RHEOLOGY OF POWER LAW FLUID FLOW AROUND A STAGNATION POINT IN POROUS MEDIUM WITH ENERGY DISSIPATION

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***Abstract*— -- An investigation on two-dimensional stagnation point flow past a stretching or shrinking surface in a porous medium with energy dissipation using power law model is carried out in this paper. By applying some similarity transformations, the governing partial differential equations are converted to non-linear ordinary differential equations. Consequently, numerical calculations of these equations are done by using MATLAB built- in bvp4c method. Impact of various parameters such as Prandtl number, permeability parameter and magnetic parameter are depicted graphically on velocity and temperature distributions. Also, the numerical values for velocity gradient and shear stress are shown in tabular form. From the analysis, it is noted that Prandtl number helps in reducing the shear stress, Also, as the power law parameter increases, a decrease in velocity is observed.**

***Keywords*— stagnation point flow, stretching surface, shrinking surface power law model, bvp4c method.**

I. INTRODUCTION

Fluid is divided into two categories viz., Newtonian and non- Newtonian fluids. The study of MHD non- Newtonian fluid flow past a stretched sheet has been a very traditional problem and gained concern amid authors in the recent years. It has wide applications theoretically as well as technically in many manufacturing processes such as drying of paper, crystal cooling, melting processes etc. In addition to these studies, shrinking effect was the one which attracted many researchers as this effect has a backward flow and was found useful and significant in textile industries such as glass fibre, production of paper, etc. Hot air balloon that rises up is also a practical use of shrinking effect. In polymer industries, wide applications of stretching/ shrinking effect is observed.

The study of effects of flow and heat transfer on non-Newtonian MHD flow towards a stagnation point has fascinated many researchers. There are many favourable appliances of MHD flow past a porous medium such as in energy storage, geophysics etc. In porous medium Agbaje *et al.* (2018) have investigated MHD stagnation point flow along with heat transfer towards a stretching sheet. Aladdin *et al.* (2019) havediscussed the impact of radiation parameter on MHD stagnation point past a stretching sheet using MATLAB built- in bvp4c method. In presence of induced magnetic field, Ali *et al.* (2011) have extended the work of Agbaje *et al.* (2018). Consequences of transfer of heat on visco- elastic MHD flow past a wavy channel with slip velocity have been explored by Choudhury *et al.* (2011). Choudhury *et al.* (2014) have studied the heat and mass transfer in a vertical channel. Also, a study on MHD stagnation point flow in a porous medium has been studied by Chaudhury *et al.* (2016) using the shooting method. Dey (2017) has studied the influence of Lorentz force on visco-elastic fluid flow.

Power law fluid has also been studied by many due to its wide range of studies in shear stresses. Followed by Power law fluid model, Dey and Deb (2018) have beautifully studied the MHD flow with dissipation of energy past a porous medium using MATLAB built – in bvp4c method. Using the same method, Dey and Nath (2018) have studied two dimensional binary mixture flow past a porous stretching surface. The findings of influence of convective mixed MHD stagnation point flow past a stretching sheet with effects of heat generation using MATLAB built- in bvp4c method was deliberated by Khashi *et al.* (2019). Kumar *et al.* (2019) studied the two dimensional MHD stagnation flow of micropolar fluid past a stretching surface using Runge- Kutta based shooting method. The effects of variable thermal conductivity in a porous medium due to stretched sheet in MHD flow of power law fluid using the same method has been investigated by Mishra *et al.* (2020). Narender *et al.* (2019) have considered nanofluid as a base fluid and studied the effects of reaction of chemical parameter on stagnation point flow past a stretching sheet. With the effects of sink and heat radiation, Nasir *et al.* (2019) have studied the stagnation point flow of a viscous fluid past a stretching/ shrinking sheet and received appealing results. The solution of power law fluid past a stretching sheet has been carried by Yang and Lin (2019) with variable thermal conductivity using the shooting method. Ghasemi and Hatami (2021) have considered the magnetic field and investigated thoroughly of effects of solar radiation on 2-D stagnation flow of nanofluid past a stretching sheet. Ullah *et al.* (2021) analysed the effects of radiation on MHD free convection flow past a verticle plate.

In view of the above ample investigations and motivated by their works, our present paper aims to study the rheology of viscous MHD flow near a stagnation point in a porous medium past a stretching/ shrinking surface with dissipation of energy by taking power law fluid model.

With defined boundary conditions and similarity transformation, concerned partial differential equations are converted to ordinary differential equations and solved numerically by using MATLAB built- in bvp4c methodology. The major significance of the power law fluid model is that it helps in describing the behaviour of the fluid and correlates various investigational data extensively across the diverse range of shear stress. The originality of our paper aims in studying the rheology of MHD stagnation point flow in a porous medium with energy dissipation using Power law model which is different as compared to Chaudhury *et al* (2016). The flow analysis of the problem is described in the next section. Followed by Section III, the methodology used to solve the current problem is explained. Results in the form of graphs and tables obtained from the above work is depicted and expressed in Section IV. In Section V, conclusions of the paper so far is written in a short and concise way for the better understanding, followed by the final section which contains the references that helped us for this paper.

II. METHODS

We consider a 2-D boundary layer flow which is steady, viscous and incompressible fluid near a stagnation point in a porous medium over a stretching/ shrinking surface. The assumptions taken to formulate the problem are:

(i) Flow is restricted in the plane $y > 0$.

(ii) Magnetic Reynolds number is very small.

The constitutive equations are headed by power law model. Let u and v be the velocity components along x and y directions respectively. The governing equations of fluid motion in porous medium are as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_e\frac{du_e}{dx} + v\frac{\partial^2 u}{\partial y^2} + \left(\frac{v}{K_1} + \frac{\sigma_e B_0^2}{\rho}\right)(u_e - u) + \frac{K_2}{\rho}\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)^n \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho C_p}\left(\frac{\partial u}{\partial y}\right)^{n+1} \tag{3}$$

with the relevant boundary conditions as:

$$\left.\begin{array}{c} y = 0: u = u_w(x) = bx, v = 0, T = T_w \\ y \to \infty: u \to u_e(x), T \to T_\infty \end{array}\right\} \tag{4}$$

Equation (1) is trivially satisfied by the stream function $\psi(x, y)$ given by:

$$u = \frac{\partial \psi}{\partial x}, v = -\frac{\partial \psi}{\partial y} \tag{5}$$

The momentum Eq. (2) and energy Eq. (3) is converted into equivalent ordinary differential equations by introducing some similarity transformation as given below:

$$\psi = \sqrt{a\alpha x} f(\eta), \eta = \sqrt{\frac{a}{\alpha}} y, \theta = \frac{T - T_\infty}{T_w - T_\infty} \tag{6}$$

Substituting the data given in (5) and (6) in Eqs. (2) and (3) we have the following set of dimensionless equations:

$$f'^2 - ff'' - Prf''' - (K+M)(1-f') - Ef'''f''^{(n-1)} - 1 = 0 \tag{7}$$

$$\theta'' + f\theta' + Af''^{(n+1)} = 0 \tag{8}$$

where, prime indicates differentiation w.r.t η and

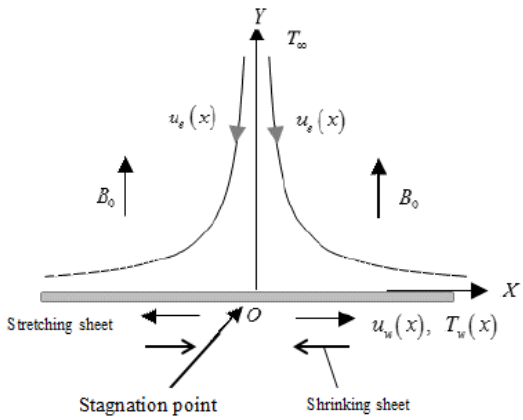

Fig. 1. Physical model of the analysis

$$Pr = \frac{\mu}{\rho\alpha}, K = \frac{\mu}{\rho\alpha K_1}, M = \frac{\sigma_e B_0^2}{a\rho}, E = \frac{K_2}{\rho\alpha^{\frac{n}{2}}} a^{\frac{3n-4}{2}} x^{n-1} n$$

The appropriate boundary conditions are:

$$\left.\begin{array}{c} \eta = 0: f = 0, f' = \frac{b}{a} = c, \theta = 1 \\ \eta \to \infty, f' \to 1, \theta \to 0 \end{array}\right\} \tag{9}$$

where c is the stretching/ shrinking parameter i.e., when $c > 0$ it is stretching parameter, when $c < 0$ it is shrinking parameter and when $n = 0$ it is the stagnation point flow

III. METHOD OF SOLUTION

We adopt MATLAB built- in bvp4c solver technique (Dey and Hazarika, 2020; Dey and Borah, 2020; and Dey and Chutia, 2020) in this problem of the form $y' = f(x, y, c)$ with the applicable boundary conditions $\theta(y(a), y(b), c)$ where $a \le x \le b$. We have the set of non- linear ordinary differential equations as:

Here, we suppose $f = y_1, f' = y_2 = y_1', f'' = y_3 = y_2'$ then,

$$y_3' = f''' = \frac{y_2^2 - y_1 y_3 - (K+M)(1-y_2) - 1}{Pr + Ey_3^{n-1}} \tag{10}$$

This is obtained by extracting f''' from Eq. (7). Similarly, we again suppose

$$\theta = y_4, \theta' = y_5 = y_4',$$

Then

$$y_5' = \theta'' = -y_1 y_5 - Ay_3^{n+1} \tag{11}$$

This is obtained by separating θ' from Eq. (8).

In appliance with the above supposition, the formulated boundary conditions are:

$$y_1(0) = 0, y_2(0) = c, y_4(0) = 1, \quad y_1(1) = 1, y_4(1) = 0 \tag{12}$$

Using the boundary conditions prepared in (10) along with Eqs. (11) and (12) in MATLAB software we get results in form of graphs and tables which are shown in the next section.

IV RESULTS AND DISCUSSION

An inspection on rheology of MHD stagnation point flow using power law model in porous medium near stretching/ shrinking surface has been shown graphically using a numerical method known as MATLAB built- in bvp4c

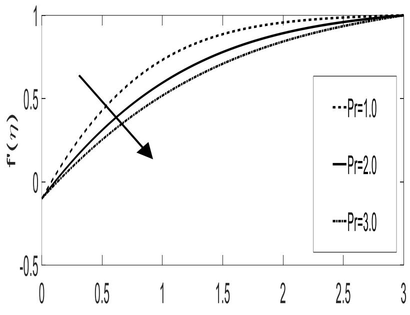

Fig. 2: Velocity distribution against η for various values of Pr when $K = 0.1$, $M = 0.1$, $A = 0.1$, $E = 0.1$, $c = -0.1$ ($n < 1$).

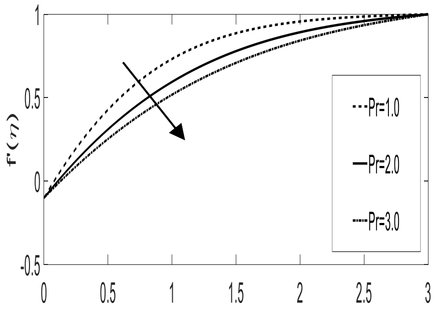

Fig. 3: Velocity distribution against η for various values of Pr when $K = 0.1$, $M = 0.1$, $A = 0.1$, $E = 0.1$, $c = -0.1$ ($n > 1$).

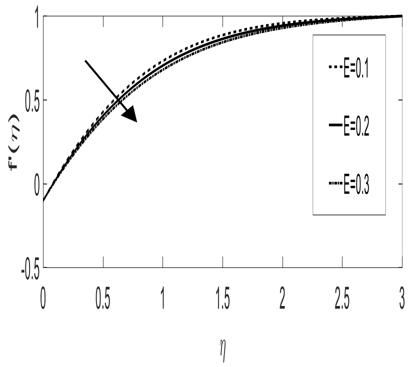

Fig. 4: Velocity distribution against η for various values of E when $K = 0.1$, $M = 0.1$, $A = 0.1$, $\mathrm{Pr} = 1.0$, $c = -0.1$ ($n < 1$).

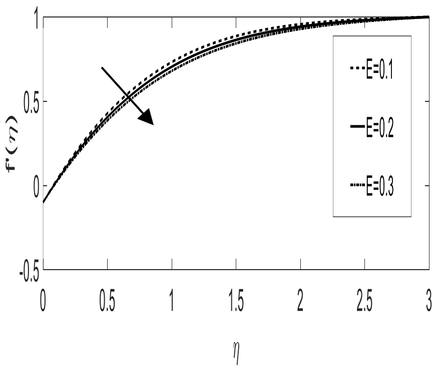

Fig. 5: Velocity distribution against η for various values of E when $K = 0.1$, $M = 0.1$, $A = 0.1$, $\mathrm{Pr} = 1.0$, $c = -0.1$ ($n > 1$).

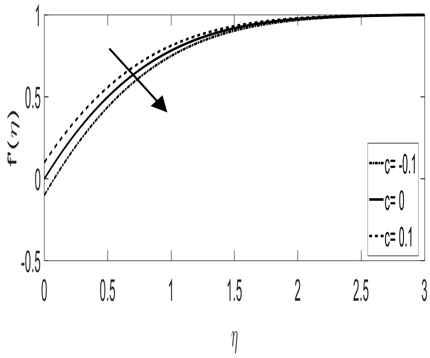

Fig. 6: Velocity distribution against η for various values of c when $K = 0.1$, $M = 0.1$, $A = 0.1$, $\mathrm{Pr} = 1.0$, $E = 0.1$ ($n < 1$).

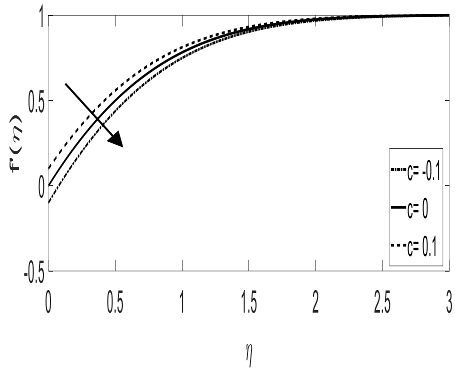

Fig. 7: Velocity distribution against η for various values of c when $K = 0.1$, $M = 0.1$, $A = 0.1$, $\mathrm{Pr} = 1.0$, $E = 0.1$ ($n > 1$).

method which is used to solve the final obtained partial differential equation as described in the earlier section. The numerical calculations were solved for different values of the parameters involved in the present problem such as Prandtl number, Magnetic parameter, permeability parameter and power law parameter. The impacts of these parameters emphasized our present work and their effects on velocity and temperature distribution are analysed below.

As we know, Prandtl Number (Pr) is the ratio of momentum diffusivity to the thermal diffusivity, so as Pr leads, an increase in thickness of velocity boundary layer is observed as compared to thermal boundary layer. So, with increase in momentum diffusivity, increase in viscosity is noticed which results in lowering of the speed of the flow. This behaviour of curve can be seen in both the cases of n i.e., $n < 1$ (shear thinning) and $n > 1$ (shear thickening) as depicted in Figs. 2 and 3, respectively.

Figure 4 represents velocity curve with respect to η when $n < 1$ for various values of power law parameter (E). Increase in boundary layer thickness is cleared from the figure as E increases. Due to this increment, there is an increase in the viscosity, thus lowering in the speed of the fluid i.e., deceleration of fluid motion is observed. A similar deviation of curve can also be drawn from Fig. 5 when $n > 1$. Also, as energy dissipation increases temperature rises as shown in Fig. 10.

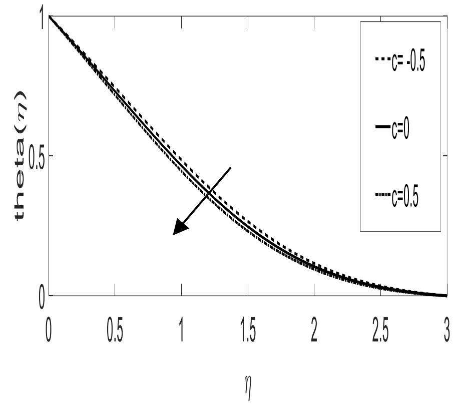

Fig. 8: Temperature distribution against η for various values of c when $K = 0.1$, $M = 0.1$, $A = 0.1$, $\mathrm{Pr} = 1.0$, $E = 0.1$ ($n < 1$).

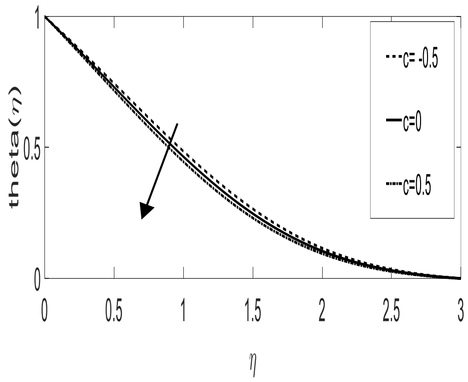

Fig. 9: Temperature distribution against η for various values of c when $K = 0.1$, $M = 0.1$, $A = 0.1$, $\mathrm{Pr} = 1.0$, $E = 0.1$ ($n > 1$).

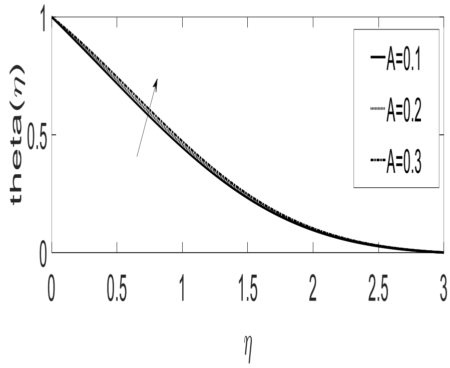

Fig. 10: Temperature distribution against η for various values of A when $K = 0.1$, $M = 0.1$, $\mathrm{Pr} = 1.0$, $c = 0.1$, $E = 0.1$.

Table 1: Numerical values for velocity gradient f'' when $n < 1$.

Pr	K	M	c	A	E	f'' (Choudhury *et al.*, 2014)	f'' (Present paper)	*Er* %
0.7	0.1	0.1	0.1	0.1	0.1	1.67031	1.6539	0.9
1						1.39749	1.3922	0.3
2						0.98815	0.9816	0.6
1.0	0.1	1	0.1	0.1	0.1	1.74282	1.7228	1.1
		3				2.33510	2.3300	0.2
		6				2.98427	2.9542	1.0

Table 2: Numerical values for velocity gradient f'' when $n > 1$.

Pr						f''
0.7	0.1	0.1	0.1	0.1	0.1	0.3377
1						-0.3881
2						-1.6136
1.0	0.1	1	0.1	0.1	0.1	0.3030
		3				1.4787
		6				2.5221

Table 3: Numerical values for shear stress f''^{n}.

Pr	K	M	c	A	E	f''^{n} ($n < 1$)	f''^{n} ($n > 1$)
0.7	0.1	0.1	0.1	0.1	0.1	1.0905	1.7096
1						1.0558	1.4146
2						0.9909	0.9652
1.0	0.1	1	0.1	0.1	0.1	0.8269	0.1636
		3				1.0955	1.5582
		6				1.2102	3.0407

Figures 6 and 7 represents effects of change of stretching/ shrinking parameter in velocity profile. Clearly, from the figures it can be said that enrichment of this parameter increases the velocity profile. Here, the negative value of c implies the shrinking and the positive value of it indicates the stretching for a surface. This observable fact can be easily elucidated as the positive values of c triggers the free stream of velocity as compared to the negative values of c. Reduction in velocity boundary layer is also seen and as such pressure seems to be high near the stagnation point. Also, with decrease in shrinking parameter, velocity decreases and as such friction force comes into action which releases thermal energy resulting hike in temperature as can be seen from Figs. 8 and 9.

To ensure the correctness of our results, a comparison of our data with the data given in Choudhury *et al.* (2011) when $K = 0.1$, $A = 0.1$, $c = -0.1$ and $E = 0.1$ for $n < 1$ is shown through Table 1, i.e., numerical values for velocity gradient is shown. This table illuminates a good deal with the compared data and the results authenticate our calculated numerical values for velocity gradient.

In Table 1 and 2, velocity gradient across the stretching/ shrinking surface towards a stagnation point flow is calculated for various values of Prandtl number (Pr) and Magnetic parameter (M). The study of variation in flow of velocity between adjacent layers which is called velocity gradient is very important in the study of fluid dynamics. It also plays a characteristic role in drag formation. When velocity gradient is zero, the flow is undisturbed and as the value increases variation in flow displacement variables is seen. The physical meaning of velocity gradient is the deformation rate per unit area. So, we can infer from Table 1 that velocity gradient decreases as Pr increases and an increment of rate of deformation is noted as Magnetic parameter M increases which is tabulated in Table 2.

Knowledge in importance of shear stress is worth during the study of non- Newtonian viscous fluid in the research field. It is due to the fact that shear stress is directly proportional to the viscosity of the fluid; the shear stress is responsible for the damage at the surface due to more friction. The main aim of our study is to lower the shear stress and to study about the factors and parameters which can lower the shear stress. In Table 3, we can see that Prandtl Number (Pr) helps to reduce the shear stress while an opposite behaviour is seem with Magnetic parameter (M).

V. CONCLUSIONS

In our present study, numerical investigations on rheology on stagnation point flow toward a stretching/ shrinking surface using power law model has been widely carried out. MATLAB built- in bvp4c methodology is used in this study to solve the ordinary differential equations which was converted from partial differential equations. The study on curves of velocity profile and temperature profile due to effect of various parameters linked with this problem were scrutinised graphically and numerically. Following are the major points which are found noteworthy during the study when $n < 1$ and $n > 1$.

(i) As Prandtl number increases, decrease in the velocity is noted. Similarly, same trend in velocity profile is seen when power law parameter increases.

(ii) Enhancement of the stretching/Shrinking parameter boosts the velocity while it lowers the temperature.

NOMENCLATURE

a	Strength of stagnation flow
b	$b > 0$ (Stretching case), $b < 0$ (Shrinking case)
Pr	Prandtl Number
K	Permeability parameter
M	Magnetic parameter
E	Power law parameter
A	Energy dissipation parameter
u_e	Velocity of external flow
u_w	Velocity of stretching/ shrinking surface
υ	Kinematic viscosity of the fluid
K_1	Permeability of the porous medium
σ_e	Electrical conductivity
B_0	Magnetic field (constant strength)
ρ	Fluid density
K_2	Flow consistency index
n	Flow behaviour index
T	Temperature of the fluid
α	Thermal Diffusivity
μ	Coefficient of viscosity
C_p	Specific heat at constant pressure
T_w	Surface temperature
T_∞	Temperature of the fluid far away from the surface

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