

OPTIMAL FEEDBACK LINEARIZATION CONTROL OF A FLEXIBLE CABLE ROBOT

M.H. KORAYEM, H. TOURAJIZADEH, M. TAHERIFAR and A.H. KORAYEM

Robotics Research Laboratory, Center of Excellence in Experimental Solid Mechanics and Dynamics,

School of Mechanical Engineering, Iran University of Science and Technology, Tehran, Iran

hkorayem@iust.ac.ir

Abstract— In this paper the flexible cable robot tracking is controlled using optimal feedback linearization method. Feedback linearization is used to control the robot within a predefined trajectory while its controlling gains are optimized using LQR method to achieve the maximum payload of the end-effector in presence of flexibilities. Required motors' torque and tracking error caused by flexibility uncertainties are calculated for a predefined trajectory of an under constrained cable robot with six Degrees of Freedom (DOF) and six actuating cables while its cables are considered elastic. Robust controller is also designed and added to the controller to ensure the accuracy and stability of the system and cancel any disturbing effects of the uncertainties. A series of analytic simulation study is done for the mentioned spatial cable robot to show the flexibility effect on dynamic performance of the robot and also prove the superiority of the proposed optimal control strategy to compensate these flexibilities. Finally the results are compared and verified with experimental results of the cable robot of ICaSbot to verify the proposed controlling strategy for controlling the mentioned flexible robot and also prove the correctness of the simulations.

Keywords— Cable Robot, Flexible Cables, Optimal Feedback Linearization Control

I. INTRODUCTION

Cable suspended robots provide lighter manipulators which can carry higher loads compared to their weight (Albus *et al.*, 1993). Considering parallel nature and non linear dynamics of this kind of robots and positive tension restriction of the cables, the control procedure of them is more challenging. On the other hand cable robots cannot always be considered as a rigid system (Williams and Gallina, 2001; Alp and Agrawal, 2002) while parametric uncertainties like elasticity at the joints and also cables can make considerable effect on its performance. In order to predict the behavior of the robots with flexible cables, it is highly significant to model and simulate these elasticities and design suitable controller to compensate their probable uncertainties.

Optimal control of rigid cable robots, is performed (Korayem *et al.*, 2012a) using open loop approach i.e. Hamilton-Jacobi-Bellman Method and in another research (Korayem and Tourajizadeh, 2011) using closed loop approach i.e. LQR. Korayem *et al.* (2010a) considered flexibility of the motors and dynamics of this flexibility is modeled. Then the optimal path of this flexible

joint cable robot is obtained for the open loop condition. Elasticity of the cables and controlling this elasticity has become one of the most challenging studies recently. Flexible dynamics of a cable robot is coupled with the dynamics of the end-effector (Zhang *et al.*, 2006). Also a workspace study of this kind of robots is conducted by Korayem *et al.* (2007). A different method to control a flexible cable robot is presented by Baicu *et al.* (1996) using active boundary control. State Dependent Riccati Eq. (SDRE) is used to optimal control of a flexible cable robot with variable length (Zhang *et al.*, 2005). Zhang (2004) has used H-infinity and delta flatness method to damp the vibrating response of the flexible cables. LQG is used to perform the optimization process here. Optimal force distribution is considered for a crane with flexible cables (Shiang *et al.*, 2000). Also optimal control of cable robot is studied in an open loop way using Iterative Linear Programming (ILP) which is not robust against flexibility uncertainties and its performance is not suitable since its controller is not closed loop (Korayem *et al.*, 2010b). Korayem *et al.* (2012b) implemented optimal sliding mode controller for cable robot, however, the uncertainties and flexibility of the cables were not considered in their study.

In this paper the control of closed loop rigid cable robot which acts based on optimal feedback linearization theory is extended for robots with flexible cable. Flexibilities are modeled and proper optimal controlling strategy is proposed to increase the accuracy and stability. Converting the system to linear states makes it possible to use LQR as the optimizer tool. While the system is supposed to be flexible, the states of the system increases and vibrating equation of the flexible parts needs to be coupled with the dynamics of the main system. Because of existence of flexibility, the system might be faced to parametric uncertainties since the exact value of the stiffness is not always exactly available. While these uncertainties can cause instability in the system, additive robust control is added to the system to cancel the effects of mentioned uncertainties or external disturbances. Thus, in this paper not only the flexibilities are modeled, but also their parametric uncertainties are compensated using robust feedback linearization. Feedback linearization is extremely applicable in experimental applications, since other possible nonlinear controllers have heavy calculation processes which are not suitable for online applications. By the aid of modeling the flexibilities and also using the proposed robust feedback linearization method, not only the large displacement of the end-effector can be controlled fast and in an

online way (which is suitable for online applications) but also its flexible uncertainties can be compensated using the proposed robust methodology.

Some analytic simulation studies are provided for an under constrained spatial cable robot with six DOFs and six supporting cables for which the cables are flexible. These analytic simulations help us to investigate the necessity of modeling the flexibility, its destructive effect on the performance of the robot and also verify the efficiency of the designed controller for improving the trajectory. Finally in the last section, simulation results of flexible cable robot are verified and analyzed by the aid of experimental tests which is conducted on the under constrained cable robot of Iran University of Science and Technology (IUST) called ICaSbot with six DOFs and six actuating cables for which the cables practically have flexible specifications. Not only correctness of the simulations and necessity of modeling the flexibility is proved by the aid of these comparisons, but also the efficiency of the proposed controlling strategy is verified. The flexibility characteristics of the ICaSbot are also estimated using a parameter identification study as a useful output of modeling the flexibility.

II. DYNAMIC MODELING OF FLEXIBLE CABLE

The dynamic modeling of rigid cable robot was done in previous works (Alp and Agrawal, 2002; Korayem *et al.*, 2012a; Korayem and Tourajizadeh, 2011). For modeling the cable robots with flexible cables, it is supposed that each cable has m modes of longitudinal vibration. The number of this parameter depends on the rate of required accuracy. Lateral vibrations are neglected since the cables should always be under enough tensional stress and so lateral vibration does not occur seriously. It is obvious that the number of system's DOFs increase to $6+6*m$. Supposing two vibrating modes for each cable, dynamic DOFs is:

$$DOF = \{x, y, z, \Psi, \Theta, \phi, q_{11}, q_{12}, q_{21}, q_{22}, q_{31}, q_{32}, q_{41}, q_{42}, q_{51}, q_{52}, q_{61}, q_{62}\} \quad (1)$$

where q_{im} is the m th vibrating mode of i th cable. Again the fourth order of flexible cables system is converted to two dynamic systems of order two. Here our goal is to calculate the tension of the cables at the beginning of the cable in where it is attached to the pulley. This tension is not equal to the tension of its end-side in where the cable is attached to the end-effector, since the cables are vibrating. So, free vibration equation of a cable is used to start the solution since the tension applied at the end of the cable is supposed as its boundary condition. Using Newton-Euler method we have (Zhang *et al.*, 2006):

$$E_i A_i \frac{\partial^2 w_i(Z_i, t)}{\partial Z_i^2} = \rho_i A_i \frac{\partial^2 w_i(Z_i, t)}{\partial t^2}; \quad c_i = \sqrt{\frac{E_i}{\rho_i}} \quad (2)$$

where t is time, Z is the coordinate of the cable's length, E is the Young modulus of elasticity of the cables, A is the area of cross section of the cable, ρ is the density of the cable, c_i is the wave propagation velocity of the cable and w_i is the vibrating displacement of each point of the cable i . Quasi steady state strategy is employed to

solve the mentioned vibration PDE with variable cable length since the speed of cables' elongation is considerably lower than the speed of cable vibration frequency. Thus the path and time is divided into a lot of segments (j segments) with little time interval so that each part follows the vibrating formula of Eq. (2) which is applicable only to linear continuous dynamic systems and also the length and boundary conditions can be supposed constant during each interval. Assumed mode method and separation of variables is used to solve this PDE by the aid of the following boundary and initial conditions: The tension at the end of the cable is considered as one of the boundary conditions while it is determined by the aid of inverse dynamics and the goal is to find the tension at the other end side of the cable. The second boundary condition is the zero displacement of the beginning point of the cable for the beginning instance of each interval since it is tangent to the pulley. So the boundary conditions can be written as:

$$\begin{cases} \partial w_{i,j}(Z, t_j) / \partial Z \big|_{Z=l(t_j)} = (1/E_i A_i) u_{i,j}(t_j) = g_{i,j}(t_j) \\ w_{i,j}(0, t_j) = 0 \end{cases} \quad (3)$$

where l is the overall length of the cables, $g_{i,j}(t_j)$ is a function of time as the boundary condition of displacement velocity of the end-side of the cable, $u_{i,j}(t_j)$ is the i th cable's tension at its end side during j th time interval which is small enough for $u_{i,j}(t_j)$ to be considered as a constant value. Static displacement of the cable's length due to tension applied at its end at each time interval is considered as its initial displacement at each interval. Also the initial velocity of each point of the cable is estimated for every interval by an interpolation between the determined velocity values of the beginning and the end of the cable (we are permitted to do this because E_i and A_i are not function of Z), so initial condition of the PDE can be explained in this way:

$$\begin{cases} w_{i,j}(Z, t_j(0)) = f_{i,j}(Z) = \frac{1}{E_i A_i} u_{i,j}(t_j) Z \\ \partial w_{i,j}(Z, t_j) / \partial t \big|_{t_j=0} = h_{i,j}(Z) \end{cases} \quad (4)$$

where $h_{i,j}(Z)$ is the approximate value of i th cable's velocity at point Z and j th time interval and $f_{i,j}(Z)$ is a function of Z as the initial condition of the displacement of the cable at the start of vibration of each interval. The last step is calculating the length of the cable for each time interval which varies with time and can be supposed constant during each interval. Because static displacement was imposed as the cable initial displacement for each step, it is not required to be considered as the length, however, because the extension of the cables due to vibration of all of previous steps are not considered as initial condition, they should be added to the rigid cable's length of each step:

$$l_{i,j} = l_{i,j,s} + \sum_{j=1}^{j-1} w_{i,j}(l_i, t_j) \quad (5)$$

where $w_{i,j}$ is the vibrating displacement of the end point of the cable for i th cable during j th interval which can be calculated for previous intervals, $l_{i,j}$ is the overall length of cable i during time interval j and $l_{i,j,s}$ is the rig-

id length of cable i during time interval j . Now everything is ready to solve the vibrating PDE using separation of variables approach. It will results in:

$$w_{i,j}(Z, t_j) = \quad (6)$$

$$\sum_{n=0}^m \left\{ \left(\frac{2}{c_i l_{i,j} \lambda_{i,n}} \right) \int_0^{l_{i,j}(t_j)} h_{i,j}(Z) \sin(\lambda_{i,n} Z) dZ \right\} \sin(\lambda_{i,n} Z) \cos(\lambda_{i,n} c_i t_j) + g_{i,j}(t_j) Z$$

where n is the number of mode shapes which is considered up to m , $c_i = \sqrt{E_i / \rho_i}$ is the wave propagation velocity of the cables and $\lambda_{i,n} = (2n+1)\pi / (2l_i(t_j))$ is the eigenvalue of mode shapes of the cables. By calculating the vibrating displacement of cables, motors' torque and the tension of beginning of the cables can be obtained:

$$T_{i,j}(t_j) = E_i A_i \frac{\partial w_{i,j}(Z, t_j)}{\partial Z} \Big|_{z=0} = u_{i,j}(t_j) + \sum_{n=0}^m E_i A_i \lambda_{i,n} C_{i,n} \sin(\lambda_{i,n} c_i t_j) = \quad (7)$$

$$u_{i,j}(t_j) + \sum_{n=0}^m \sin(\lambda_{i,n} c_i t_j) \left[\frac{2}{c_i l_{i,j-1}(\Delta t)} \{ u_{i,j}(t_{j-1}) (1 / \lambda_{i,n}^2) \sin(\lambda_{i,n} l_{i,j-1}) \} + \right. \quad (8)$$

where $T_{i,j}(t_j)$ is the cable tension of the beginning side of the cable attached to the pulley and $C_{i,n}$ is substituted in the second statement. The required cable tension and cable displacement is calculated through the inverse dynamics of the end-effector. Solving the mentioned PDE in the inverse dynamics of the cable provides the tension of the beginning of the cable together with the vibration of the cables. Afterward the actual end-side tension of the cable and its vibration can be calculated through the direct dynamics of the cables. Finally the actual path is evaluated in the direct dynamics of the end-effector.

III. CONTROL SCHEME

A. Flexible Cable Control Scheme

In dynamics section the DOFs of the system with two vibrating modes for each cable was explained as Eq. (1), so the state variables and state space can be defined as:

$$z = [X, \dot{X}, q_{im}, \dot{q}_{im}]^T; \quad \begin{cases} \dot{z}_i = z_{i+1} & \text{if } i : \text{odd} \\ \dot{z}_i = D(X)^{-1} (-S_j^T u - C(X, \dot{X}) \dot{X} - g(X))_i & \text{if } i : \text{even}, i \leq 12 \\ \dot{z}_i = -[AA_{i,j} \times (\lambda_{i,n} c_i)^2 \cos(\lambda_{i,n} c_i t_j) + BB_i \times \ddot{u}_{i,j}] & \text{if } i : \text{even}, i > 12 \end{cases} \quad (9)$$

q_{ij} is the term of w that is time dependent, and according to previous section can be written as:

$$q_{i,j} = AA_{i,j} \cos \lambda_{i,n} c_i t_j + BB_i u_{i,j} \quad (10)$$

where:

$$AA_{i,j} = \left(\frac{2}{c_i l_{i,j} \lambda_{i,n}} \right) \int_0^{l_{i,j}(t_j)} h_{i,j}(Z) \sin(\lambda_{i,n} Z) dZ \sin(\lambda_{i,n} c_i t_j); \quad (11)$$

$$BB_i = 1 / (E_i A_i); \quad \lambda_{i,n} = \frac{(2n+1)\pi}{2l_i(t_j)}$$

Also the end side tension of the cables (T_i) according to feedback linearization is (Korayem and Tourajizadeh, 2011):

$$T_i = \left(-\frac{1}{S_j^T} (D(X) \ddot{v} + C(X, \dot{X}) \dot{X} + g(X)) \right)_i \Rightarrow \tau_i(t) = (rT + J\ddot{\beta} + c\dot{\beta}); \quad i = 1, \dots, 6 \quad (12)$$

So the linearized state space is:

$$\begin{cases} \dot{z}_i = z_{i+1} & \text{if } i : \text{odd} \\ \dot{z}_i = v_i & \text{if } i : \text{even}, i \leq 12 \\ \dot{z}_i = -[AA_{i,j} (\lambda_{i,n} c_i)^2 \cos(\lambda_{i,n} c_i t_j) + BB_i \times \ddot{u}_{i,j}] & \text{if } i : \text{even}, i > 12 \end{cases} \quad (13)$$

Again v should be replaced by the aid of optimal feedback linearization method. Implementing the explained optimal gains which are derived using LQR method for the flexible cable case results in the optimal cables' tension and motors' torque like:

$$T_{i,j}(t_j) = \left(-\frac{1}{S_j^T} (D(X) (\ddot{X}_{id} - R^{-1} B^T S_e(X)) + C(X, \dot{X}) \dot{X} + g(X)) \right) + \quad (14)$$

$$\sum_{n=0}^m E_i A_i \lambda_{i,n} \times C_{i,n} \sin(\lambda_{i,n} c_i t_j)_i \quad (15)$$

Also the uncertainty specifications of a flexible cable robot can be explained as:

$$\dot{\alpha} = (-rS_j^{-T} (C\dot{X} + g) + \sum_{n=0}^m E_i A_i \lambda_{i,n} \times C_{i,n} \sin(\lambda_{i,n} c_i t_j) + J\ddot{\beta} + c\dot{\beta}); \quad (16)$$

$$\dot{\gamma} = -rS_j^{-T} D$$

Therefore the final optimal motors' torque of a flexible cable robot for which the robust feature is also considered can be defined as follow:

$$\begin{cases} \text{if } \|B^T P_e\| \neq 0: \\ \tau_{i,j}(t_j) = \left(r \times \left[\left(-\frac{1}{S_j^T} (D(X) (\ddot{X}_{id} - R^{-1} B^T S_e(X)) - (u \frac{B^T P_e}{\|B^T P_e\|}) \right) + C(X, \dot{X}) \dot{X} + g(X) \right] \right) + \\ \sum_{n=0}^m E_i A_i \lambda_{i,n} \times C_{i,n} \sin(\lambda_{i,n} c_i t_j) + J\ddot{\beta} + c\dot{\beta}_i \\ \text{if } \|B^T P_e\| = 0: \\ \tau_{i,j}(t_j) = \left(r \times \left[\left(-\frac{1}{S_j^T} (D(X) (\ddot{X}_{id} - R^{-1} B^T S_e(X)) + C(X, \dot{X}) \dot{X} + g(X) \right) \right] + \right. \\ \left. \sum_{n=0}^m E_i A_i \lambda_{i,n} \times C_{i,n} \sin(\lambda_{i,n} c_i t_j) + J\ddot{\beta} + c\dot{\beta}_i \right) \end{cases} \quad (17)$$

To sum up, the summation of the uncertainty errors related to the payload of the end-effector and cable elasticities are measured here by comparing the desired path with actual one which is evaluated using installed sensors. Thus not only the linear error of each joint can be measured and controlled by a linear PID controller, but also the norm of these errors should be controlled by a central nonlinear controller of the end-effector for stability, increasing the accuracy of the system based on the explained proposed controlling strategy.

IV. SIMULATION OF CONTROL PROCEDURE

The simulations are done for the cable robot with main characteristics, controlling gains and elastic cable specifications of Table 1.

$$t \leq 4: \begin{cases} x = 0.05 \cos(4\pi t^2 / 64); \\ y = 0.05 \sin(4\pi t^2 / 64); \end{cases} \quad (18)$$

$$t \leq 4: \begin{cases} x = 0.05 \cos(4\pi(8-t)^2 / 64); \\ y = 0.05 \sin(4\pi(8-t)^2 / 64); \end{cases} \begin{cases} z = 0.45 \\ \psi = \theta = \varphi \end{cases}$$

The reference input is the circle of Eq. (18). The Young's module of the cables are increased from $165 \times (10^6)$ to $165 \times (10^9) \text{ N/m}^2$ with a fixed controlling gain of Table 1 to show the effect of flexibility of the cables on the dynamic performance of the system. Then controlling gains of optimal control are increased up to 100 for a fixed Young's module of $165 \times (10^7) \text{ N/m}^2$ to investigate the efficiency of the proposed optimal control. The comparison result of tracking for four different Young's module can be seen in Fig. 1. Since the longitudinal vibration is just modeled, the effect of flexibility of the cables can be seen mostly in the z direction. It can

Table 1: Parameter specifications of the simulated cable robot and elastic cable

Name	Symbol	Value	Unit
Motor's specifications			
Moment of inertia of the triangle end-effector	I	$I_{xx}=I_{yy}=0.0018, I_{zz}=0.0036$	$kg.m^2$
Radius of the motor	r	$diag[0.015]$	m
Rotary inertia of the pulley	J	$diag[0.0008]$	$kg.m^2$
Mass of the end-effector	m_e	1.09	kg
Error gain matrix	Q	$diag[3]$	
Input gain matrix	R	$diag[1]$	
Elastic cable specification			
Young's module of the cables	R	$165*(10^9)-165*(10^6)$	N/m^2
Density of the cable	ρ	66	kg/m^3
Number of considered longitudinal mode shapes	m	20	
Diameter of the cable	d	3	mm

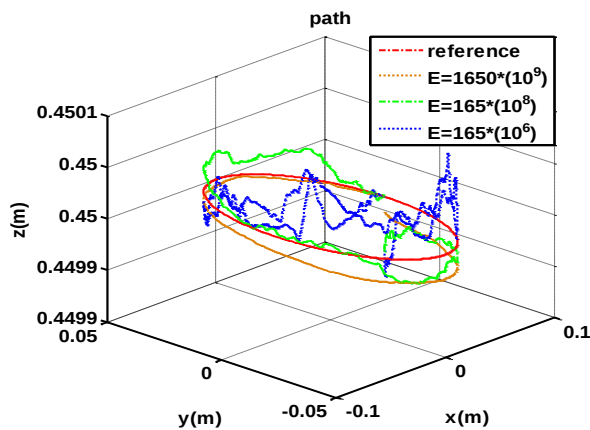


Fig. 1: Comparison of tracking of the end-effector for different modules of elasticity

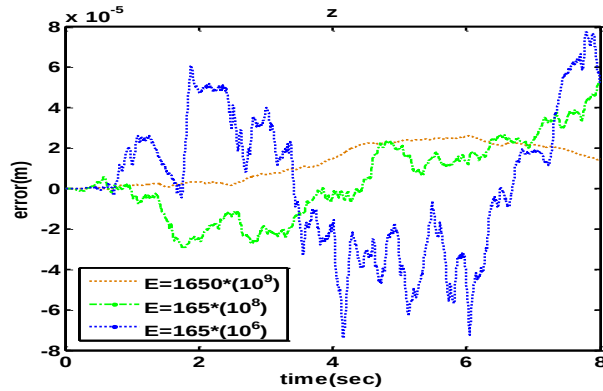


Fig. 2: Comparison of z for different modules of elasticity

be seen that the less the cables' Young's module is, the bigger vibrating amplitude occurs and vice versa. The comparison of error of the system in the z direction for different Young's modules is plotted in Fig. (2) which agrees the mentioned conclusion.

However, the vibrating amplitude and frequency which is produced in the cables' tension or motors' torque is considerable (Figs (3, 4)).

Also required motors' torque shows a little vibrating behavior around its mean value of rigid profile by decreasing the cables' module because of high frequency of cables' vibration (Fig. (3)). Finally the comparison of the required cables' tension for different error gain matrix of LQR and a fixed cable module of $165 \times (10^7)$

N/m^2 is plotted in Fig. (4) for two samples of motors. It can be concluded that for bigger error matrix, bigger vibrating response can be seen from the motors. Since this vibrating response has bigger amplitude, the maximum point of motors' torque increases while its error decreases. The LQR is responsible to find out the optimal gain which compromises between these two paradoxical parameters. Here the vibrations are steady since no structural damping is modeled for the system. Also it can be seen that the un-damped vibrating behavior of the presented closed loop system is stable while this fluctuations might be unstable for a forced vibration in open loop approach of (Korayem *et al.*, 2010b).

By the aid of these results it can be concluded that this elasticity makes a considerable oscillatory response in the torque of the motor with a high frequency of order 10^3 Hz if it wants to be controlled in a closed loop way. These results show their effect more seriously for the open loop system. Also these considerations are extremely necessary while choosing the required motors.

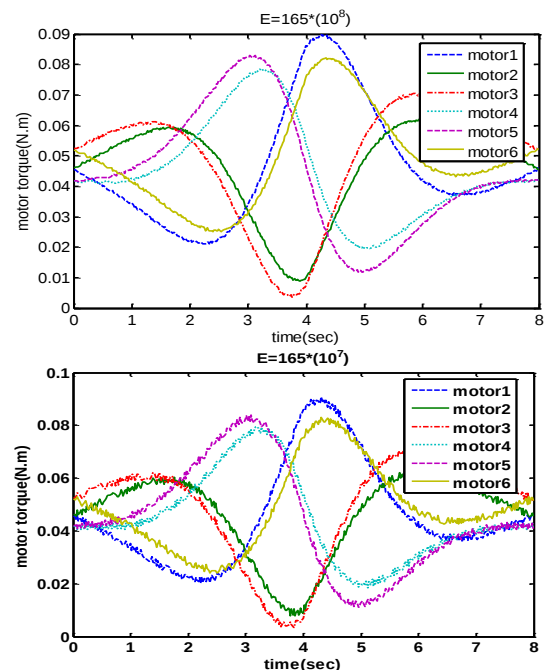


Fig. 3: Comparison of required motors' torque of flexible cable robot for different modules

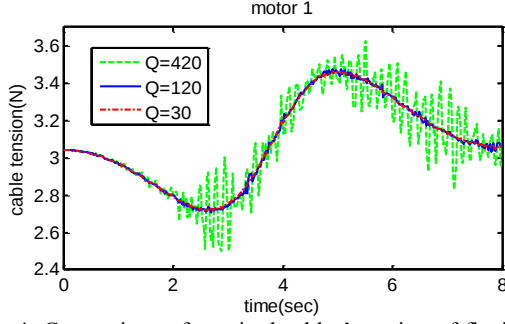


Fig. 4: Comparison of required cables' tension of flexible cable robot for different controlling gains.



Fig. 5: Overall scheme of ICaSbot (Korayem *et al.*, 2013)

V. EXPERIMENTAL VERIFICATION

A. The ICaSbot Cable Robot

In order to verify the simulation results, show the flexibility characteristics of a real cable robot and prove the efficiency of the proposed controlling strategy for handling a flexible cable robot, a prototype of under constrained spatial cable robot has been designed and manufactured at Iran University of Science and Technology (IUST) called ICaSbot with six DOFs and six actuating cables (Fig. 5). The mentioned robot was developed using open loop controlling strategy (using solely motor feedback) (Korayem *et al.*, 2013).

B. Verification of the Results

The simulation results of an ISO predefined trajectory which is a 3D circle in an inclined plane with 45 degrees angle in the space (based on ISO9283, Slamani *et al.*, 2012) are compared and analyzed with the experimental tests of the robot which is equipped by the proposed controller. Controlling gains of Table 1 is employed. The path has the formula as:

$$\begin{aligned} x &= 0.05 \times \cos(\pi \times (5.37 - t)^2 / 7.2) \\ y &= \cos(\pi / 4) \times (-0.05 \times \sin(\pi \times (5.37 - t)^2 / 7.2)) \\ z &= \sin(\pi / 4) \times (-0.05 \times \sin(\pi \times (5.37 - t)^2 / 7.2)) + \cos(\pi / 4) \times 0.8 + 0.3 \end{aligned} \quad (19)$$

In simulation flexibilities is considered and the mentioned controlling strategy is implemented. The same controller is employed for the ICaSbot which has practically flexibility characteristics. Comparing the trajectory for the simulation and experiment can be seen in Fig. 6.

Comparison is performed between four data: 1. Experiment 2. Simulation with rigid structure but closed loop nature, 3. Simulation with flexible structure and

closed loop nature, 4. Simulation with flexibility but open loop nature. Comparing data related to the systems two and three shows the effect of flexibility on the system performance. Comparing data related to the systems three and four shows the positive effect of the designed optimal closed loop system in compensating the flexibility uncertainties.

And finally, comparing data related to the system one with data related to the systems two and three helps us to conclude that modeling the flexibility of the system provides a more accurate and realistic model of the real system practically.

It can be seen that an acceptable compatibility exists for tracking the trajectory for both cases of flexible profiles which illustrates the correctness of both simulation and experimental setups. As it was expected a deviation can be observed respect to the desired trajectory in both cases which shows the existence of flexibilities in reality. However, the proposed controlling strategy has successfully minimized the unwanted errors related to these flexibilities since an acceptable consistency can be seen between the flexible profiles and rigid case. The superiority of the proposed closed loop method is obvious compared to (Korayem and Bamdad, 2009) since the errors related to flexibilities are not considered in this research and these errors can't be compensated automatically. Also the difference of deviations between the simulation and experimental results can be referred to several remained uncertainties of the robot which are not yet modeled in the simulation including of frictions,

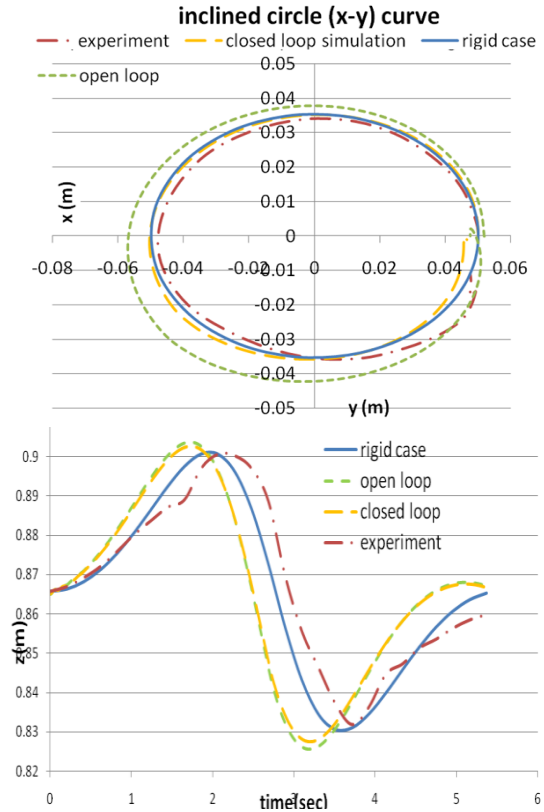


Fig. 6: Comparison of trajectory for simulation and experiment

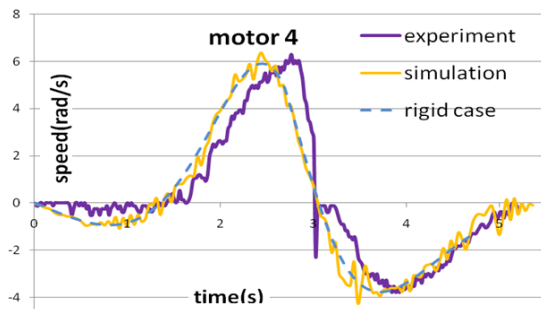


Fig. 7. Comparison of motors' angular velocity for simulation and experiment

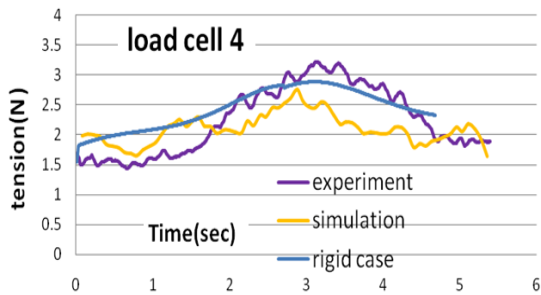


Fig. 8. Comparison of cables' tension for simulation and experiment

inertia of the motors, flexibility of the structure, limitations of using DC motors and PWM controlling strategy and lateral vibrations of the cables. Comparison of angular velocity of the motors is also shown in Fig. 7. The vibrating response of the robot are compared with simulation results in which cables are modeled flexible. Controlling strategy is also implemented for both cases. In order to estimate the flexibility of the manufactured robot, the flexibility parameters of the cables are changed in the simulation till the nearest results to experimental test can be gained corresponding to amplitude and frequency ratio of the vibrating response. This estimation shows the necessity and applicability of modeling the flexibility through the parameter identification of a flexible robot system.

Finally comparing the kinetic results of load cells which show the tension of the cables for simulation and experiment can be observed in Fig. 8.

The Young's module of the cables are estimated to be $E=9.27 \times 10^7 \text{ N/m}^2$ which is acceptable based on the cables' module of ICaSbot ($E=12 \times 10^7 \text{ N/m}^2$). Differences are related to un-modeled uncertainties. Again not only compatibility of the flexible profiles proves the correctness of flexibility modeling, but also their good consistency respect to the rigid results shows the efficiency of the proposed algorithm.

VI. CONCLUSION

One of the most important uncertainties of cable robots i.e. flexibility of cables is considered and controlled in this paper in an optimal way. An optimal control was designed based on feedback linearization method and LQR. Not only tracking performance of the robot was improved by the aid of this controller but also its gains were optimized by the LQR which provides the best ac-

curacy using the least energy. Considering the risk of uncertainties of the flexible system, the stability of the end-effector was assured by the aid of additive robust control. It was discussed that only longitudinal vibration of a cable suspended robot is considerable since the cables are always under enough tension. The error of a flexible cable tracking is of order 10-8 m. It was seen that the vibrating response the vibration of cables are steady since no structural damper is modeled for the cables. On the other hand the vibration of kinetic response of a flexible cable system is severe because of its high frequency. In finale, the simulation studies of rigid and flexible cases were compared with experimental test conducted on cable robot of ICaSbot to show the validity of the proposed theories and simulations, study the flexibility of a real robot and its effects, estimate its flexibility parameters and prove the efficiency of the proposed optimal control for compensating the uncertainties. It was seen that although modeling these flexibility provides results with better consistency with reality, there are still some deviations which are related to un-modeled uncertainties. So modeling the flexibilities can provide a chance of parameter identification through evaluating the amount of flexibility of the cables of the robot by comparing the frequency and amplitude of vibrating response of its dynamic profiles with the modeled flexible simulations.

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