

MATHEMATICAL MODELING FOR POROUS MEDIA TRANSPORT IN NEWTONIAN RADIATING/CHEMICALLY REACTING FLUID OVER AN IMPULSIVELY-STARTED VERTICAL PLATE: A FINITE DIFFERENCE APPROACH

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Abstract— Theoretical investigation is presented here for the model of unsteady MHD thermal convection flow of a viscous incompressible absorbing-emitting optically thin gray gas along an impulsively-started semi-infinite vertical plate adjacent to the Darcian porous regime in the presence of a first order chemical reaction and significant thermal radiation effects. The conservation equations are nondimensionalized and are solved by an accurate and unconditionally stable implicit finite difference scheme of Crank-Nicholson type. The flow is found to be accelerated with increasing porosity parameter (K), whereas the temperature and concentration distributions are reduced in the Darcian regime. Flow velocity and Temperature are found to be depressed with progression of thermal radiation (R_a) contribution, but enhanced the concentration distribution. Applications of the model arise in solar energy collector analysis, magneto-fluid dynamics and industrial materials processing.

Keywords— Optically thick Gray Gas; Magneto-fluid dynamics; Thermal radiation; Solar energy collectors; Crank-Nicolson scheme; Darcian regime; Chemically Reacting fluid.

I. INTRODUCTION

Transport processes through porous media play important roles in diverse applications such as petroleum industries, chemical catalytic reactors and many others. Moreover, the combined forced and free convection flow (mixed convection flow) is encountered in several industrial and technical applications such as nuclear reactors cooled during emergency shutdown, electronics devices cooled by fans, heat exchangers placed in a low velocity environment, solar central receivers exposed to wind currents, etc. The heat transfer phenomenon can be described by means of the hydrodynamics, the convective heat transfer mechanism and the electromagnetic field as they have a symbiotic relationship (Ingham and Pop, 1998; Ingham and Pop, 2002; and Nield and Bejan, 2006). In many transport processes existing in nature and in industrial applications in which heat and mass transfer is a consequence of buoyancy effects caused by diffusion of heat and chemical species. The study of such processes is useful for improving a number of chemical technologies such as polymer production and food processing. In addition, chemical reactions can be classified as either heterogeneous or homogeneous processes. This depends on whether they occur at an inter-

face or as a single phase volume reaction. In well-mixed systems, the reaction is heterogeneous if it takes place at an interface and homogeneous if it takes place in solution. In most cases of chemical reactions, the reaction rate depends on the concentration of the species itself. A reaction is said to be of first-order, if the rate of reaction is directly proportional to the concentration itself (Cussler, 1988). A few representative fields of interest in which combined heat and mass transfer along with chemical reaction plays an important role is the chemical process industries such as food processing and polymer production. For example, formation of smog is a first order homogeneous chemical reaction. Because of its importance and possible applications, combined heat and mass transfer problems with chemical reaction effect received a considerable amount of attention. Jaiswal and Soundalgekar (2001) obtained an approximate solution to the problem of an unsteady flow past an infinite vertical plate with constant suction and embedded in a porous medium with oscillating plate temperature. Chamkha (2003) investigated the chemical reaction effects on heat and mass transfer laminar boundary layer flow in the presence of heat generation/absorption effects. Muthucumaraswamy and Kulaivel (2003) presented an analytical solution to the problem of flow past an impulsively started infinite vertical plate in the presence of heat flux and variable mass diffusion, taking into account the presence of a homogeneous chemical reaction of first order. Ahmed (2008) investigated the effect of transverse periodic permeability oscillating with time on the heat transfer flow of a viscous incompressible fluid through a highly porous medium bounded by an infinite vertical porous plate, by means of series solution method. Ahmed (2010) studied the effects of transverse periodic permeability oscillating with time on the free convective heat transfer flow of a viscous incompressible fluid through a highly porous medium bounded by an infinite vertical porous plate subjected to a periodic suction velocity. The effects of radiation and chemical reaction on natural convection flows of a Newtonian fluid along a vertical surface embedded in a porous medium presented by Mahmud and Chamkha (2010). The study of heat and mass transfer on the free convective flow of a viscous incompressible fluid past an infinite vertical porous plate in presence of transverse sinusoidal suction velocity and a constant free stream velocity was presented by Ahmed (2009). Bég *et al.* (2009) have also derived

closed-form solutions for the velocity, temperature and magnetic induction functions in Rayleigh free convection hydromagnetic flow. Ahmed and Liu (2010) analyzed the effects of mixed convection and mass transfer of three-dimensional oscillatory flow of a viscous incompressible fluid past an infinite vertical porous plate in presence of transverse sinusoidal suction velocity oscillating with time and a constant free stream velocity.

In case of large temperature differences between the surface and the ambient causes, the radiation effect may become important in natural convection, as in many engineering processes. An analysis of the thermal radiation effects on stationary mixed convection from vertical surfaces in saturated porous media for both a hot and a cold surface has been presented by Bakier (2001). Hussain and Pop (2001) studied the radiation effects on stationary free convection of an optically thin dense fluid along an isothermal vertical surface embedded in a porous medium with highly porosity. Raptis and Perdakis (2003) studied the effects of thermal radiation on moving vertical plate in the presence of mass diffusion. The governing equations were solved by the Laplace transform technique. The unsteady flow through a highly porous medium in the presence of radiation was studied by Raptis and Perdakis (2004). Zueco (2007) used Network Simulation method to study the effects of viscous dissipation and radiation on unsteady MHD free convection flow past a vertical porous plate. The effects of radiation and viscous dissipation on the transient natural convection-radiation flow of viscous dissipation fluid along an infinite vertical surface embedded in a porous medium, by means of network simulation method, investigated by Zueco (2008). Loganathan *et al.* (2008) investigated first order chemical reaction on flow past an impulsively started semi-infinite vertical plate in the presence of thermal radiation by an implicit finite-difference scheme of the Crank-Nicolson type. Recently, Ahmed and Kalita (2013) presented the magnetohydrodynamic transient convective radiative heat transfer in an isotropic, homogenous porous regime adjacent to a hot vertical plate using the *Laplace transform technique*. Ahmed and Batin (2013) studied the problem of a steady magnetohydrodynamic free convective boundary layer flow over a porous vertical isothermal flat plate with constant suction where the effects of the induced magnetic field as well as viscous and magnetic dissipations of energy are considered.

In the present study we consider the effects of thermal radiation, porosity and chemical reaction on unsteady hydromagnetic free convection flow past an impulsively-started semi-infinite vertical plate embedded in a **Darcian** regime in the presence of first order chemical reaction and thermal radiation. The dimensionless governing boundary layer partial differential equations are solved by an efficient, accurate, extensively validated and unconditionally stable finite difference scheme of the *Crank-Nicolson type*. Such a study is of relevance to thermo-fluid simulation of inclined solar plate collectors.

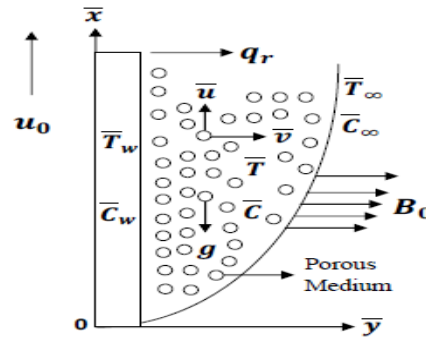


Figure 1: flow model

II. MATHEMATICAL ANALYSIS

A two-dimensional, transient, laminar, natural convection flow of a viscous incompressible fluid through a porous medium bounded by an impulsively-started semi-infinite vertical plate in the presence of conduction-radiation has been considered. It is assumed that a first-order chemical reaction between the diffusing species and the fluid exists. Magnetic Reynolds number is small enough to neglect induced hydromagnetic effects. Here, the x -axis is taken along the plate in the vertically upward direction and the y -axis is taken normal to the plate. Initially, it is assumed that the plate and the fluid are of the same temperature and the temperature of the plate and the concentration level are also raised to $\bar{T}_w = \bar{T}_\infty + ax^n$ and $\bar{C}_w = \bar{C}_\infty + bx^m$. They are maintained at the same level for all time $\bar{t} > 0$. All the fluid properties are assumed constant except the influence of the density variation with temperature and concentration are considered in the body force term.

Then under the above assumptions, the governing boundary layer equations for free convective MHD flow through porous medium with usual Boussinesq's approximation are as follows:

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0 \quad (1)$$

$$\frac{\partial \bar{u}}{\partial \bar{t}} + \bar{u} \frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{u}}{\partial \bar{y}} = g\beta(\bar{T} - \bar{T}_\infty) + g\beta(\bar{C} - \bar{C}_\infty) + \bar{v} \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} - \left(\frac{\sigma B_0^2}{\rho} + \frac{\bar{v}}{K} \right) \bar{u} \quad (2)$$

$$\frac{\partial \bar{T}}{\partial \bar{t}} + \bar{u} \frac{\partial \bar{T}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{T}}{\partial \bar{y}} = \frac{1}{\rho C_p} \left(\kappa \frac{\partial^2 \bar{T}}{\partial \bar{y}^2} - \frac{\partial q_r}{\partial \bar{y}} \right) \quad (3)$$

$$\frac{\partial \bar{C}}{\partial \bar{t}} + \bar{u} \frac{\partial \bar{C}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{C}}{\partial \bar{y}} = D \frac{\partial^2 \bar{C}}{\partial \bar{y}^2} - \bar{C}_r \bar{C} \quad (4)$$

The initial and boundary conditions are

$$\left. \begin{aligned} \bar{t} \leq 0: \bar{u} = 0, \bar{v} = 0, \bar{T} = \bar{T}_\infty, \bar{C} = \bar{C}_\infty \\ \bar{t} > 0: \bar{u} = u_0, \bar{v} = 0, \bar{T} = \bar{T}_\infty + ax^n, \\ \bar{C} = \bar{C}_\infty + bx^m \quad \text{at } \bar{y} = 0 \\ \bar{u} = 0, \bar{T} = \bar{T}_\infty, \bar{C} = \bar{C}_\infty \quad \text{at } \bar{x} = 0 \\ \bar{u} \rightarrow 0, \bar{T} \rightarrow \bar{T}_\infty, \bar{C} \rightarrow \bar{C}_\infty \quad \text{as } \bar{y} \rightarrow \infty \end{aligned} \right\} \quad (5)$$

For the case of an optically thin gray gas, the local radiant absorption is expressed by

$$\frac{\partial q_r}{\partial y} = -4\bar{a}\bar{\sigma}(\bar{T}_\infty^4 - \bar{T}^4) \quad (6)$$

We assume that the temperature differences within the flow are sufficiently small such that $\bar{T}^4$ may be expressed as a linear function of the temperature. This is accomplished by expanding $\bar{T}^4$ in a Taylor series about $\bar{T}_\infty$ and neglecting higher-order terms, thus

$$\bar{T}^4 \cong 4\bar{T}_\infty^3\bar{T} - 3\bar{T}_\infty^4. \quad (7)$$

By using Eqs. (6) and (7), Eq. (3) reduces to

$$\frac{\partial \bar{T}}{\partial t} + \bar{u} \frac{\partial \bar{T}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{T}}{\partial \bar{y}} = \frac{1}{\rho C_p} \left[\kappa \frac{\partial^2 \bar{T}}{\partial \bar{y}^2} - 16\bar{a}\bar{\sigma}\bar{T}_\infty^3(\bar{T} - \bar{T}_\infty) \right] \quad (8)$$

On introducing the following non-dimensional quantities:

$$\begin{aligned} x &= \frac{u_0 \bar{x}}{v}, y = \frac{u_0 \bar{y}}{v}, u = \frac{\bar{u}}{u_0}, v = \frac{\bar{v}}{u_0}, t = \frac{\bar{t} u_0^2}{v}, Sc = \frac{v}{D}, \\ \theta &= \frac{\bar{T} - \bar{T}_\infty}{\bar{T}_w - \bar{T}_\infty}, C_r = \frac{v \bar{C}_r}{u_0^2}, M = \frac{\sigma B_0^2 v}{\rho u_0^2}, \phi = \frac{\bar{C} - \bar{C}_\infty}{\bar{C}_w - \bar{C}_\infty}, Pr = \frac{\rho v C_p}{\kappa}, \\ K &= \frac{u_0^2 \bar{K}}{v^2}, Gr = \frac{v g \beta (\bar{T}_w - \bar{T}_\infty)}{u_0^3}, Gr_m = \frac{v g \beta (\bar{C}_w - \bar{C}_\infty)}{u_0^3}, \\ R_a &= \frac{64 \bar{a} v^2 \bar{\sigma} \bar{T}_\infty^3}{\kappa u_0^2} \end{aligned} \quad (9)$$

Equations (1) to (4) are reduced to the following non-dimensional form:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (10)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = Gr\theta + Gr_m\phi + \frac{\partial^2 u}{\partial y^2} - (M + K^{-1})u \quad (11)$$

$$\frac{\partial \theta}{\partial t} + u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} = \frac{1}{Pr} \left(\frac{\partial^2 \theta}{\partial y^2} - R_a \theta \right) \quad (12)$$

$$\frac{\partial \phi}{\partial t} + u \frac{\partial \phi}{\partial x} + v \frac{\partial \phi}{\partial y} = \frac{1}{Sc} \frac{\partial^2 \phi}{\partial y^2} - C_r \phi. \quad (13)$$

The corresponding initial and boundary conditions in non-dimensional form are

$$\begin{aligned} t \leq 0: u &= 0, v = 0, \theta = 0, \phi = 0 \\ t > 0: u &= 1, v = 0, \theta = x^n, \phi = x^m \quad \text{at } y = 0 \\ u &= 0, \theta = 0, \phi = 0 \quad \text{at } x = 0 \\ u &\rightarrow 0, \theta \rightarrow 0, \phi \rightarrow 0 \quad \text{as } y \rightarrow \infty \end{aligned} \quad (14)$$

The local skin friction (wall shear stress), local Nusselt number (surface heat transfer gradient) and the local Sherwood number (surface species transfer gradient) are given respectively by

$$\tau_x = -\mu \left(\frac{\partial \bar{u}}{\partial \bar{y}} \right)_{\bar{y}=0} = -\left(\frac{\partial u}{\partial y} \right)_{y=0} \quad (15)$$

$$Nu_x = -\frac{\bar{x}}{\bar{T}_w - \bar{T}_\infty} \left(\frac{\partial \bar{T}}{\partial \bar{y}} \right)_{\bar{y}=0} = -x \left(\frac{\partial \theta}{\partial y} \right)_{y=0} \quad (16)$$

$$Sh_x = -\frac{\bar{x}}{\bar{C}_w - \bar{C}_\infty} \left(\frac{\partial \bar{C}}{\partial \bar{y}} \right)_{\bar{y}=0} = -x \left(\frac{\partial \phi}{\partial y} \right)_{y=0} \quad (17)$$

III. METHOD OF SOLUTION

In order to solve the Eqs. (10) to (13) under the conditions (14), an implicit finite difference scheme of the *Crank-Nicolson* type has been employed. The finite difference equations corresponding to Eqs. (10) to (13) are as follows:

$$\begin{aligned} \frac{1}{4\Delta x} [u_{i,j}^{n+1} - u_{i-1,j}^{n+1} + u_{i,j}^n - u_{i-1,j}^n - u_{i-1,j-1}^{n+1} + u_{i,j-1}^n - u_{i-1,j-1}^n] + \\ \frac{[v_{i,j}^{n+1} - v_{i-1,j}^{n+1} + v_{i,j}^n - u_{i,j-1}^n]}{2\Delta y} = 0 \end{aligned} \quad (18)$$

$$\begin{aligned} \frac{[u_{i,j}^{n+1} - u_{i,j}^n]}{\Delta t} + u_{i,j}^n \frac{[u_{i,j}^{n+1} - u_{i-1,j}^{n+1} + u_{i,j}^n - u_{i-1,j}^n]}{2\Delta x} + \\ v_{i,j}^n \frac{[u_{i,j+1}^{n+1} - u_{i,j-1}^{n+1} + u_{i,j+1}^n - u_{i,j-1}^n]}{4\Delta y} = \frac{Gr[\theta_{i,j}^{n+1} + \theta_{i,j}^n]}{2} + \\ \frac{Gr_m[\phi_{i,j}^{n+1} + \phi_{i,j}^n]}{2} + \frac{1}{2(\Delta y)^2} [u_{i,j-1}^{n+1} - 2u_{i,j}^{n+1} + u_{i,j+1}^{n+1} + u_{i,j-1}^{n+1} \\ - 2u_{i,j}^n + u_{i,j+1}^n] - \frac{[u_{i,j}^{n+1} + u_{i,j}^n]}{2} (M + K^{-1}) \end{aligned} \quad (19)$$

$$\begin{aligned} \frac{[\theta_{i,j}^{n+1} - \theta_{i,j}^n]}{\Delta t} + u_{i,j}^n \frac{[\theta_{i,j}^{n+1} - \theta_{i-1,j}^{n+1} + \theta_{i,j}^n - \theta_{i-1,j}^n]}{2\Delta x} + \\ v_{i,j}^n \frac{[\theta_{i,j+1}^{n+1} - \theta_{i,j-1}^{n+1} + \theta_{i,j+1}^n - \theta_{i,j-1}^n]}{4\Delta y} = -\frac{R_a[\theta_{i,j}^{n+1} + \theta_{i,j}^n]}{2Pr} \\ + \frac{[\theta_{i,j-1}^{n+1} - 2\theta_{i,j}^{n+1} + \theta_{i,j+1}^{n+1} + \theta_{i,j-1}^n - 2\theta_{i,j}^n + \theta_{i,j+1}^n]}{2Pr(\Delta y)^2} \end{aligned} \quad (20)$$

$$\begin{aligned} \frac{[\phi_{i,j}^{n+1} - \phi_{i,j}^n]}{\Delta t} + u_{i,j}^n \frac{[\phi_{i,j}^{n+1} - \phi_{i-1,j}^{n+1} + \phi_{i,j}^n - \phi_{i-1,j}^n]}{2\Delta x} + \\ v_{i,j}^n \frac{[\phi_{i,j+1}^{n+1} - \phi_{i,j-1}^{n+1} + \phi_{i,j+1}^n - \phi_{i,j-1}^n]}{4\Delta y} = -\frac{C_r[\phi_{i,j}^{n+1} + \phi_{i,j}^n]}{2Sc} \\ + \frac{[\phi_{i,j-1}^{n+1} - 2\phi_{i,j}^{n+1} + \phi_{i,j+1}^{n+1} + \phi_{i,j-1}^n - 2\phi_{i,j}^n + \phi_{i,j+1}^n]}{2Sc(\Delta y)^2} \end{aligned} \quad (21)$$

The region of integration is considered as a rectangle with sides $x_{max}(=1)$ and $y_{max}(=14)$, where y_{max} corresponds to $y=\infty$ which lies very well outside both the momentum and energy boundary layers. The maximum of y was chosen as 14 after some preliminary investigations so that the last two of the boundary conditions (14) are satisfied within the tolerance limit 10^{-5} . After experimenting with a few set of mesh sizes, the mesh sizes have been fixed at the level $\Delta x=0.05$, $\Delta y=0.25$ with time step $\Delta t=0.01$. In this case, the spatial mesh sizes are reduced by 50% in one direction, and later in both directions, and the results are compared. It is observed that, when the mesh size is reduced by 50% in the y -direction, the results differ in the fifth decimal place while the mesh sizes are reduced by 50% in x -direction

or in both directions; the results are comparable to three decimal places.

Hence, the above mesh sizes have been considered as appropriate for calculation. The coefficients $u_{i,j}^n$ and $v_{i,j}^n$ appearing in the finite-difference equations are treated as constants in any one time step. Here i designates the grid point along the x -direction, j along the y -direction. The values of u , v and θ are known at all grid points at $t=0$ from the initial conditions.

The computations of u , v , θ and ϕ at time level $(n+1)$ using the values at previous time level (n) are carried out as follows: The finite difference Eq. (21) at every internal nodal point on a particular i -level constitutes a tridiagonal system of equations. Such systems of equations are solved by using Thomas algorithm as discussed in Carnahan *et al.* (1969). Thus, the values of ϕ are found at every nodal point for a particular i at $(n+1)$ th time level. Similarly, the values of θ are calculated from Eq. (20). Using the values of ϕ and θ at $(n+1)$ th time level in the Eq. (19), the values of u at $(n+1)$ th time level are found in a similar manner. Thus, the values of ϕ , θ and u are known on a particular i -level. Finally, the values of v are calculated explicitly using the Eq. (18) at every nodal point on a particular i -level at $(n+1)$ th time level. This process is repeated for various i -level. Thus the values of ϕ , θ , u and v are known, at all grid points in the rectangular region at $(n+1)$ th time level.

Computations are repeated until the steady-state is reached. The steady state solution is assumed to have been reached, when the absolute difference between the values of u , as well as temperature θ and concentration ϕ at two consecutive time steps are less than 10^{-5} at all grid points.

IV. STABILITY ANALYSIS

The stability criterion of the finite difference scheme for constant mesh sizes are examined using Von-Neumann technique as explained by Carnahan *et al.* (1969). The general term of the *Fourier expansion* for u , θ and ϕ at a time arbitrarily called $t=0$, are assumed to be of the form $\exp(i\alpha x)\exp(i\beta y)$ (here $i = \sqrt{-1}$). At a later time t , these terms will become,

$$\left. \begin{aligned} u &= F(t)\exp(i\alpha x)\exp(i\beta y) \\ \theta &= G(t)\exp(i\alpha x)\exp(i\beta y) \\ \phi &= H(t)\exp(i\alpha x)\exp(i\beta y) \end{aligned} \right\} \quad (22)$$

Substituting Eq. (22) in Eqs. (19) to (21) under the assumption that the coefficients u , θ and ϕ are constants over any one time step and denoting the values after one time step by F' , G' and H' . After simplification, we get

$$\frac{F'-F}{\Delta t} + \frac{u(F'-F)[1-\exp(-i\alpha\Delta x)]}{2\Delta x} + \frac{v(F'+G')i\sin(\beta\Delta y)}{2\Delta y} = \frac{(G'+G)Gr + (H'+H)Gr_m}{2} +$$

$$(F'+F)\left[\frac{\{\cos(\beta\Delta y)-1\}}{(\Delta y)^2} - \frac{(M+K^{-1})}{2}\right] \quad (23)$$

$$\frac{G'-G}{\Delta t} + \frac{u(G'-G)[1-\exp(-i\alpha\Delta x)]}{2\Delta x} + \frac{v(G'+G)i\sin(\beta\Delta y)}{2\Delta y} = \frac{(G'+G)[\cos(\beta\Delta y)-1]}{Pr(\Delta y)^2} - \frac{R_o(G'+G)}{2Pr} \quad (24)$$

$$\frac{H'-H}{\Delta t} + \frac{u(H'-H)[1-\exp(-i\alpha\Delta x)]}{2\Delta x} + \frac{v(H'+H)i\sin(\beta\Delta y)}{2\Delta y} = \frac{(H'+H)[\cos(\beta\Delta y)-1]}{Sc(\Delta y)^2} - \frac{C_r(H'+H)}{2Pr} \quad (25)$$

Equations (23) to (25) can be rewritten as,

$$(1+A)F' = (1-A)F + \frac{\Delta t}{2}[Gr(G'+G) + Gr_m(H'+H)] \quad (26)$$

$$(1+B)G' = (1-B)G \quad (27)$$

$$(1+E)H' = (1-E)H \quad (28)$$

where

$$A = \frac{u}{2} \frac{\Delta t}{\Delta x} [1 - \exp(-i\alpha\Delta x)] + \frac{v}{2} \frac{\Delta t}{\Delta y} i\sin(\beta\Delta y) - [\cos(\beta\Delta y) - 1] \frac{\Delta t}{(\Delta y)^2} + \frac{(M+K^{-1})\Delta t}{2}$$

$$B = \frac{u}{2} \frac{\Delta t}{\Delta x} [1 - \exp(-i\alpha\Delta x)] + \frac{v}{2} \frac{\Delta t}{\Delta y} i\sin(\beta\Delta y) - [\cos(\beta\Delta y) - 1] \frac{\Delta t}{Pr(\Delta y)^2} + \frac{R_o\Delta t}{2Pr}$$

$$E = \frac{u}{2} \frac{\Delta t}{\Delta x} [1 - \exp(-i\alpha\Delta x)] + \frac{v}{2} \frac{\Delta t}{\Delta y} i\sin(\beta\Delta y) - [\cos(\beta\Delta y) - 1] \frac{\Delta t}{Sc(\Delta y)^2} + \frac{C_r\Delta t}{2Sc}$$

After eliminating G' and H' in Eq. (26) using Eqs. (27) and (28), the resultant equation is given by,

$$(1+A)F' = (1-A)F + G \frac{Gr\Delta t}{(1+B)} + H \frac{Gr_m\Delta t}{(1+E)} \quad (29)$$

Equations (27) to (29) can be written in matrix form as follows:

$$\begin{pmatrix} F' \\ G' \\ H' \end{pmatrix} = \begin{pmatrix} \frac{1-A}{1+A} & D_1 & D_2 \\ 0 & \frac{1-B}{1+B} & 0 \\ 0 & 0 & \frac{1-E}{1+E} \end{pmatrix} \begin{pmatrix} F \\ G \\ H \end{pmatrix} \quad (30)$$

where $D_1 = \frac{Gr\Delta t}{(1+A)(1+B)}$ and $D_2 = \frac{Gr_m\Delta t}{(1+A)(1+E)}$.

Now, for stability of the finite difference scheme, the modulus of each *Eigen* value of the amplification matrix does not exceed unity. Since the matrix Eq. (30) is triangular, the *Eigen* values are its diagonal elements. The *Eigen* values of the amplification matrix are $(1-A)/(1+A)$, $(1-B)/(1+B)$ and $(1-E)/(1+E)$.

Assuming that, u is everywhere non-negative and v is everywhere non-positive, we get

$$A = 2a_1 \sin^2\left(\frac{\alpha\Delta x}{2}\right) + 2c_1 \sin^2\left(\frac{\beta\Delta y}{2}\right) + i[a_1 \sin(\alpha\Delta x) - b_1 \sin(\beta\Delta y)] + \frac{(M + K^{-1})\Delta t}{2}.$$

$$a_1 = u\Delta t/(2\Delta x), \quad b_1 = v\Delta t/(2\Delta y), \quad c_1 = \Delta t/(\Delta y)^2.$$

Since the real part of A is greater than or equal to zero, $|(1-A)/(1+A)| \leq 1$ always. Similarly, $|(1-B)/(1+B)| \leq 1$ and $|(1-E)/(1+E)| \leq 1$. Hence, the finite difference scheme is unconditionally stable. The local truncation error is $O(\Delta t^2 + \Delta y^2 + \Delta x)$ and it tends to zero as Δt , Δx and Δy tend to zero. Hence, the scheme is compatible. Stability and compatibility ensures convergence.

V. ACCURACY:

In order to ascertain the accuracy of the numerical results, the results for the present study are compared with the available solution in the literature. The velocity profiles for $R_a=1.5$, $K=0.5$, $Sc=0.60$, $m=n=0.7$, $t=1.75$ and $Pr=0.71$ are compared with the available exact solution of Loganathan *et al.* (2008) in Table 1. It is clearly observed from this table that the present results are in good agreement with the available theoretical solution at lower time level. This favorable comparison lends confidence in the numerical results reported subsequently.

Table 1. velocity profile

Loganathan <i>et al.</i> (2008) for $Gr=Gr_m$			
y	0.0	5.0	10.0
0	1	1	1
2	1.472185	1.717228	1.874410
4	0.146033	0.193744	0.248211
6	0.028831	0.052325	0.057653
8	0.004672	0.012958	0.012920
10	0.000729	0.003152	0.002830
Present work for $Gr=Gr_m$			
y	0.0	5.0	10.0
0	1	1	1
2	1.471693	1.717183	1.874381
4	0.147580	0.193670	0.248146
6	0.028706	0.052321	0.057638
8	0.004661	0.012943	0.012905
10	0.000728	0.003147	0.002814

VI. RESULTS AND DISCUSSIONS:

Extensive computations have been performed throughout the discussions for $Gr_m=Gr=5$ (strong species and thermal buoyancy forces), $Sc=0.60$ approximately simulates lower molecular weight gases diffusing in air ($Pr=0.71$) and $m=n=1$ (temperature and concentration exponents) unless otherwise stated.

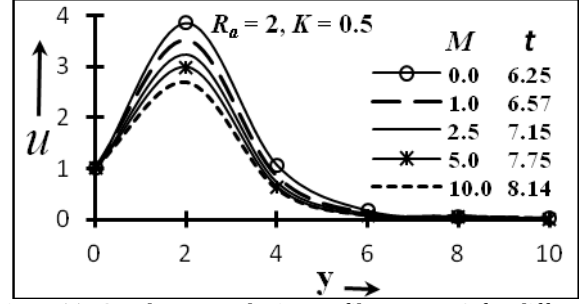


Fig. 1 (a): Steady state velocity profiles at $x=1.0$ for different M and t .

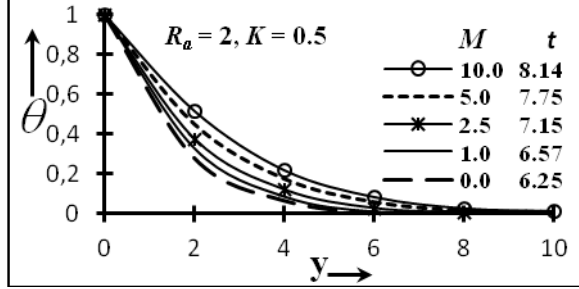


Fig. 1 (b): Steady state temperature profiles at $x=1.0$ for different M and t .

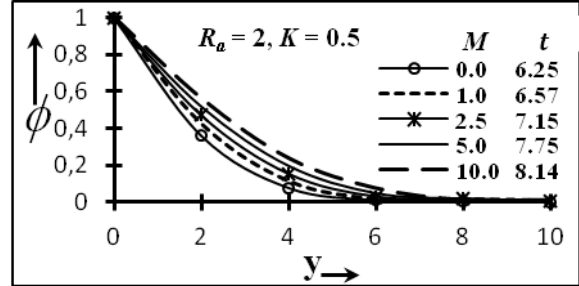


Fig. 1 (c): Steady state concentration profiles at $x=1.0$ for different M and t .

Figures 1a to 1c present the response of velocity (u), temperature (θ) and concentration (ϕ) to magnetohydrodynamic body force parameter (M). Here $M=(\sigma\nu B_0^2)/(\rho\nu_0^2)$ and it signifies the ratio of *Lorentz hydromagnetic body force* to *viscous hydrodynamic force*. Increasing M from 0 (non-conducting case), to 1.0 (magnetic body force and viscous force equal), 2.5, 5.0 through to 10.0 (very strong magnetic body force) induces a distinct reduction in velocities as shown in Fig. 1a. With higher M values since the magnetic body force, $(-Mu)$ in the momentum Eq. (11) is amplified, this serves to increasingly retard the flow. The imposition of an external magnetic field is therefore a powerful mechanism for inhibiting flow in the regime. The maximum velocities as before arise close to the surface of the plate, a *short distance* from it (at the surface, $y=0$); with further distance into the boundary layer, the profiles converge i.e. the magnetic body force has a weaker effect in the far field regime than in the near-field regime. Conversely with increasing M , temperature, (Fig. 1b) is observed to be markedly increased. This is physically explained by the fact that the extra work expended in dragging the fluid against the magnetic field is dissipated as thermal energy in the boundary layer, as elucidated by Sutton and Sherman (1965), Pai (1962) and

Hughes and Young (1966). This results in heating of the boundary layer and an ascent in temperatures, an effect which is maximized some distance away from the surface. The magnetic field influence on temperatures while noticeable is considerably less dramatic than that on the velocity field, since the *Lorentz body force* only arises in the momentum Eq. (11) and influences the temperature (θ) and concentration (ϕ) fields, only via the thermal and buoyancy terms. Magnetic effects do not feature in either the temperature (12) or species diffusion Eqs. (13). The *deceleration in flow* serves to enhance species diffusion in the regime and this causes a rise also in the concentration profiles (Fig. 1c) with increasing magnetic parameter. Applied external magnetic field, therefore while *counteracting* the momentum development in the regime, serves to *enhance* the heat and species diffusion, and this is of immense benefit in chemical engineering operations, where designers may wish to elevate transport in a regime *without* accelerating the flow.

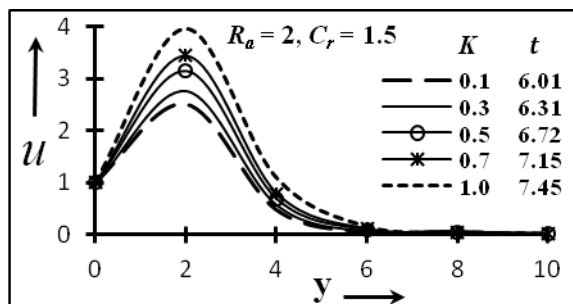


Fig. 2 (a): Steady state velocity profiles at $x=1.0$ for different K and t .

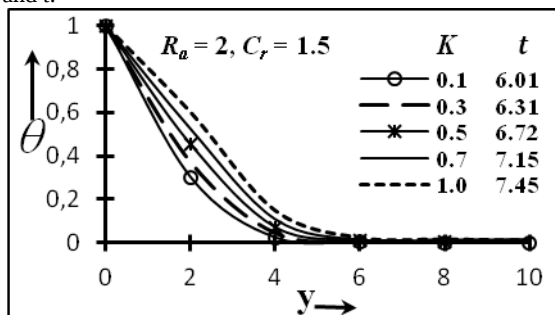


Fig. 2 (b): Steady state temperature profiles at $x=1.0$ for different K and t .

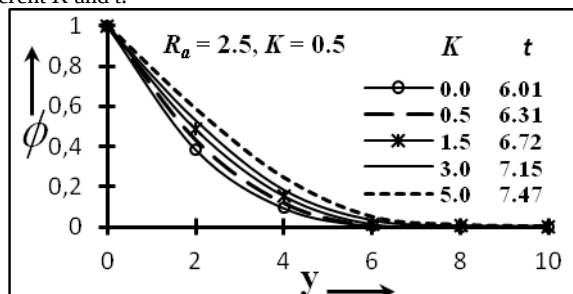


Fig. 2 (c): Steady state concentration profiles at $x=1.0$ for different K and t .

Figures 2a to 2c illustrate the influence of the porosity (K) on the boundary layer variables, u , θ and ϕ , respectively. Increasing the porosity of the porous medi-

um clearly serves to enhance the flow velocity (Fig. 2a) i.e. accelerates the flow. This effect is accentuated close to the surface where a peak in the velocity profile arises. With further distance transverse to the surface, the velocity profiles are all found to decay into the free stream. An increased porosity clearly corresponds to a reduced presence of matrix fibers in the flow regime which, therefore, provides a lower resistance to the flow and in turn, boosts the momentum. For increasing K values, the time t required to attain the steady-state scenario is also elevated considerably. As such, the steady state is achieved faster for higher values of K . Conversely, with increasing porosity values, the temperature profile (Fig. 2b) in the regime is found to be decreases i.e. the boundary layer is cooled. A reduction in the volume of solid particles in the medium implies a lower contribution via thermal conduction. This serves to decrease the fluid temperature. As with the velocity field (Fig. 2a), the time required to attain the steady-state condition decreases substantially with a positive increase in K . In Fig. 2c, a similar response for the concentration field is observed, as with the temperature distributions. The values of ϕ are continuously decreased with increasing values of porosity, also decreases with positive increases in the values of K , but reaches the steady state progressively faster.

Figures 3a to 3c present the effects of the chemical reaction parameter, C_r , on the velocity (u), temperature (θ) and concentration (ϕ) profiles, respectively. Physically, the mass diffusion Eq. (13) can be adjusted to represent a destructive chemical reaction (means endothermic, i.e., heat is absorbed) if $C_r > 0$ or a generative chemical reaction (means exothermic, i.e., heat is generated) if $C_r < 0$. Endothermic reactions cannot occur spontaneously, due to that a work must be done in order to get these reactions to occur. When endothermic reactions absorb energy, a temperature drop is measured during the reaction. Endothermic reactions are characterized by positive heat flow (into the reaction) and an increase in enthalpy. Exothermic reactions may occur spontaneously and result in higher randomness or entropy of the system. They are denoted by a negative heat flow (heat is lost to the surroundings) and decrease in enthalpy. There is a clear increase in the velocity values at the wall accompanying a rise in C_r from -1.5 through -1.0 , 0.0 , 1.0 to 1.5 , i.e. the flow is accelerated throughout the boundary layer. This shows that velocity increases during generative reaction and decreases in destructive reaction. Irrespective of the values of C_r or t , it is important to highlight that there is no flow reversal condition, i.e. no back flow in the boundary layer regime. The velocity u sustains positive values throughout the flow regime. With an increase in the value of C_r , the time taken to attain the steady-state condition does not follow a direct increase or decrease. For $C_r = -1.5$, $t = 10.51$ a value which decreases to 9.27 for $C_r = -1.0$ but then increases to 9.67 for $C_r = 0.0$ and then continues to increase to 10.71 for $C_r = 1.0$ and finally increases to 11.01 for $C_r = 1.5$. The steady-state condition is therefore,

achieved fastest for $C_r = -1.0$. Conversely, with an increase in the value of C_r , the temperature, θ , as shown in Fig. 3b, increases continuously through the boundary layer. Again the steady-state condition is attained fastest for $C_r = -1.0$ but slowest for $C_r = 1.5$. Figure 3c indicates that a rise in the value of C_r strongly suppresses the concentration levels in the boundary layer regime. All profiles decay monotonically from the surface (wall) to the free stream. The concentration boundary layer thickness is therefore, considerably greater for $C_r = -1.5$ than for $C_r = 1.5$.

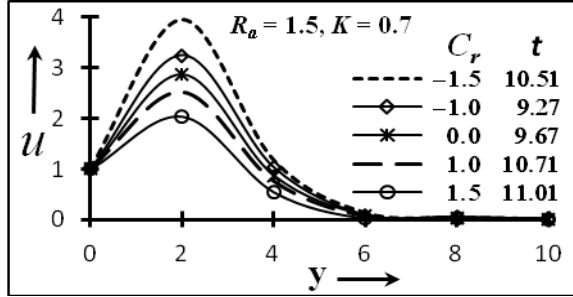


Fig. 3 (a): Steady state velocity profiles at $x=1.0$ for different C_r and t .

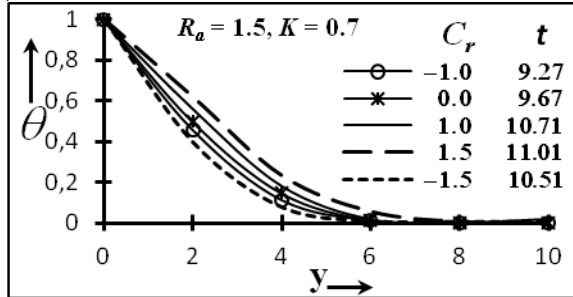


Fig. 3 (b): Steady state temperature profiles at $x=1.0$ for different C_r and t .

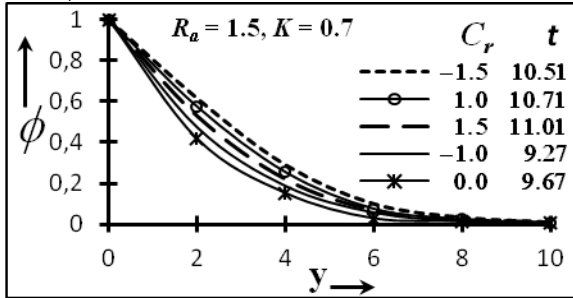


Fig. 3 (c): Steady state concentration profiles at $x=1.0$ for different C_r and t .

In Figures 4a to 4c, the influence of the radiation parameter, R_a on the steady-state velocity, temperature and concentration distributions with distance transverse to the surface (i.e. with y -coordinate) are presented, respectively. The parameter R_a defines the ratio of thermal conduction contribution relative to thermal radiation. For $R_a = 1$, the thermal radiation and the thermal conduction contributions are equivalent. For $R_a > 1$, the thermal radiation effect is dominant over the thermal conduction effect and vice versa for $R_a < 1$. An increase in the value of R_a from 0 (non-radiating) through 0.5 (thermal conduction is dominant over radiation), 1.0, 3.0, 5.0 to 8.0 (radiation is dominant over thermal conduction), causes

a significant decrease in the velocity with distance into the boundary layer i.e. decelerates the flow. The velocities in all cases ascend from the surface, peak close to the wall and then decay smoothly to zero in the free stream. It is also noted that with increasing values of the parameter R_a , the time taken to attain the steady state condition is reduced. Therefore, it is concluded that the thermal radiation flux has a de-stabilizing effect on the transient flow regime. This is important in polymeric and other industrial flow processes since it shows that the presence of thermal radiation while decreasing temperatures, will affect flow control from the surface into the boundary layer regime. As expected, the temperature values are also significantly reduced with an increase in the value of R_a as there is a progressive decrease in the thermal radiation contribution accompanying this. All profiles show monotonic decays from the wall to the free stream. The maximum reduction in temperatures is witnessed relatively close to the surface since thermal conduction effects are prominent closer to the surface, rather than further into the free stream. The concentration (ϕ) profile is conversely boosted with an increase in the value of R_a (i.e. decrease in thermal radiation contribution). The parameter R does not arise in the species conservation Eq. (13) and therefore, the concentration field is indirectly influenced by R_a via the coupling of the energy Eq. (12) with the momentum Eq. (11), the latter also being coupled with the convective acceleration terms in the species Eq. (13). However, as with the response of the velocity and temperature fields, an increase in the value of R_a decreases the time that elapses to achieve the steady-state condition. Therefore, while greater thermal radiation augments diffusion of species in the regime, it requires greater time to achieve the steady-state condition.

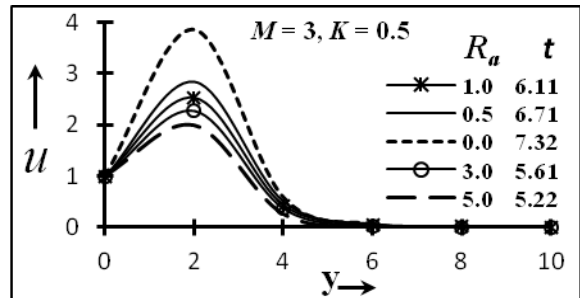


Fig. 4 (a): Steady state velocity profiles at $x=1.0$ for different R_a and t .

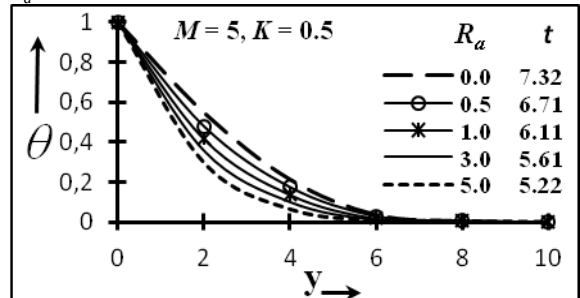


Fig. 4 (b): Steady state temperature profiles at $x=1.0$ for different R_a and t .

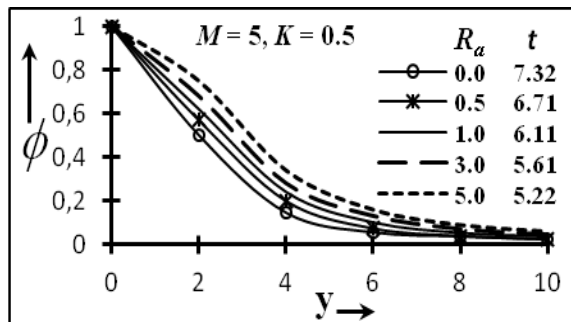


Fig. 4 (c): Steady state concentration profiles at $x=1.0$ for different R_a and t .

Figure 5 show the effects of porosity (K) and magnetic parameter (M) on shear stress function ($-u'$) plotted against time variable (t). Increasing the porosity parameter (K) clearly enhances the skin friction since with progressively decreasing solid matrix fibers to resist the flow, the fluid will be accelerated and shear faster past the surface of the wall. However, it is interesting to note that for each profile, the shear skin friction initially grows from the lower value of time (here $t=0.01$, where it vanishes owing to the no-slip condition), peaks at $t=0.05$ and thereafter descends in value when $t>0.06$ (which although not included in the graph scale, clearly indicates that profiles are decaying towards it). Therefore, the maximum shear stress arises for any porosity value at the time $t=0.05$ point on the surface of the wall. The converse response is seen for M , wherein a sharp decrease in skin friction is induced by increasing the magnetic parameter, M from 0 (electrically non-conducting case) through 1.0, to 5.0. Here $M=(\sigma\nu B_0^2)/(\rho u_0^2)$ and is directly proportional to the applied radial magnetic field, B_0 , for constant electrical conductivity (σ), fluid density (ρ), kinematic viscosity (ν) and plate velocity (u_0). Therefore greater retarding effect is generated in the flow with greater M values (i.e. stronger magnetic field strengths) which causes the prominent depression in skin friction. For $M=1$, the magnetic drag force will be of the same order of magnitude as the viscous hydrodynamic force. For $M>1$, hydromagnetic drag will dominate and vice versa for $M<1$. In magnetic materials processing, the flow can therefore be very effectively controlled with a magnetic field.

Figure 6 show the influence of porosity (K) and magnetic parameter (M) on surface heat transfer rate i.e. Nusselt number function ($-\theta'$) plotted against time variable (t). An increase in porosity clearly enhances ($-\theta'$) values which are consistent with the reduction of temperatures in the boundary layer regime, computed earlier. The negative sign for ($-\theta'$) indicates that heat is being conducted away from the boundary layer to the surface with increasing porosity and this causes a simultaneous decrease in temperatures in the boundary layer i.e. cools the regime. Peak Nusselt number function is located consistently at the plate (i.e. at the smaller time) and in all cases, profiles decay smoothly to a minimum value (i.e. at the greater time). A contrary response is computed for the effect of magnetic parameter, M , on

the Nusselt number function ($-\theta'$) distributions in same figure. Increasing magnetic field enhances the *Lorentzian hydromagnetic drag* which serves to decelerate the flow and warm the fluid. Therefore heat will be transferred at progressively lower rates from the fluid to the surface.

Finally in Fig. 7, the response of surface mass transfer rate i.e. Sherwood number function ($-\phi'$) to a change in porosity (K) and magnetic parameter (M), respectively, are shown. As with the Nusselt number function distributions, ($-\phi'$) is found to be strongly enhanced with increasing porosity, but reduced with an increase in magnetic parameter. Again, the peak surface mass transfer rates arise at the surface of the wall for smaller time and decrease continuously towards zero for $t>0.6$.

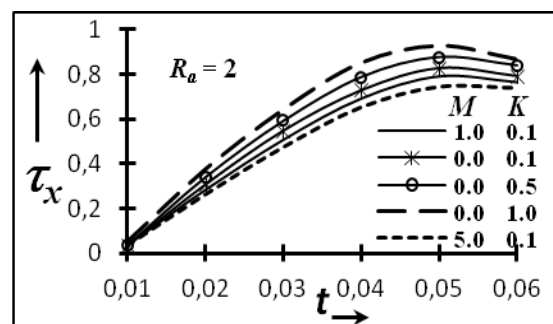


Fig. 5: Effects of M and K on local skin friction for different t .

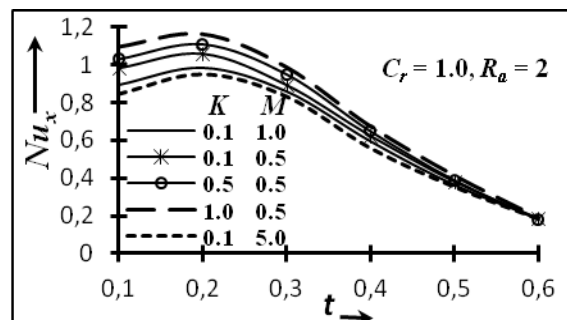


Fig. 6: Effects of K and M on local Nusselt number for different t .

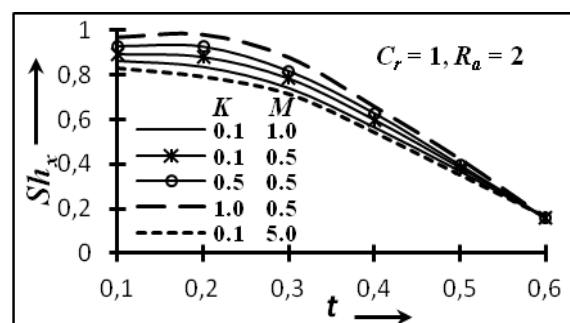


Fig. 7: Effects of K and M on local Sherwood number for different t .

VII. CONCLUSIONS

A numerical model has been presented for the radiative-convective transient MHD flow in a gray absorbing emitting fluid adjacent to an impulsively started vertical plate in a *Darcian* regime with variable temper-

ature and concentration in presence of homogeneous chemical reaction of first order. The dimensionless governing equations were solved by an *implicit finite-difference scheme* of the *Crank-Nicolson* type. Few significant outcomes are

1. Lorentz force opposes the motion of the fluid more effectively in the presence of Darcian porous regime. Presence of heavier diffusing species with destructive reaction reduces the velocity and concentration in a cooled plate. This is obvious due to absorption of heat energy both for destructive reaction and cooling of the plate.
2. The decrease of temperature may be attributed to the loss of heat energy due to radiation as well as low diffusion.
3. An increase in Lorentz force with the presence of porous matrix reduces the skin friction as well as the Nusselt and Sherwood numbers for cooling of the plate.

NOMENCLATURE

$\bar{C}$	Species concentration (Kg. m ⁻³)
C_p	Specific heat at constant pressure (J. kg ⁻¹ . K)
$\bar{C}_\infty$	Species concentration in the free stream (Kg. m ⁻³)
$\bar{C}_w$	Species concentration at the surface (Kg. m ⁻³)
D	Chemical molecular diffusivity (m ² .s ⁻¹)
g	Acceleration due to gravity (m.s ⁻²)
Gr	Thermal Grashof number
Gr_m	Mass Grashof number
M	Hartmann number
q_r	Radiation parameter
E_c	Eckert number
K	Permeability parameter
n	Surface temperature power law exponent
m	Surface concentration power law exponent
Nu_x	Nusselt number
Sh_x	Sherwood number
$\bar{p}$	Pressure (Pa)
Pr	Prandtl number
Sc	Schmidt number
$\bar{T}$	Temperature (K)
$\bar{T}_w$	Fluid temperature at the surface (K)
$\bar{T}_\infty$	Fluid temperature in the free stream (K)
u	Dimensionless velocity component in x-direction (m. s ⁻¹)
v	Dimensionless velocity component in y-direction (m. s ⁻¹)
u_0	Plate velocity (m. s ⁻¹)
$\bar{a}$	Absorption coefficient
$\bar{\sigma}$	Stefan-Boltzmann constant,

Greek symbols

β	Coefficient of volume expansion for heat transfer (K ⁻¹),
$\bar{\beta}$	Coefficient of volume expansion for mass transfer (K ⁻¹)
α	Thermal diffusivity
θ	Dimensionless fluid temperature (K),
κ	Thermal conductivity (W. m ⁻¹ . K ⁻¹),
μ	Coefficient of viscosity (kg. m ⁻³)
ν	Kinematic viscosity (m ² .s ⁻¹),
ρ	Density (kg. m ⁻³),
σ	Electrical conductivity (VA ⁻¹ m ⁻¹)
τ_x	Shearing stress (N. m ⁻²)
ϕ	Dimensionless species concentration (Kg.m ⁻³)

Subscripts

w	Conditions on the wall
∞	Free stream conditions

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