*., 2007; Chu *et al*., 2007). As system performance depends on the sensitivity of vibrating motion of the MC in AFM and MEMS applications, the study of vibrating motion and sensitivity analysis of MC is of high importance.

Some studies have already been conducted on sensitivity analysis of AFM and MEMS beams. Korayem *et al.* (2012) did sensitivity analysis of a rectangular MC for manipulating biologic cells. They performed the sensitivity analysis using the Sobol method and applying different nano-contact models. Lee and Chang (2011) studied bending sensitivity of a V-shaped MC using the modified couple stress theory and compared it with the results of the classical theory. Matviykiv and Lobur (2010) studied the principles of MEMS piezoelectric sensors design and carried out sensitivity analysis on the two parameters of quality factor and sensitivity. Yang *et al.* (2010) made the beam of vibrator piezoelectric sensor for a three-axis tactile sensor application and studied its axial sensitivity at the first three vibrating modes under the influence of an external force. Kovacs and Vizvary (2001) carried out sensitivity analysis of cantilever-type and bridge-type piezoresistive and capacitive accelerators. They extracted the relationship between geometrical parameters and the lowest eigenfrequency and conducted sensitivity analysis to obtain the optimal geometrical parameters using a closed-form relation. Yamashita *et al.* (2011) made a force sensor using a piezoelectric cantilever and used that as a tactile sensor. They used sensitivity analysis to study the axial vibrating behavior of this cantilever and concluded that frequency of cantilever changes linearly under the influence of the three axial forces.

Piezoelectric MCs, which are used in AFM, are usually different geometrically form the rest of piezoelectric MCs. Due to the presence of piezoelectric layer, main bodies of piezoelectric MCs are usually made wider and their tips are considered narrower (Mahmoodi and Jalili, 2009; Mahmoodi *et al.*; 2010). Piezoelectric layer only covers a part of MC. Therefore, due to the presence of piezoelectric layer and tip in these types of MCs, MC has geometrical discontinuity. To determine how each layer and MC section affects vibrating motion, it is necessary to done sensitivity analysis on MC's geometrical parameters. In addition, it is possible to obtain the optimal quantities of geometrical parameters to maximize MC's actuation ability using the results of sensitivity analysis. The sensitivity analysis of vibrating motion of piezoelectric MC in the presence of geometrical discontinuities is carried out in the present paper for the first time using the Sobol method. As resonance amplitude and natural frequency are highly important in vibrating motion and engineering applications (AFM & MEMS), they were selected as the parameters of sensitivity analysis and the influence of geometrical analysis of all layers and MC parts on them is studied.

II. DYNAMIC MODELING

The discrete MC (Fig. 1) is selected for dynamic modeling. Piezoelectric layer is a mechanical part that bends by applying electric voltage; therefore, imposing stress on MC will make it bend. This layer is placed on the MC and encompassed by two electrodes.

If MC's deformations are as much as a nanometer, a linear theory provides a favorable accuracy in calculating amplitude and natural frequency (Bashash *et al*., 2010). Simulation results show that in case MC's deformation is as much as a micrometer, it is better to use a non-linear theory to achieve more accurate results (Mahmoodi *et al*., 2008). As deformation is at nano scales in AFM applications, it is better to use linear theory. Using Hamilton's method and with respect to the Euler-Bernoulli theory, the differential equation of MC vibrating motion can be defined as follows:

$$m(x)\ddot{v} + \left[K(x)v''\right]'' + C\dot{v} + C_e''(x)P(t) = 0 \tag{1}$$

where

$$m(x) = \sum_{i=1}^{4}\rho_i W_i h_i \left(H_0 - H_{L_1}\right) + \rho_1 W_1 h_1 \left(H_{L_1} - H_{L_2}\right) + \rho_t W_t h_t \left(H_{L_2} - H_L\right)$$

$$K(x) = EI\left(H_0 - H_{L_1}\right) + \frac{E_1 W_1 h_1}{12}\left(H_{L_1} - H_{L_2}\right) + \frac{E_t W_t h_t}{12}\left(H_{L_2} - H_L\right)$$

$$EI = \sum_{i=1}^{4} E_i h_i W_i \left\{ \frac{h_i^2}{12} + \left[y_n - \left(\sum_{j=1}^{i} h_j - \frac{h_i}{2} \right) \right]^2 \right\} \tag{2}$$

In these equations, indices 1 to 4 indicate base layer, lower electrode, piezoelectric layer and upper electrode, respectively. As layer axial deformation was overlooked in the calculations, layers thicknesses should be much smaller than their length as far as geometry is concerned. v is MC deformation. C and $P(t)$ respectively indicate damping coefficient and input voltage. Viscous damping coefficient (C), which was considered in the calculations, is due to the damping of air environment and MC structure. The coefficient can be calculated based on environmental conditions and using an experiment. This article selected the coefficient value as per the experiments carried out by Mahmoodi *et al.* (2010).

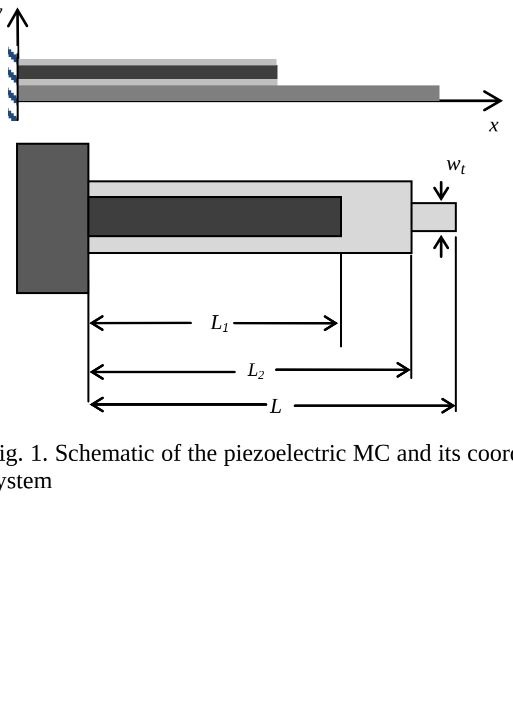

Fig. 1. Schematic of the piezoelectric MC and its coordinate system

Y_n specifies the position of neutral axis, which can be obtained as follows:

$$y_n = \frac{\sum_{i=k}^{n} E_i h_i W_i \left(\sum_{j=1}^{i} h_j - \frac{h_i}{2} \right)}{\sum_{i=k}^{n} E_i h_i W_i} \tag{3}$$

The electric voltage applied to a piezoelectric layer, which vibrates the MC, enters the equation of motion through electromechanical coupling coefficient C_e that is as follows:

$$C_e = E_3 d_{31} W_3 (h_1 + h_2 + \frac{1}{2}h_3 - y_n)(H_0 - H_{L_1}) \tag{4}$$

In this equation, h_1, h_2 and h_3 are the thicknesses of MC, electrode, and piezoelectric, respectively. W_3 is the width of piezoelectric layer, E_3 is Yang's modulus of piezoelectric layer, d_{31} is piezoelectric constant and H_i specifies Heaviside function. The equation of motion can be solved using the separation of variables method:

$$v(x,t) = \sum_{n=1}^{\infty} \varphi_n(x)\, q_n(t), \tag{5}$$

where q_n is the generalized time-dependent coordinates and ϕ_n is the n^{th} form of the mode. With respect to the geometrical discontinuities to improve accuracy of the calculations (Salehi-Khojin *et al.*, 2010) it is necessary to divide MC into three homogeneous beams. Part one of MC with piezo-electric layer and electrodes, part two of MC without piezoelectric layer, and part three is the tip. Therefore, ϕ_n can be stated as below:

$$\varphi_n(x) = \begin{cases} A_{1n} Sin\beta_{1n}x + B_{1n} Cos\beta_{1n}x + C_{1n} Sinh\beta_{1n}x \\ + D_{1n} Cosh\beta_{1n}x, & 0 \le x \le L_1 \\ A_{2n} Sin\beta_{2n}x + B_{2n} Cos\beta_{2n}x + C_{2n} Sinh\beta_{2n}x \\ + D_{2n} Cosh\beta_{2n}x, & L_1 \le x \le L_2 \\ A_{3n} Sin\beta_{3n}x + B_{3n} Cos\beta_{3n}x + C_{3n} Sinh\beta_{3n}x \\ + D_{3n} Cosh\beta_{3n}x, & L_2 \le x \le l \end{cases} \tag{6}$$

where $\beta_{mn}^4 = \omega_n^2 m_m / (EI)_m$. A_{rn}, B_{rn}, C_{rn}, D_{rn}, β_{rn} are unknown values which can be calculated through boundary conditions, continuity at each step point, and normalization condition. Boundary condition and beam continuity at each stepped point are:

$$\varphi_{1n}(0) = 0 \quad \varphi_{1n}'(0) = 0 \quad \varphi_{3n}''(L) = 0 \quad \varphi_{3n}'''(L) = 0$$
$$\varphi_{in}(L_i) = \varphi_{i+1,n}(L_i) \quad \varphi_{in}'(L_i) = \varphi_{i+1,n}'(L_i) \quad i = 1,\ldots,3 \tag{7}$$
$$(EI)_i \varphi_{in}''(L_i) = (EI)_{i+1} \varphi_{i+1,n}''(L_i) \quad (EI)_i \varphi_{in}'''(L_i) = (EI)_{i+1} \varphi_{i+1,n}'''(L_i)$$

By separation variables of the differential equation of motion using Eq. 5, we can write:

$$\ddot{q}_n + \omega_n^2 q_n + \mu_n \dot{q}_n + J_n.P(t) = 0 \tag{8}$$

where:

$$\omega_n^2 = \int_0^L \varphi_n (K(x)\varphi_n'')'' dx \tag{9}$$

$$\mu_n = 2\xi_n \omega_n \tag{10}$$

$$J_n = \int_0^L \varphi_n C_e''(x) dx \tag{11}$$

μ_n is modal damping coefficient.

III. SENSITIVITY ANALYSIS RESULTS

Sensitivity of tip deformation, the first natural frequency, and actuation ability into 9 main parameters, including thickness, width and length of MC, piezoelectric layers and electrode and tip are studied using Sobol method. The Sobol method is efficient for analysis of complex models and can conduct absolute sensitivity analysis of the inputs. Additionally, this method can determine the sensitivity of the MC deformation and natural frequency to input variables. Simlab software was employed to use the Sobol's method. Here, all the geometrical parameters and their ranges are specified on the software. Then inputs of the main programming code, which are used for calculating amplitude and frequency, are determined.

Since electrodes are used to cover the piezoelectric layer, their width and length are equal to the width and length of the piezoelectric layer. Thicknesses of the two electrodes are also considered equal (Mahmoodi and Jalili, 2009). As the thicknesses of tip and MC are considered equal while making MC (Mahmoodi *et al.*, 2010; Bashash *et al.*, 2010), here, the thicknesses of tip and MC are considered equal. Table 1 shows the properties of piezoelectric MC.

A. Sensitivity Analysis of Amplitude

In self-actuator piezoelectric MCs, in spite of the normal beams, the geometrical dimension of each layer not only affects stiffness, mass of unit length of MC, but also affects ability of MC actuation, with respect to Eq. 4. Therefore, the sensitivity of parameters of vibrating motion (amplitude, natural frequency) in this MC is different from the rest of beams.

Figure 2 shows how tip's vibrating motion amplitude changes in proportion to MC geometrical dimensions. Sobol is a statistical method, which calculates values of amplitude motion and natural frequency for any statistical sets. The statistical sets include MC's geometrical dimensions. It is possible to specify how variables are changed after calculating variables for different statistical sets using trend line. Therefore, the blue points in this figure show the amplitude values, which were calculated by selecting each statistical set using Sobol method. According to the results (Fig. 2-a), with the increase of MC thickness, amplitude of vibrating motion decreases. Therefore, it can be concluded that in piezoelectric MCs, the effect of MC thickness on stiffness is greater than the ability of actuation. Vibrating motion in piezoelectric MCs is performed by applying electric voltage to a piezoelectric layer. Undoubtedly, the more the amplitude of vibrating motion is for applying a certain voltage to piezoelectric, the more is the actuation ability for that certain voltage. Therefore ability of actuation can be determined based on size of amplitude motion in a certain actuation voltage.

Low gradient of the amplitude in terms of width (Fig. 2-b) and high gradient of the amplitude in terms of length (Fig. 2-c) indicate low effect of MC thickness and high effect of MC length on vibrating motions.

Table 1. Material properties of simulated MC

	E (Gpa)	ρ (Kg/m^3)
MC and Tip	170	2330
Electrode Layers	78	19300
Piezoelectric Layer (ZnO)	130	6390

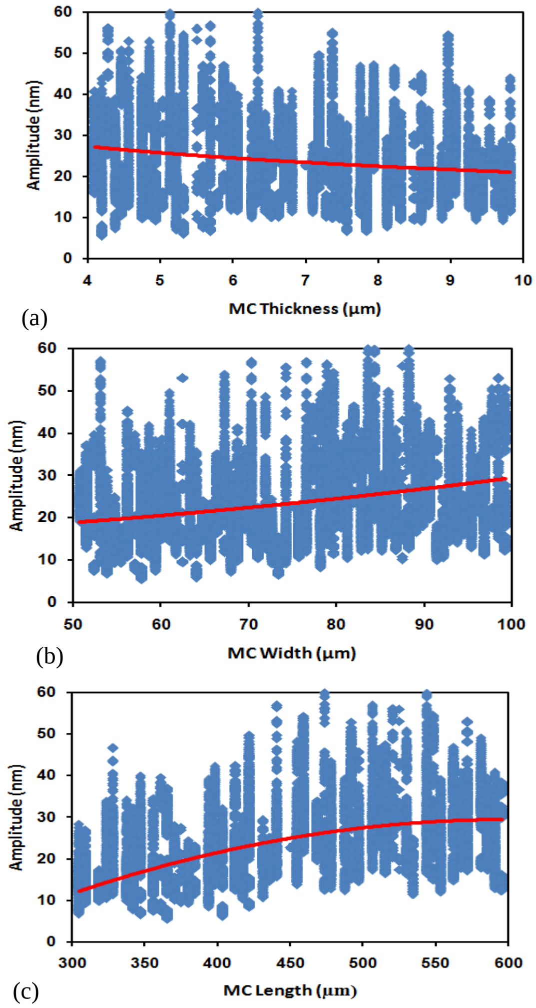

Fig. 2 Effect of geometrical dimensions of MC's base layer on amplitude of vibrating motion of tip

In self-actuator piezoelectric MCs, piezoelectric layer causes MC to vibrate by AC input voltage. Actuation ability not only depends on the input voltage but also, according to Eq. 4, the geometrical dimensions of the piezoelectric layer. With respect to Eq. 4, there is a direct relationship between actuation ability and width of this layer. That is, width increase, on the one hand, leads to the increase of MC stiffness and on the other hand increases actuation ability. This is also true for the thickness and length of this layer.

The obtained results of Figs. 3-b and 3-c indicate that the effect of width and length of piezoelectric layer on the ability of actuation was more as compared with MC stiffness; therefore, with the increase of these two quantities, amplitude of vibrating motion will increase as well. According to the results of Fig. 3-a, despite width and length, thickness of piezoelectric layer has more effects on MC stiffness and increase of thickness will lead to the decrease of amplitude of vibrating motion. It can be concluded that to improve performance of piezoelectric in MCs, it is better to select a long, wide, and thin form of base layer.

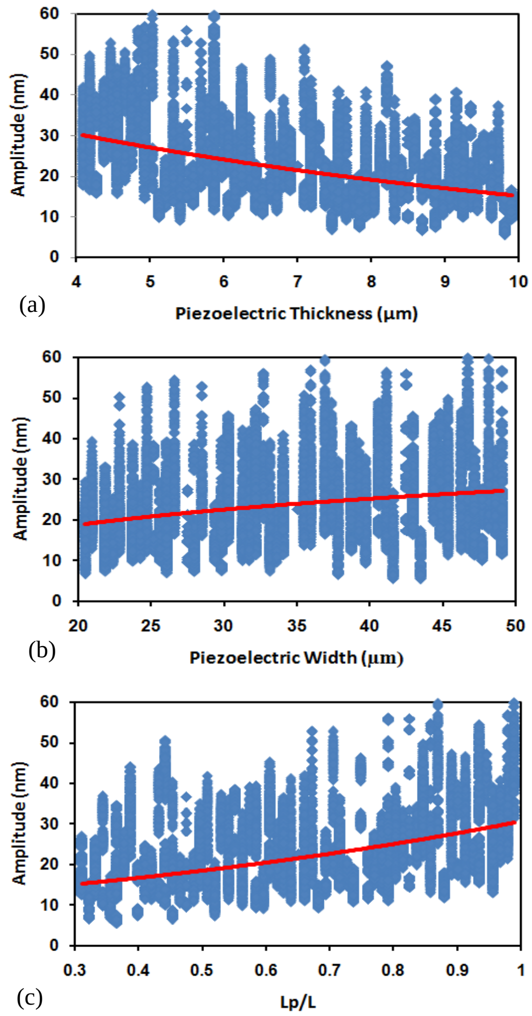

Fig. 3 Effect of geometrical dimensions of piezoelectric layer on the amplitude

Piezoelectric MCs, due to present of piezoelectric layer, are usually made in a wide form (Mahmoodi *et al.*, 2010). In case of using these MCs in AFM for surface topography and measuring the interaction force acting between probe and sample surface, it is necessary to make tip thinner to increase MC's sensitivity to these forces. The geometrical dimensions of tip affect both sensitivity of force measurement and vibrating motion of MC. Figure 4 shows the effect of geometrical dimensions of tip (width, length) on amplitude. As thicknesses of MC and tip are considered equal in making piezoelectric MCs (Mahmoodi *et al.*, 2010; Bashash *et al.*, 2010), here, we only study the effect of width and length of tip on amplitude. The results of Figs. 4-a and 4-b show that increase of length and width of tip leads to decrease of amplitude of vibrating motion.

In piezoelectric self-actuator MCs, electrodes are used to create electric voltage on both sides of piezoelectric layer. To cover the piezoelectric layer completely, the width and length of these electrodes are selected equal to the width and length of a piezoelectric layer. Therefore, it is only the thickness of electrode layer that is selected independently. Figure 5 shows the effect of the thickness of this layer on amplitude. As shown, the thickness of the electrode layer has a little effect on amplitude.

Figure 6 shows the percentage of amplitude sensitivity to changes of parameters in piezoelectric MC. As it is shown, the length of MC, the length of piezoelectric layer and the MC thickness has the maximum effects on the amplitude vibrating motion of tip. The least effects are related to the width of tip and MC.

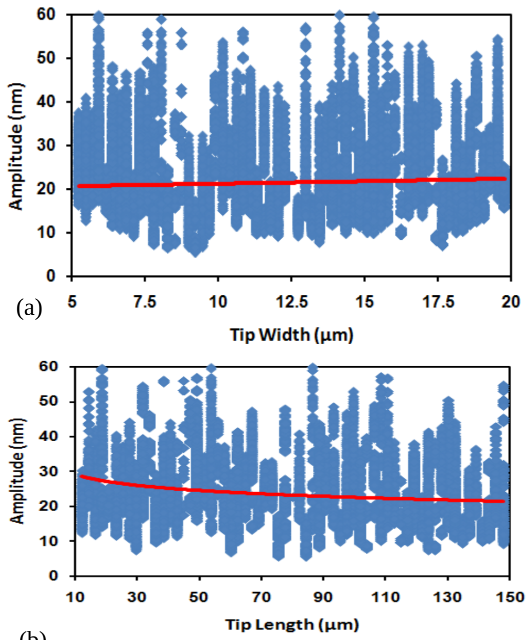

Fig. 4. Effect of geometrical dimensions of tip on the amplitude

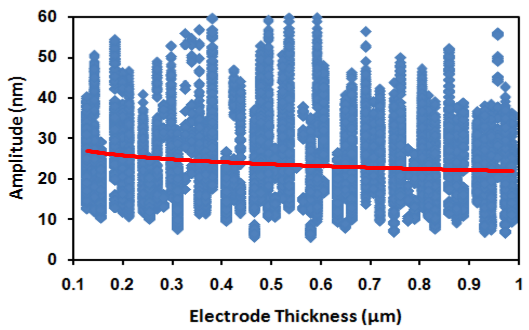

Fig. 5. Effect of electrode thickness on the amplitude

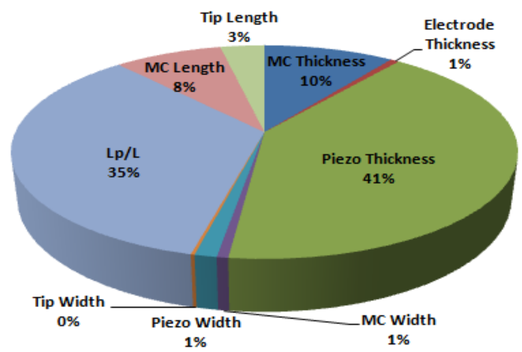

Fig. 6. Percentage of amplitude sensitivity to changes of geometrical parameters

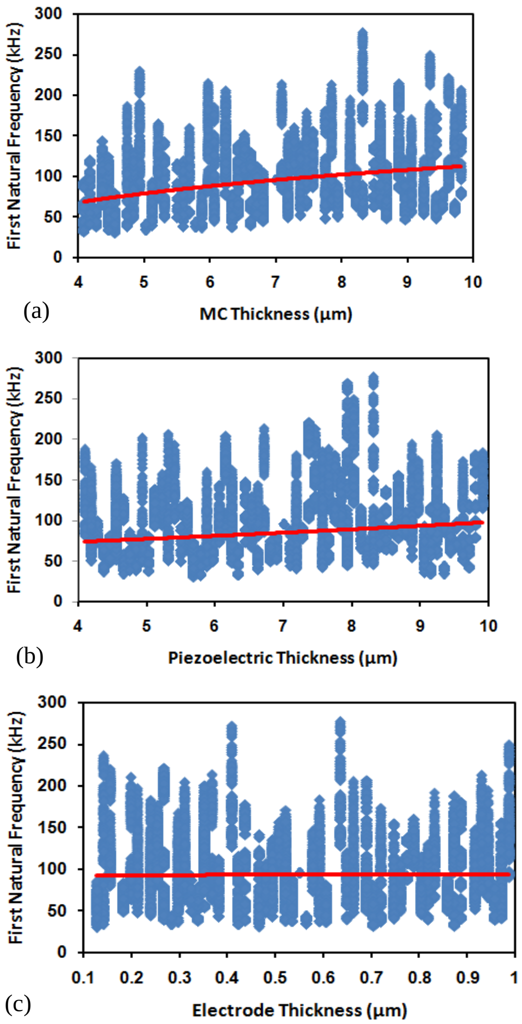

Fig. 7. Effect of thickness of each layer on natural frequency

B. Sensitivity Analysis of Natural Frequency

As mentioned earlier, piezoelectric MC has geometrical discontinuities due to the presence of piezoelectric layer and tip and the non-uniform beam method is used for its vibrating analysis. Therefore, with respect to the discontinuities of this MC, natural frequency can be calculated by solving the determinant of coefficients of Eq. 5.

Figure 7 shows the effect of thickness of each layer on the first natural frequency of piezoelectric MC. With the increase of thickness, MC stiffness increases; therefore, the increase of thickness leads to the increase of natural frequency. According to the obtained results of Fig. 7-a, thickness of MC layer, due to being wider, causes significant change on natural frequency. However, piezoelectric layer and electrodes, due to having less width and thickness, have less effect on MC stiffness. Therefore, the increase of their thickness does not cause many changes in natural frequency.

Figure 8 shows the effect of width of each layer on the first natural frequency. As shown, the width of each layer (MC, piezoelectric) has less effect as compared with the thickness on natural frequency. Contrasting the results of Figs. 4-b and 8-c indicates that in vibrating motion of MC, tip is more effective in natural frequency than in the amplitude.

Figure 9 shows the effect of MC, piezoelectric layer and tip lengths on the first natural frequency of MC. As it is shown in this Fig., with the increase of lengths of MC and tip, natural frequency decreases. As geometrical dimensions of tip are less than MC, it has less effect on natural frequency. As Fig. 9-c shows, the effect of piezoelectric layer on natural frequency is different from the lengths of MC and tip. When the length of piezoelectric layer is smaller than 0.66 of MC length, with the increase of length of this layer, natural frequency increases. When the length of this layer exceeds 0.66 of MC length, natural frequency decreases with the increase of length.

Figure 10 shows the percentage of natural frequency sensitivity to different parameters of piezoelectric MC. The most and the least effective parameters in natural frequency are MC length and tip width, respectively. By comparing the effect of geometrical dimensions of MC, piezoelectric, and tip one can conclude that the length of each part is the most effective parameter while its width has the lowest effect on natural frequency.

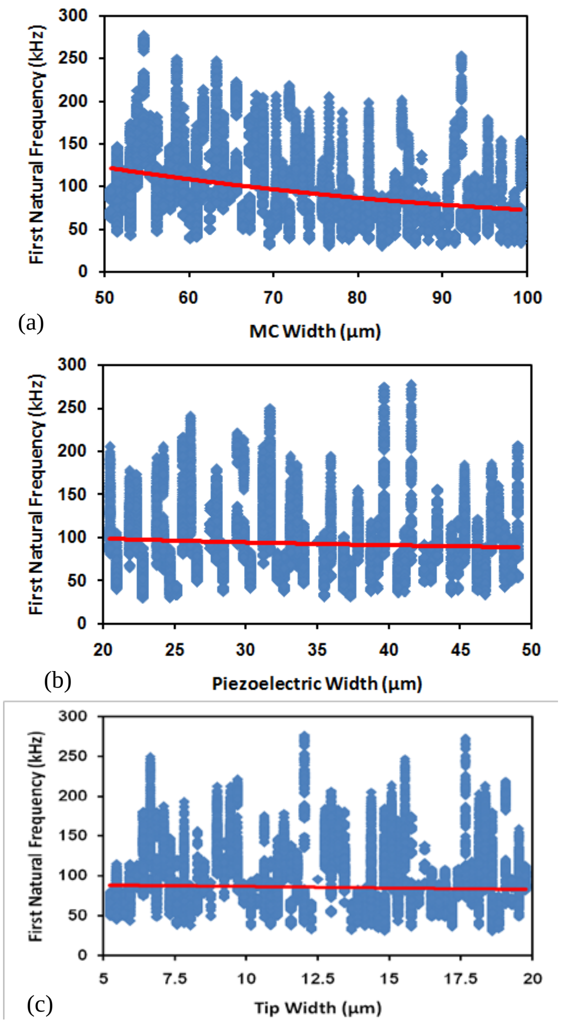

Fig. 8. Effect of width of each layer on natural frequency

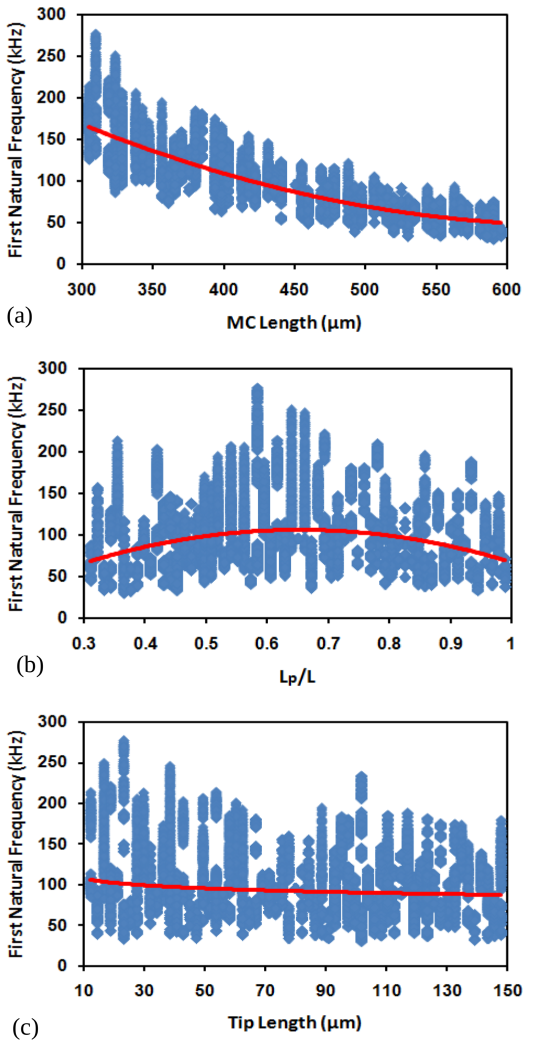

Fig. 9. Effect of length of each layer on natural frequency

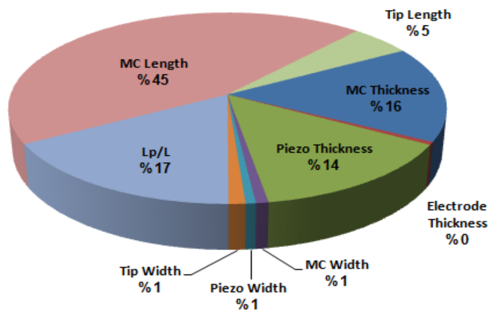

Fig. 10. Percentage of natural frequency sensitivity to changes of geometrical parameters

IV. CONCLUSIONS

The present paper conducts sensitivity analysis of vibrating motion of piezoelectric MC with the presence of geometrical discontinuities. In addition, it studies the effect of geometrical dimensions of each MC layer and tip on amplitude, natural frequency and actuation ability. As the geometrical dimensions of each layer (MC, electrodes, and piezoelectric) in a self-actuator piezoelectric MC not only affect stiffness and mass per unit length (the effective coefficients in motion equation), but also change the ability of actuation of piezoelectric layer. Due to the presence of piezoelectric layer and tip, the geometrical discontinuities in beam will affect vibrating motion. Geometrical dimensions of piezoelectric layer not only affect stiffness and mass per unit length of MC, but also have an effect on actuation ability. Simulation results show that the major effect of width and length of piezoelectric layer is on actuation ability than stiffness and mass per unit length of MC; whereas, the effect of thickness of this layer is mainly on stiffness and mass per unit length of MC, that is, with the increase of width and length of this layer, actuation ability increases and amplitude decreases with the increase of thickness. Therefore, to achieve higher amplitude, it is necessary to choose a wide, long and thin piezoelectric layer.

The effect of piezoelectric layer on natural frequency is different as compared with the other parts. The simulation results show that when L_p/L is equal to 0.66, natural frequency reaches its maximum amount. Meanwhile, the effect of the other geometrical dimensions on natural frequency is either ascending and/or descending.

REFERENCES

Adams, J.D., L. Manning, B. Rogers, M. Jones and S.C. Minne, "Self-sensing tapping mode atomic force microscopy", *Sen. Act. A.*, **121**, 262-266 (2005).

Bashash, S., R. Saeidpourazar and N. Jalili, "Development, analysis and control of a high-speed laser-free atomic force microscope," *Rev. Sci. Inst.*, **81**, article id. 023707 (2010).

Chu, C.-H., W.-P. Shih, S.-Y. Chung, H.-C. Tsai, T.-K. Shing and P.-Z. Chang, "A low actuation voltage electrostatic actuator for RF MEMS switch applications," *J. Micromech. Microeng.*, **17**, 1649–56 (2007).

Korayem, M.H., Z. Rastegar and M. Taheri, "Sensitivity Analysis of Nano-contact Mechanics Models in Manipulation of Biological Cell," *Nanoscie. Nanotech.*, **2**, 49-56 (2012).

Kovacs, A. and Z. Vizvary, "Structural parameter sensitivity analysis of cantilever and bridge-type accelerometers," *Sens. and Act. A: Phys.*, **89**, 197-205 (2001).

Lee, H.-C., J.-H. Park and Y.-H. Park, "Development of shunt type ohmic RF MEMS switches actuated by piezoelectric cantilever," *Sen. Act. A.*, **136**, 282–90 (2007).

Lee, H.-W. and W.-J. Chang, "Sensitivity of V-shaped atomic force microscope cantilevers based on a modified couple stress theory," *Microelec. Eng.* 88(11), 3214-3218 (2011).

Mahmoodi, S.N., M. Afshari and N. Jalili, "Nonlinear vibrations of piezoelectric microcantilevers for biologically-induced surface stress sensing," *Comm. in Non. Scie. Num. Sim.*, **13**, 1964–1977 (2008).

Mahmoodi, S.N. and N. Jalili, “Piezoelectrically actuated microcantilevers: An experimental nonlinear vibration analysis”, *Sens. and Act. A*, **150**, 131–136 (2009).

Matviykiv, O. and M. Lobur, “Design principles and sensitivity analysis of MEMS cantilever sensors,” *Proc. of VIth Int. Conf. on Perspective Technologies and Methods in MEMS Design (MEMSTECH)* (2010).

Mahmoodi, S.N., N. Jalili and M. Ahmadian, “Subharmonics analysis of nonlinear flexural vibrations of piezoelectrically actuated microcantilevers,” *Nonlinear Dyn.*, **59**, 397–409 (2010).

Rogers, B., L. Manning, T. Sulchek and J.D. Adams, “Improving tapping mode atomic force microscopy with piezoelectric cantilevers,” *Ultramicroscopy*, **100**, 267-276 (2004).

Salehi-Khojin, A., S. Bashash and N. Jalili, “Modeling and experimental vibration analysis of nanomechanical cantilever active probes,” *J. Micromech. Microeng.*, **18**, article id. 085008 (2008).

Yamashita, K., Y. Yang, T. Nishimoto, K. Furukawa and M. Noda, “Piezoelectric vibratory-cantilever force sensors and axial sensitivity analysis for individual triaxial tactile sensing,” *IEEE Conf. on Sensors* (2011).

Yang, Y., K. Yamashita, T. Nishimoto, K. Furukawa and M. Noda, “Fabrication and axial sensitivity analysis of vibratory piezoelectric cantilever sensors for three-axis tactile sensing,” *Int. Meet. for Fut. of Elect. Dev.* (2011).

Received: February 4, 2013. Sent to new Subject Editor: May 15, 2014. Accepted: July 14, 2015. Recommended by Subject Editor: Walter Tuckart.