

MAGNETOHYDRODYNAMIC HEAT AND MASS TRANSFER FLOW WITH INDUCED MAGNETIC FIELD AND VISCOUS DISSIPATIVE EFFECTS

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Abstract— An approximate solution to the problem of steady free convective MHD flow of an incompressible viscous electrically-conducting fluid over an infinite vertical isothermal porous plate with mass convection is presented here. A uniform magnetic field is assumed to be applied transversely to the direction of the flow, taking into account the induced magnetic field with viscous and magnetic dissipations of energy. The dimensionless governing equations are solved by using the series solution method. The induced magnetic field, current density, temperature gradient and flow velocity are studied for magnetohydrodynamic body force, magnetic Prandtl number, Schmidt number and Eckert number. It is observed that the induced magnetic field is found to increase with a rise in magnetic Prandtl number. Current density is strongly reduced with increasing magnetic Prandtl number, but enhanced with Schmidt number. The acquired knowledge in our study can be used by designers to control MHD flow as suitable for a certain applications such as laminar magneto-aerodynamics, and MHD propulsion thermo-fluid dynamics.

Keywords— Newtonian Fluid, Thermo-fluid dynamics, Heat transfer control, MHD energy generators, Viscous and Magnetic dissipative heat.

I. INTRODUCTION

The study of magneto-hydrodynamic flow for an electrically conducting fluid past a porous vertical surface has attracted the interest of many researchers in view of its important applications in many engineering problems such as plasma studies, petroleum industries, MHD power generators, cooling of nuclear reactors, the boundary layer control in aerodynamics, and crystal growth. In all these applications understanding the behaviour of MHD convective flow and the various problem parameters that influence is a very important asset to designers developing applications that aim to control this flow. In view of these applications, several investigators have made model studies on the effect of magnetic field free convection flows. Some of them are Georgantopoulos (1979), Nanousis *et al.* (1980) and Raptis and Singh (1983). Along with the effects of magnetic field as well as transpiration parameter, being an effective method of controlling the boundary layer has been considered by Kafoussias *et al.* (1979). Raptis

and Soundalgekar (1984) investigated the steady laminar free convection flow of an electrically conducting fluid along a porous hot vertical plate in presence of heat source/sink. The combined heat and mass transfer of an electrically conducting fluid in MHD natural convection adjacent to a vertical surface was analyzed by Chen (2004). The study of heat and mass transfer on the free convective oscillatory flow of a viscous incompressible fluid past an infinite vertical porous plate in presence of transverse sinusoidal suction velocity and a constant free stream velocity was presented by Ahmed (2009). Seddeek (2004) investigated the effects of heat source in the presence of suction and blowing on mixed free-forced convective flow and mass transfer over an accelerating surface in the case of variable viscosity. Zueco *et al.* (2009) studied the problem of thermophoretic hydromagnetic dissipative heat and mass transfer by network simulation numerical technique. Steven *et al.* (2012) studied the magnetic hydrodynamic free convective flow past an infinite vertical porous plate with the effect of viscous dissipation subject to a constant suction velocity.

The effects of external temperature-dependent sources on the unsteady free convective flows past an infinite vertical porous plate were presented by Pop and Soundalgekar (1979). Zueco Jordán (2006) presented a numerical analysis of an unsteady free convective MHD flow of a dissipative fluid along a vertical plate subject to a constant heat flux. Ahmed and Liu (2010) analyzed the effects of mixed convection and mass transfer of three-dimensional oscillatory flow of a viscous incompressible fluid past an infinite vertical porous plate in presence of transverse sinusoidal suction velocity oscillating with time and a constant free stream velocity. Chamkha and Camille (2000) investigated the effects of heat generation/absorption and thermophoresis on hydromagnetic flow with heat and mass transfer over a flat surface. Free convection flow past a vertical plate with surface temperature oscillations was considered by Li *et al.* (1982). Ahmed (2010) investigated the effect of periodic heat transfer on unsteady MHD mixed convection flow past a vertical porous flat plate with constant suction and heat sink when the free stream velocity oscillates in about a non-zero constant mean. Chamkha (2003) investigated the chemical reaction effects on heat and mass transfer laminar boundary layer flow in presence of heat generation/absorption. The effect of the lo-

cal acceleration term on the MHD transient free convection flow over a vertical plate was studied by Aldoss and Al-Nimir (2005). Muthucumaraswamy *et al.* (2001) studied the heat and mass transfer effects on flow past an impulsively started infinite vertical plate. Asghar *et al.* (2005) investigated the effects of Hall current and heat transfer on the steady flow of an electrically conducting, Oldroyd-B and incompressible fluid. The fluid is between two electrically insulating disks maintained at two constant but different temperatures. The flow is due to a pull with constant velocities of eccentric rotating infinite disks and an external uniform magnetic field is applied perpendicular to the disks. The magnetic Reynolds number is assumed small so that the induced magnetic field is neglected.

The above studies have generally been confined to very small magnetic Reynolds numbers, allowing *magnetic induction effects* to be neglected. Such effects must be considered for larger values of magnetic Reynolds number. Glauert (1962) presented a seminal analysis for hydromagnetic flat plate boundary layers along a magnetized plate with uniform magnetic field in the streamwise direction at the plate. He obtained series expansion solutions (for both large and small values of the electrical conductivity parameter) for the velocity and a magnetic field, indicating that for a critical value of applied magnetic field, boundary-layer separation arises. Dorrepaal and Moosavizadeh (1998) studied the oblique stagnation-point magnetohydrodynamic flow in the vicinity of a location where a separation vortex reattaches itself to a rigid boundary, obtaining a similarity solution for the velocity and induced magnetic field distributions near the point of reattachment. Raptis and Massalas (1998) investigated the effects of radiation on the oscillatory flow of a gray gas, absorbing-emitting in presence of induced magnetic field and analytical solutions were obtained with the help of perturbation technique. The hydrodynamic free convective flow of an optically thin gray gas in the presence of radiation, when the induced magnetic field is taken into account was studied by Raptis *et al.* (2003) using perturbation technique. More recently Bég *et al.* (2009) obtained local non-similarity numerical solutions for the velocity, temperature and induced magnetic field distributions in forced convection hydromagnetic boundary layers, over an extensive range of magnetic Prandtl numbers and Hartmann numbers. Bég *et al.* (2009) have also derived closed-form solutions for the velocity, temperature and magnetic induction functions in Rayleigh free convection hydromagnetic flow. Alom *et al.* (2008) investigated the steady MHD heat and mass transfer by mixed convection flow from a moving vertical porous plate with induced magnetic, thermal diffusion, constant heat and mass fluxes, and the non-linear coupled equations are solved by shooting iteration technique. Ahmed and Zueco (2010) studied the effect of the transverse magnetic field on a steady mixed convective heat and mass transfer flow of an incompressible viscous electrically conducting fluid past an infinite vertical isothermal po-

rous plate taking into account the induced magnetic field, viscous and magnetic dissipations of energy in presence of chemical reaction of first order and heat generation/absorption, and the non-linear coupled equations are solved by network simulation technique. Ahmed and Chamkha (2010) investigated the effects of radiation and chemical reaction on steady mixed convective heat and mass transfer flow of an optically thin gray gas over an infinite vertical porous plate with constant suction taking into account the induced magnetic field, and viscous dissipation of energy, and the non-linear coupled equations are solved by series solution method. The steady magnetohydrodynamic (MHD) mixed convection stagnation point flow over a vertical flat plate is investigated by Ali *et al.* (2011). Ahmed *et al.* (2012) investigated the effect of the transverse magnetic field on a transient free and forced convective flow over an infinite vertical plate impulsively held fixed in free stream taking into account the induced magnetic field.

The present paper aims to study the problem of a steady magnetohydrodynamic free convective boundary layer flow over a porous vertical isothermal flat plate with constant suction where the effects of the induced magnetic field as well as viscous and magnetic dissipations of energy has been considered. The present study may have useful applications to several transport processes as well as magnetic materials processing. The analytical results for some particular cases are matched with Raptis *et al.* (2003) and found them in excellent agreement.

II. MATHEMATICAL ANALYSIS

The steady MHD mixed convective heat and mass transfer flow of an electrically conducting, viscous incompressible fluid over a porous vertical infinite isothermal plate with viscous dissipative heat and magnetic dissipation of energy has been considered (Fig. 1) by making the following assumptions:

- All the fluid properties except the density in the buoyancy force term are constant;
- The Eckert number Ec is small;
- The plate is subjected to a constant suction velocity;
- A magnetic field of uniform strength (H_0) is applied transversely to the direction of the main stream taking into account the induced magnetic field;

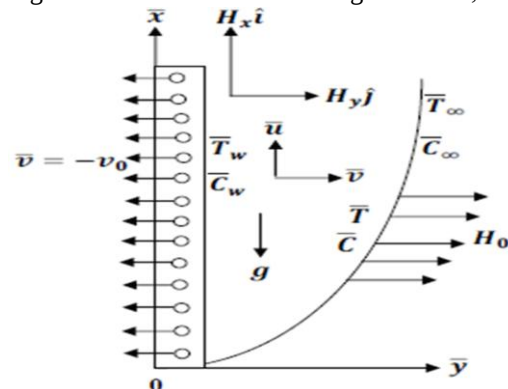


Fig. 1: Physical configuration.

- The magnetic Prandtl number is so chosen in comparison to the Hartmann number such that the induced magnetic is not negligible;
- The concentration of the diffusing species in the binary mixture is assumed to be very small in comparison with the other chemical species, which are present, and hence the Soret and Dufour effects are negligible;
- The equation of conservation of electric charge is $\nabla \cdot J = 0$ where $J = (J_x, J_y, J_z)$. The direction of propagation is considered only along the y-axis and does not have any variation along the y-axis and so $\partial J_y / \partial y = 0$ which gives $J_y = \text{constant}$. Since the plate is electrically non-conducting, this constant is zero and hence $J_y = 0$ everywhere in the flow;
- The wall is maintained at constant temperature $\bar{T}_w$ and concentration $\bar{C}_w$ higher than the ambient temperature $\bar{T}_\infty$ and concentration $\bar{C}_\infty$ respectively;
- The fluid is non-magnetic, neglecting the thermoelectric effect together with the short circuit condition.

In case of electrically conducting fluid, the flow and heat transfer are induced by an imposed magnetic field. An e.m.f. is produced in the fluid flowing across the transverse magnetic field. The resultant effect of current and magnetic field produces a force, resisting the fluid motion. The fluid velocity also affects the magnetic field by producing an induced magnetic field which perturbs the original field.

We introduce a coordinate system $(\bar{x}, \bar{y}, \bar{z})$ with $\bar{x}$ -axis vertically upwards along the plate, $\bar{y}$ -axis normal to the plate into the fluid region and $\bar{z}$ -axis along the width of the plate. Let the plate be long enough in $\bar{x}$ -direction for the flow to be parallel. Let $(\bar{u}, \bar{v}, 0)$ be the fluid velocity and $(\bar{H}_x(y), \bar{H}_y, 0)$ be the magnetic induction vector at a point $(\bar{x}, \bar{y}, \bar{z})$ in the fluid. Since the plate is infinite in length in $\bar{x}$ -direction, therefore all the physical quantities except possibly the pressure are assumed to be independent of $\bar{x}$.

Within the frame of such assumptions and under the **Boussinesq's** approximation and in consistency with boundary layer theory, the governing equations relevant to the problem are:

$$\frac{d\bar{v}}{d\bar{y}} = 0 \quad (1)$$

which is satisfied with $\bar{v} = -v_0 = \text{a constant}$.

$$\frac{d\bar{H}_y}{d\bar{y}} = 0 \quad (2)$$

which holds for $\bar{H}_y = -H_0 = \text{a constant} = \text{strength from applied magnetic field}$.

$$\rho \bar{v} \frac{d\bar{u}}{d\bar{y}} = -\frac{d\bar{p}}{d\bar{x}} - \rho g + \mu \frac{d^2 \bar{u}}{d\bar{y}^2} + \eta \sigma H_0 \frac{d\bar{H}_x}{d\bar{y}} \quad (3)$$

$$\bar{v} \frac{d\bar{T}}{d\bar{y}} = \frac{\lambda}{\rho C_p} \nu \frac{d^2 \bar{T}}{d\bar{y}^2} + \frac{\nu}{C_p} \left(\frac{d\bar{u}}{d\bar{y}} \right)^2 + \frac{\sigma \eta^2}{\rho C_p} \left(\frac{d\bar{H}_x}{d\bar{y}} \right)^2 \quad (4)$$

$$0 = \eta \frac{d^2 \bar{H}_x}{d\bar{y}^2} + H_0 \frac{d\bar{u}}{d\bar{y}} - \bar{v} \frac{d\bar{H}_x}{d\bar{y}} \quad (5)$$

$$\bar{v} \frac{d\bar{C}}{d\bar{y}} = D \frac{d^2 \bar{C}}{d\bar{y}^2} \quad (6)$$

Since there is no large velocity gradient here, the viscous term in the Eq. (3) vanishes for small μ and hence for the outer flow, beside there is no induced magnetic field along x-direction gradient, so we have

$$0 = -\frac{d\bar{p}}{d\bar{x}} - \rho_\infty g \quad (7)$$

By eliminating the pressure term from the Eqs. (3) and (7), we obtain

$$\rho \bar{v} \frac{d\bar{u}}{d\bar{y}} = (\rho_\infty - \rho)g + \mu \frac{d^2 \bar{u}}{d\bar{y}^2} + \eta \sigma H_0 \frac{d\bar{H}_x}{d\bar{y}}. \quad (8)$$

The Boussinesq approximation gives

$$\rho_\infty - \rho = \rho_\infty \beta (\bar{T} - \bar{T}_\infty) + \rho_\infty \beta (\bar{C} - \bar{C}_\infty). \quad (9)$$

On using (1) and (9) in the Eq. (8) and noting that ρ_∞ is approximately equal to 1, the momentum equation reduces to

$$-v_0 \frac{d\bar{u}}{d\bar{y}} = g\beta(\bar{T} - \bar{T}_\infty) + g\beta(\bar{C} - \bar{C}_\infty) + \nu \frac{d^2 \bar{u}}{d\bar{y}^2} + \frac{\eta \sigma H_0}{\rho} \frac{d\bar{H}_x}{d\bar{y}}. \quad (10)$$

The boundary conditions are:

$$\bar{y} = 0 : \bar{u} = 0, \frac{d\bar{T}}{d\bar{y}} = -\frac{\bar{q}}{\lambda}, \bar{H}_x = 0, \bar{C} = \bar{C}_w \quad (11)$$

$$\bar{y} \rightarrow \infty : \bar{u} \rightarrow 0, \bar{T} \rightarrow \bar{T}_\infty, \frac{d\bar{H}_x}{d\bar{y}} \rightarrow 0, \bar{C} \rightarrow \bar{C}_\infty$$

The non-dimensional quantities are:

$$y = v_0 \bar{y} / \nu, u = \bar{u} / v_0, \nu = \mu / \rho, Pr = \nu C_p / \lambda, Pr_m = \nu / \eta, \theta = \lambda v_0 (\bar{T} - \bar{T}_\infty) / (\nu \bar{q}), \phi = D v_0 (\bar{C} - \bar{C}_\infty) / (\nu \bar{m}), Gr = \nu^2 g \beta \bar{q} / (\lambda v_0^4), Gr_m = \nu^2 g \beta \bar{m} / (D v_0^4), Sc = \nu / D, Ec = \lambda v_0^3 / (\nu \bar{q} C_p), M = \sigma \nu H_0^2 / (\rho v_0^2), H_x = \bar{H}_x / H_0.$$

The non-dimensional forms of (4) to (6) and (10) are

$$-\frac{du}{dy} = Gr\theta + Gr_m + \frac{d^2 u}{dy^2} + \frac{M}{Pr_m} \frac{dH_x}{dy}. \quad (12)$$

$$-\frac{d\theta}{dy} = \frac{1}{Pr} \frac{d^2 \theta}{dy^2} + Ec \left(\frac{du}{dy} \right)^2 + \frac{M Ec}{Pr_m^2} \left(\frac{dH_x}{dy} \right)^2. \quad (13)$$

$$-\frac{d\phi}{dy} = \frac{1}{Sc} \frac{d^2 \phi}{dy^2}. \quad (14)$$

$$\frac{d^2 H_x}{dy^2} + Pr_m \frac{du}{dy} + Pr_m \frac{dH_x}{dy} = 0. \quad (15)$$

The corresponding boundary conditions are:

$$y = 0 : u = 0, \frac{d\theta}{dy} = -1, H_x = 0, \phi = 1 \quad (16)$$

$$y \rightarrow \infty : u \rightarrow 0, \theta \rightarrow 0, \frac{dH_x}{dy} \rightarrow 0, \phi \rightarrow 0$$

III. METHOD OF SOLUTION

Perturbation theory leads to an expression for the desired solution in terms of a *power series* in some “small” parameter that quantifies the deviation from the exactly solvable problem. The leading term in this power series is the solution of the exactly solvable problem, while further terms describe the deviation in the solution, due to the deviation from the initial problem. Perturbation theory is applicable if the problem at hand can be formulated by adding a “small” term (**Eckert** number “*Ec*” in this work) to the mathematical description of the exactly solvable problem.

The solution of the Eq. (14) subject to the boundary condition (16) is

$$\phi = e^{-Scy} \quad (17)$$

Now, in order to solve the Eqs. (12), (13) and (15) under the boundary condition (16), we note that $Ec \ll 1$ for all the incompressible fluids and it is assumed the solutions of the equations to be of the form

$$f(y) = f_0(y) + Ec f_1(y) + O(Ec^2) \quad (18)$$

where f stands for u , θ or H_x .

Substituting (18) into the Eqs. (12), (13) and (15) and equating the coefficients of the same degree terms and neglecting terms of $O(Ec^2)$, the following differential equations are obtained:

$$u_0'' + u_0' = -Gr \theta_0 - Gr_m \phi_0 - \frac{M}{Pr_m} H_{x_0}' \quad (19)$$

$$u_1'' + u_1' = -Gr \theta_1 - Gr_m \phi_1 - \frac{M}{Pr_m} H_{x_1}' \quad (20)$$

$$\theta_0'' + Pr \theta_0' = 0 \quad (21)$$

$$\theta_1'' + Pr \theta_1' = -Pr (u_0')^2 - \frac{Pr M}{Pr_m^2} (H_{x_0}')^2 \quad (22)$$

$$H_{x_0}'' + Pr_m H_{x_0}' = -Pr_m u_0' \quad (23)$$

$$H_{x_1}'' + Pr_m H_{x_1}' = -Pr_m u_1' \quad (24)$$

The boundary conditions (16) reduce to

$$y=0: u_0=0, u_1=0, \theta_0'=-1, \theta_1'=0, \\ H_{x_0}=0, H_{x_1}=0 \quad (25)$$

$$y \rightarrow 0: u_0 \rightarrow 0, u_1 \rightarrow 0, \theta_0 \rightarrow 0, \theta_1 \rightarrow 0, \\ H_{x_0}' \rightarrow 0, H_{x_1}' \rightarrow 0$$

The solutions of the Eqs. (19) to (24) subject to the boundary conditions (25) are:

$$\theta_0(y) = \frac{1}{Pr} e^{-Pr y} \quad (26)$$

$$u_0(y) = \Omega e^{-\delta y} + \Omega_1 e^{-Pr y} + \Omega_2 e^{-Sc y} \quad (27)$$

$$H_{x_0}(y) = \Psi e^{-Pr_m y} + \Omega_3 e^{-\delta y} + \Omega_4 e^{-Pr y} + \Omega_5 e^{-Sc y} \quad (28)$$

$$\theta_1(y) = \Gamma e^{-Pr y} + \Gamma_{11} e^{-2\delta y} + \Gamma_{12} e^{-2Pr y} + \Gamma_{13} e^{-2Sc y} \\ + \Gamma_{14} e^{-2(Pr+\delta)y} + \Gamma_{15} e^{-2(Pr+Sc)y} + \Gamma_{16} e^{-2(Sc+\delta)y} \\ + \Gamma_{17} e^{-2(Pr+Pr_m)y} + \Gamma_{18} e^{-2(\delta+Pr_m)y} + \Gamma_{19} e^{-2Pr_m y} \\ + \Gamma_{20} e^{-2(Sc+Pr_m)y}$$

$$u_1(y) = \Lambda e^{-y} + \Lambda_{13} e^{-Pr y} + \Lambda_{14} e^{-\delta y} + \Lambda_{15} e^{-2Pr y} \\ + \Lambda_{16} e^{-2\delta y} + \Lambda_{17} e^{-2Sc y} + \Lambda_{18} e^{-(Pr+\delta)y} + \quad (30)$$

$$\Lambda_{19} e^{-(Pr+Sc)y} + \Lambda_{20} e^{-(Sc+\delta)y} + \Lambda_{21} e^{-(Pr+Pr_m)y} \\ + \Lambda_{22} e^{-2(\delta+Pr_m)y} + \Lambda_{23} e^{-2Pr_m y} + \Lambda_{24} e^{-(Pr+Pr_m)y} \\ H_{x_1}(y) = \Delta e^{-\delta y} + \Gamma_{21} e^{-Pr y} + \Gamma_{22} e^{-2\delta y} + \Gamma_{23} e^{-2Pr y} \\ + \Gamma_{24} e^{-2Sc y} + \Gamma_{25} e^{-(Pr+\delta)y} + \Gamma_{26} e^{-(Pr+Sc)y} \quad (31) \\ + \Gamma_{27} e^{-(Sc+\delta)y} + \Gamma_{28} e^{-(Pr+Pr_m)y} + \Gamma_{29} e^{-(\delta+Pr_m)y} \\ + \Gamma_{30} e^{-2Pr_m y} + \Gamma_{31} e^{-(Sc+Pr_m)y}$$

These analytical approximate solutions under perturbation technique give a wider applicability in understanding the basic physics and chemistry of the problem, which are particularly important in industrial and technological fields.

IV. RATE OF HEAT TRANSFER

The Nusselt number is often used to determine heat transfer. The non-dimensional heat flux at the plate $y=0$ in terms of Nusselt number Nu is given by

$$Nu = \left(\frac{d\theta}{dy} \right)_{y=0} = Nu_0 + Ec Nu_1 \quad (32)$$

where $Nu_0 = \theta_0'(0) = -1$ and

$$Nu_1 = \theta_1'(0) = -\{Pr \Gamma + 2\delta \Gamma_{11} + 2Pr \Gamma_{12} \\ + 2Sc \Gamma_{13} + (Pr+\delta)\Gamma_{14} + (Pr+Sc)\Gamma_{15} \\ + (Sc+\delta)\Gamma_{16} + (Pr+Pr_m)\Gamma_{17} \\ + (\delta+Pr_m)\Gamma_{18} + 2Pr_m \Gamma_{19} + (Sc+Pr_m)\Gamma_{20}\}$$

V. CURRENT DENSITY

The non-dimensional current density at the plate $y=0$ is given by

$$J = \left(\frac{dH_x}{dy} \right)_{y=0} = J_0 + Ec J_1 \quad (33)$$

where

$$J_0 = H_{x_0}'(0) = -(\Psi Pr_m + \delta \Omega_3 + Pr \Omega_4 + Sc \Omega_5) \\ J_1 = H_{x_1}'(0) = -\{\delta \Delta + Pr \Gamma_{21} + 2\delta \Gamma_{22} + 2Pr \Gamma_{23} \\ + 2Sc \Gamma_{34} + (Pr+\delta)\Gamma_{25} + (Pr+Sc)\Gamma_{26} \\ + (Sc+\delta)\Gamma_{27} + (Pr+Pr_m)\Gamma_{28} + (\delta+Pr_m)\Gamma_{29} \\ + 2Pr_m \Gamma_{30} + (Sc+Pr_m)\Gamma_{31}\}$$

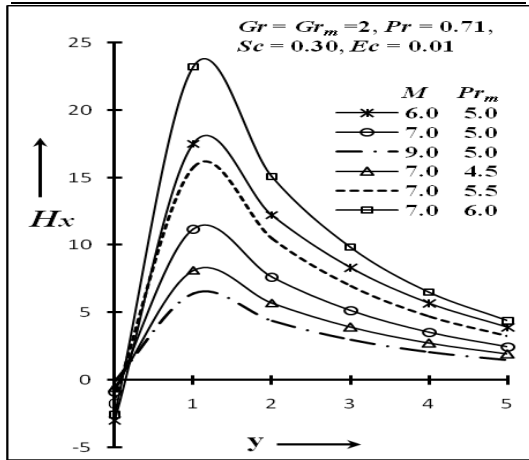
VI. VALIDATION

The method is validated by directly comparing its results with those of Raptis *et al.* (2003) without radiation, for $Pr_m=3$, $M=4$, $Ec=0=Gr_m$ as shown in Table 1, where $Pr_m=3>1$ which implies that viscous diffusion rate is dominant over the magnetic diffusion rate. From Table 1, it is seen that the results for flow velocity are in excellent agreement. Therefore, this leads to confidence in the numerical results. In Table 1, it is observed that the flow velocity is accelerated when free convection parameter increases from 2.0 through 5.0 to 10.0.

Table 1: Comparison of values of the flow velocity (u) with those obtained by Raptis *et al.* (2003) for $Pr_m=3$, $M=4$, $Ec=0=Gr_m$:

Raptis <i>et al.</i> (2003) without radiation			
Y	Gr = 2	Gr = 5	Gr = 10
0.0	0.00000	0.00000	0.00000
2.0	5.93172	7.43029	9.75438
4.0	2.58730	3.58741	4.83512
6.0	1.37572	1.87310	2.31820
8.0	0.62753	0.07595	0.08375
10.0	0.01511	0.02531	0.03914

Present work			
	Gr = 2	Gr = 5	Gr = 10
0.0	0.00000	0.00000	0.00000
2.0	5.93189	7.43103	9.75452
4.0	2.58785	3.58773	4.83530
6.0	1.37632	1.87345	2.31843
8.0	0.62774	0.07610	0.08383
10.0	0.01547	0.02563	0.03935

**Fig. 2:** Induced magnetic field distribution with Hartmann number and Magnetic Prandtl number

VII. RESULTS AND DISCUSSION

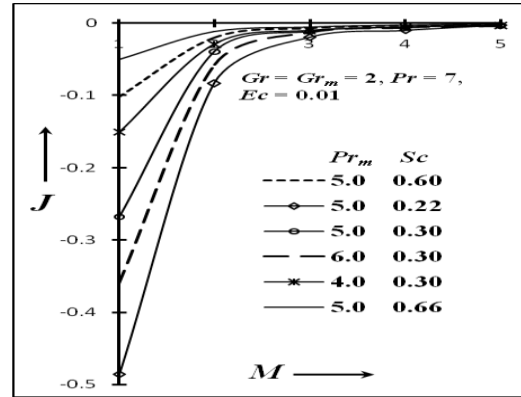
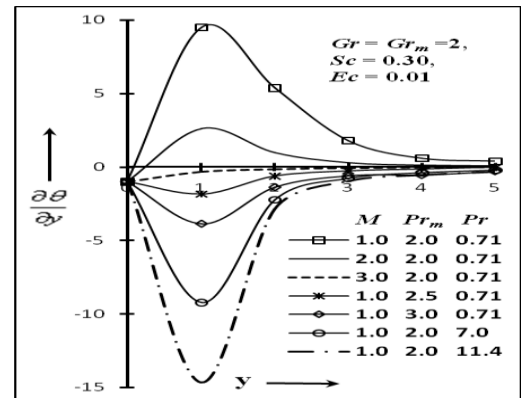
In order to get physical insight into the problem, we have carried out the numerical calculations for the velocity field, induced magnetic field, temperature gradient and current density at the plate and the values are demonstrated in graphs and Tables. The Prandtl number Pr is taken for air ($Pr=0.71$) at 20°C , water ($Pr=7.0$) and water at 4°C ($Pr=11.4$). The thermal Grashof number Gr and the Eckert number (Ec) take positive values corresponds to cooling of the plate by free convection currents. Also the values of the Schmidt number (Sc), so chosen to represent the presence of various species Hydrogen ($Sc=0.20$), Helium ($Sc=0.30$), Steam ($Sc=0.60$) and Oxygen ($Sc=0.66$).

Figure 2 illustrates the response of induced magnetic field to Hartmann number and magnetic Prandtl number for conducting air ($Pr = 0.71$) and diffusing species helium ($Sc = 0.30$). Increase in Hartmann number clearly reduces the induced magnetic field magnitudes, whereas increasing in Pr_m boosts the values of H_x through the boundary layer when $y > 1$. In this figure, Pr_m is set as greater than 1.0 and implies that the viscous diffusion rate exceeds the magnetic diffusion. In the vicinity of the plate, the values of H_x remains negative for all val-

ues of M and Pr_m , i.e., the induced magnetic flux reversal arises for $y \rightarrow 0$ into the boundary layer, and transverse to the plate. In all the cases, H_x peaks a short distance from the plate, and then decays to be zero in the free stream.

Figure 3 shows the distribution of current density at the plate for various magnetic Prandtl number and Schmidt number over Hartmann number. A rise in Pr_m from 4.0 through 5.0 to 6.0 serves to depress strongly current density magnitudes throughout the regime, but the current density rises with Schmidt number. These effects are very useful for small Hartmann numbers and insignificant in the free stream. It is also found that the current density has a large negative value throughout the flow field. Thus it is concluded that the magnetic Prandtl number or Schmidt number has a strong role on the current density.

In Fig. 4, the distribution of temperature gradient distribution ($\partial\theta/\partial y$) with Hartmann, magnetic Prandtl and Prandtl number, is presented for diffusing species helium. An increase in M , Pr_m and Pr leads to reduce sharply $\partial\theta/\partial y$ and these results are very much effective near the plate, but insignificant far away from the plate.

**Fig. 3:** Current density distribution with Magnetic Prandtl number and Schmidt number.**Fig. 4:** Temperature gradient distribution with Hartmann, Magnetic Prandtl and Prandtl number.

In Fig. 5, the influence of Hartmann number and magnetic Prandtl number on the flow velocity distributions is shown for conducting air and diffusing species helium. With an increase in M from 0.5 through 1.0 to 3.0, there is clear decrease in flow velocity i.e. flow is strongly decelerated. It is evident that the presence of

the transverse magnetic field produces a resistive force on the flow velocity. This force is called the **Lorentz force** acting on the flow velocity, which leads to slow down the motion of electrically conducting fluid. In no case however, there is a flow reversal i.e. velocity remains positive through the boundary layer. On the other hand, the flow velocity is accelerated when magnetic Prandtl number rises from 2.0 through 3.5 to 4.0.

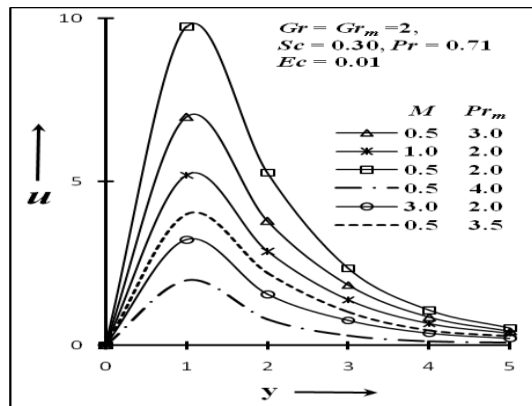


Fig. 5: Flow velocity distribution with Magnetic Prandtl number and Schmidt number

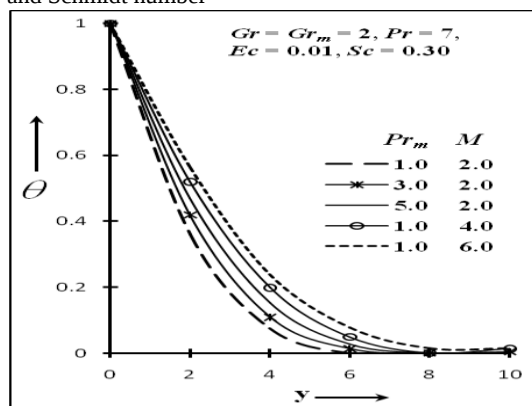


Fig. 6: Temperature distribution with Magnetic Prandtl number and Hartmann number

The effects of Hartmann number M , and magnetic Prandtl number Pr_m on the fluid temperature for the mercury flow are illustrated in Fig. 6. The trend is very consistent for all M (or Pr_m) i.e. a monotonic decay from a maximum value ($\theta = 1$) at the wall ($y=0$) to the free stream. Temperatures are strongly enhanced with increasing transverse magnetic field (or Magnetic Prandtl number). The presence of **transverse magnetic field** produces a resistive force on the fluid flow. This force is called the **Lorentz force**, which leads to slow down of the motion of electrically conducting fluid and therefore, this leads to decrease in the thickness of boundary layer and hence to increase in the temperature of fluid flow.

The response of induced magnetic field (H_x) to viscous dissipative heat (Ec) as well as Schmidt number (Sc) is shown in Table 2 for $Gr=2$, $Gr_m=2$, $Pr=0.71$, $Pr_m=5$ and $M=6$. Here magnetic Prandtl number, Pr_m , is set as 5 and implies the ratio of momentum diffusivity (viscosity) is five times the magnetic diffusivity i.e. the

viscous diffusion rate is five times the magnetic diffusion rate. Increasing Eckert number clearly increases the induced magnetic field magnitudes when $y>1$, but this trend is reversed in the vicinity of the wall. Also it is found that, the induced magnetic field decrease strongly with an increase in Schmidt number for $y>1$. Also for different values of Ec and Sc , the induced magnetic flux reversal arises for the region $0<y<1$ due to the negative values of H_x .

Table 2: Induced Magnetic distribution for different values of Sc and Ec over y , for $Gr=2=Gr_m$, $Pr=0.71$, $Pr_m=5$ and $M=6$.

y	$Sc = 0.22$	$Sc = 0.60$	$Sc = 0.66$
0	-1.8122	-0.3437	-0.2341
1	29.9975	1.4738	-0.6493
2	20.1514	0.8814	-0.2569
3	13.7878	0.4850	-0.1165
4	9.7179	0.2675	-0.0500
5	7.0248	0.1478	-0.0248

y	$Ec = 0.05$	$Ec = 0.06$	$Ec = 0.07$
0	-4.4541	-5.3450	-6.2358
1	23.7062	26.8368	29.9674
2	15.0347	16.8893	18.7440
3	9.4739	10.5562	11.6385
4	6.0377	6.1637	7.2898
5	3.9015	4.2611	4.6206

Table 3: Current density distribution for different values of Ec and Pr over M , for $Gr=2=Gr_m$, $Pr=0.71$, $Pr_m=5$ and $Sc=0.30$

M	$Ec = 0.04$	$Ec = 0.05$	$Ec = 0.07$
0	-6.5438	-8.1782	-11.4507
1	-0.6451	-0.8063	-1.1294
2	-0.1877	-0.2344	-0.3286
3	-0.0820	-0.1020	-0.1433
4	-0.0447	-0.0551	-0.0773
5	-0.0271	-0.0347	-0.0471

M	$Pr = 0.71$	$Pr = 7.0$	$Pr = 11.4$
0	-15.0734	-1.6362	-1.5352
1	-1.6511	-0.1617	-0.1517
2	-0.4302	-0.0471	-0.0449
3	-0.1647	-0.0209	-0.0193
4	-0.0755	-0.0119	-0.0103
5	-0.0408	-0.0074	-0.0061

The numerical values of current density for Eckert number and Prandtl number over Hartmann number are presented in Table 3 for $Gr=2$, $Gr_m=5$, $Pr=0.71$, $Pr_m=5$ and $Sc=0.30$. An increase in viscous dissipative heat reduces the current density, whereas current density rises with the increase of Prandtl number. It is observed that, when Hartmann number, $M \rightarrow 1$, maximum effects on current density occurred, but these effects are insignificant for large values of Hartmann number.

VIII. CONCLUSIONS

In this paper, it was demonstrated that the perturbation technique can be extended and applied to obtain a closed form expression for the problem of a steady mixed convective MHD flow of an incompressible viscous electrically conducting fluid past an infinite vertical porous plate in presence of transverse magnetic field with combined heat and mass transfer, taking into account the induced magnetic field with viscous and magnetic dissipations of energy. The present analysis brings

out the following results of physical parameters of the flow field:

- A velocity overshoot is observed with lower values of magnetic field effects but vanishes for very high values of the Hartmann number in the flow regime;
- Increasing magnetic Prandtl number (Pr_m) or magnetic field (M) serves to decelerates the flow velocity significantly for the case of conducting air.
- With an increase in Pr_m , the induced magnetic field along x-direction is progressively accelerated, but the magnetic field depress the H_x .
- Increasing Schmidt number (Sc) serves to escalate the Current density at the plate, but the current density decreases for the effect of Pr_m .
- The temperature component (θ) increases with an increase of Magnetic field or Magnetic Prandtl number.
- The induced magnetic flux reversal arises for both the effects of Ec and Sc .
- All the temperature gradients ($\partial\theta/\partial y$) have maximum values in the neighbourhood of $y=1$ comparison to the other values of distance and it is observed to decrease $\partial\theta/\partial y$ for M or Pr_m or Pr .

The study has important applications in nuclear heat transfer control, and MHD energy generators. Currently the authors are exploring extensions of this work to study rheological (non-Newtonian) working fluid effects, the results of which will be reported in the near future.

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NOMENCLATURE

H_0	Uniform magnetic field
H_x	Induced magnetic field along x-direction
$\bar{C}$	Species concentration (Kg. m ⁻³)
C_p	Specific heat at constant pressure (J. kg ⁻¹ . K)
$\bar{C}_\infty$	Species concentration in the free stream (Kg. m ⁻³)
$\bar{C}_w$	Species concentration at the surface (Kg. m ⁻³)
D	Chemical molecular diffusivity (m ² .s ⁻¹)
Ec	Eckert number/dissipative heat
g	Acceleration due to gravity (m.s ⁻²)
Gr	Thermal Grashof number
Gr_m	Mass Grashof number
M	Hartmann number/Magnetic parameter
$\bar{m}$	Mass flux per unit area
Nu	Nusselt number
$\bar{p}$	Pressure (Pa)
Pr_m	Magnetic Prandtl number
Pr	Prandtl number
$\bar{q}$	Heat flux per unit area
Sc	Schmidt number
$\bar{T}$	Dimensional Temperature
$\bar{T}_w$	Dimensional Fluid temperature at the surface
$\bar{T}_\infty$	Dimensional Fluid temperature in the free stream

u	Dimensionless velocity component in x-direction (m. s ⁻¹)
v_0	Dimensionless suction velocity (m. s ⁻¹)
J	Current density

Greek symbols

β	Coefficient of volume expansion for heat transfer (K ⁻¹),
$\bar{\beta}$	Coefficient of volume expansion for mass transfer (K ⁻¹)
η	Magnetic diffusivity
θ	Dimensionless fluid temperature (K),
λ	Thermal conductivity (W. m ⁻¹ . K ⁻¹),
μ	Coefficient of viscosity (kg. m ⁻³)
ν	Kinematic viscosity (m ² .s ⁻¹),
ρ	Density (kg. m ⁻³),
σ	Electrical conductivity
τ	Shearing stress (N. m ⁻²)
ϕ	Dimensionless species concentration (Kg. m ⁻³)

Subscripts

w	Conditions on the wall
∞	Free stream conditions

APPENDIX

$$\delta = \frac{1 + Pr_m + \sqrt{(1 - Pr_m)^2 + 4M}}{2},$$

$$\Omega_1 = \frac{Gr(1 - Pr_m / Pr)}{-Pr^3 + (1 + Pr_m)Pr^2 + (M - Pr_m)Pr},$$

$$\Omega_2 = \frac{Gr_m(Sc - Pr_m)}{-Sc^3 + (1 + Pr_m)Sc^2 + (M - Pr_m)Sc},$$

$$\Omega = -(\Omega_1 + \Omega_2), \quad \Omega_3 = \frac{\Omega Pr_m}{\delta - Pr_m}, \quad \Omega_4 = \frac{\Omega_1 Pr_m}{Pr_m - Pr},$$

$$\Omega_5 = \frac{\Omega_2 Pr_m}{Sc - Pr_m}, \quad \Psi = -(\Omega_3 + \Omega_4 + \Omega_5),$$

$$\Gamma_1 = -Pr\delta^2\Omega^2 - \frac{M Pr \delta^2 \Omega_3^2}{Pr_m^2},$$

$$\Gamma_2 = -Pr^3\Omega_1^2 - \frac{M Pr^2 \delta \Omega_4^2}{Pr_m^2},$$

$$\Gamma_3 = -Pr Sc^2\Omega_2^2 - \frac{M Pr Sc^2 \Omega_5^2}{Pr_m^2},$$

$$\Gamma_4 = -2Pr^2\delta\Omega\Omega_1 - \frac{2M Pr^2 \delta \Omega_3 \Omega_4}{Pr_m^2},$$

$$\Gamma_5 = -2Pr^3Sc\Omega_1\Omega_2 - \frac{2M Sc Pr^2 \Omega_4 \Omega_5}{Pr_m^2},$$

$$\Gamma_6 = -2ScPr\delta\Omega\Omega_2 - \frac{2M Pr Sc \delta \Omega_3 \Omega_5}{Pr_m^2},$$

$$\Gamma_7 = -\frac{2M Pr^2 \Psi \Omega_4}{Pr_m}, \quad \Gamma_8 = -\frac{2M Pr \delta \Psi \Omega_3}{Pr_m},$$

$$\begin{aligned}
\Gamma_9 &= -M Pr \Psi^2, \quad \Gamma_{10} = -\frac{2M Pr Sc \Psi \Omega_5}{Pr_m}, \\
\Gamma_{11} &= -\frac{\Gamma_1}{2\delta(2\delta - Pr)}, \quad \Gamma_{12} = \frac{\Gamma_2}{2Pr^2}, \\
\Gamma_{13} &= -\frac{\Gamma_3}{2Sc(2Sc - Pr)}, \quad \Gamma_{14} = -\frac{\Gamma_4}{(Pr + \delta)(\delta - Pr)}, \\
\Gamma_{15} &= -\frac{\Gamma_5}{(Pr + Sc)(Sc - Pr)}, \quad \Gamma_{16} = -\frac{\Gamma_4}{(Sc + \delta)(\delta + Sc - 2Pr)}, \\
\Gamma_{17} &= -\frac{\Gamma_4}{(Pr + Pr_m)(Pr_m - Pr)}, \\
\Gamma_{18} &= -\frac{\Gamma_4}{(\delta + Pr_m)(\delta + Pr_m - 2Pr)}, \quad \Gamma_{19} = -\frac{\Gamma_4}{2Pr_m(2Pr_m - 1)}, \\
\Gamma &= -\frac{1}{Pr} Pr [2\delta \Gamma_{11} + 2Pr \Gamma_{12} + 2Sc \Gamma_{13} + (Pr + \delta)\Gamma_{14} \\
&\quad + (Pr + Sc)\Gamma_{15} + (Sc + \delta)\Gamma_{16} + (Pr + Pr_m)\Gamma_{17} + (\delta + Pr_m)\Gamma_{18}, \\
\Gamma_{20} &= -\frac{\Gamma_{10}}{(Sc + Pr_m)(Sc + Pr_m - 2Pr)}, \\
\Gamma_{21} &= -\frac{Gr \Gamma Pr}{-Pr^3 + (1 + Pr_m)Pr^2 + (M - Pr_m)Pr}, \\
\Gamma_{22} &= -\frac{Gr \Gamma_{11} Pr_m}{-8\delta^3 + 4(1 + Pr_m)\delta^2 + 2(M - Pr_m)\delta}, \\
\Gamma_{23} &= -\frac{Gr \Gamma_{12} Pr_m}{-8Pr^3 + 4(1 + Pr_m)Pr^2 + 2(M - Pr_m)Pr}, \\
\Gamma_{24} &= -\frac{Gr \Gamma_{13} Pr_m}{-8Sc^3 + 4(1 + Pr_m)Sc^2 + 2(M - Pr_m)Sc}, \\
\Gamma_{25} &= -\frac{Gr \Gamma_{14} Pr_m}{-(Pr + \delta)^3 + (1 + Pr_m)(Pr + \delta)^2 + (M - Pr_m)(Pr + \delta)}, \\
\Gamma_{26} &= -\frac{Gr \Gamma_{15} Pr_m}{-(Pr + Sc)^3 + (1 + Pr_m)(Pr + Sc)^2 + (M - Pr_m)(Pr + Sc)}, \\
\Gamma_{27} &= -\frac{Gr \Gamma_{16} Pr_m}{-(\delta + Sc)^3 + (1 + Pr_m)(\delta + Sc)^2 + (M - Pr_m)(\delta + Sc)}, \\
\Gamma_{28} &= -\frac{Gr \Gamma_{17} Pr_m}{-(Pr + Pr_m)^3 + (1 + Pr_m)(Pr + Pr_m)^2 + (M - Pr_m)(Pr + Pr_m)}, \\
\Gamma_{29} &= -\frac{Gr \Gamma_{18} Pr_m}{-(\delta + Pr_m)^3 + (1 + Pr_m)(\delta + Pr_m)^2 + (M - Pr_m)(\delta + Pr_m)}, \\
\Gamma_{30} &= -\frac{Gr \Gamma_{19} Pr_m}{-8Pr_m^3 + 4(1 + Pr_m)Pr_m^2 + 2(M - Pr_m)Pr_m}, \\
\Gamma_{31} &= -\frac{Gr \Gamma_{20} Pr_m}{-(Sc + Pr_m)^3 + (1 + Pr_m)(Sc + Pr_m)^2 + (M - Pr_m)(Sc + Pr_m)}, \\
\Lambda_1 &= -Gr \Gamma + \frac{M Pr \Gamma_{21}}{Pr_m}, \quad \Lambda_2 = \frac{M \delta \Delta}{Pr_m}, \\
\Lambda_3 &= -Gr \Gamma_{11} + \frac{2M \delta \Gamma_{22}}{Pr_m}, \\
\Lambda_5 &= -Gr \Gamma_{13} + \frac{2M Sc \Gamma_{24}}{Pr_m}, \\
\Lambda_6 &= -Gr \Gamma_{14} + \frac{M(Pr + \delta) \Gamma_{25}}{Pr_m}, \\
\Lambda_7 &= -Gr \Gamma_{15} + \frac{M(Pr + Sc) \Gamma_{26}}{Pr_m},
\end{aligned}$$

$$\begin{aligned}
\Lambda_8 &= -Gr \Gamma_{16} + \frac{M(\delta + Sc) \Gamma_{27}}{Pr_m}, \\
\Lambda_9 &= -Gr \Gamma_{17} + \frac{M(Pr + Pr_m) \Gamma_{28}}{Pr_m}, \\
\Lambda_{10} &= -Gr \Gamma_{18} + \frac{M(\delta + Pr_m) \Gamma_{29}}{Pr_m}, \\
\Lambda_{11} &= -Gr \Gamma_{19} + 2M \Gamma_{30}, \\
\Lambda_{12} &= -Gr \Gamma_{20} + \frac{M(Sc + Pr_m) \Gamma_{31}}{Pr_m}, \\
\Lambda_{13} &= \frac{\Lambda_1}{Pr(Pr - 1)}, \quad \Lambda_{14} = \frac{\Lambda_2}{\delta(\delta - 1)}, \\
\Lambda_{15} &= \frac{\Lambda_3}{2Pr(2Pr - 1)}, \\
\Lambda_{16} &= \frac{\Lambda_4}{2\delta(2\delta - 1)}, \quad \Lambda_{17} = \frac{\Lambda_5}{2Sc(2Sc - 1)}, \\
\Lambda_{18} &= \frac{\Lambda_6}{(Pr + \delta)(Pr + \delta - 1)}, \quad \Lambda_{19} = \frac{\Lambda_7}{(Pr + Sc)(Pr + Sc - 1)}, \\
\Lambda_{20} &= \frac{\Lambda_8}{(Sc + \delta)(Sc + \delta - 1)}, \quad \Lambda_{21} = \frac{\Lambda_9}{(Pr + Pr_m)(Pr + Pr_m - 1)}, \\
\Lambda_{22} &= \frac{\Lambda_{10}}{(\delta + Pr_m)(\delta + Pr_m - 1)}, \quad \Lambda_{23} = \frac{\Lambda_{11}}{2Pr_m(2Pr_m - 1)}, \\
\Lambda_{24} &= \frac{\Lambda_{12}}{(Sc + Pr_m)(Sc + Pr_m - 1)}, \\
\Lambda &= -(\Lambda_{13} + \Lambda_{14} + \Lambda_{15} + \Lambda_{16} + \Lambda_{17} + \Lambda_{18} \\
&\quad + \Lambda_{19} + \Lambda_{20} + \Lambda_{21} + \Lambda_{22} + \Lambda_{23} + \Lambda_{24}).
\end{aligned}$$

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