APPLICATION OF TAGUCHI EXPERIMENTAL METHOD IN NUMERICAL SIMULATION OF VARIABLE VISCOSITY AND INTERNAL HEAT GENERATION EFFECTS ON NATURAL CONVECTION IN POROUS MEDIA

K.M. TU[†‡], K.A. YIH[‡] and J.H. CHOU[†]

† *Department of Electrical Engineering, National Kaohsiung University of Science and Technology, No. 415, Jiangong Rd., Sanmin Dist., Kaohsiung City 8077, Taiwan (R.O.C.).*
‡ *Department of Aircraft Engineering, Air Force Institute of Technology, No.1, Julun Rd., Gangshan Dist., Kaohsiung City 82063, Taiwan (R.O.C.).*
Corresponding Author: choujh@nkust.edu.tw, choujhtaiwan@gmail.com

***Abstract* –– This study uses an optimization approach representation of the numerical solution for the variable viscosity, the uniform blowing/suction and the internal heat source effects on the free convection flow over a vertical permeable plate in porous media with a non-linear Boussinesq approximation. The internal heat source is of an exponential decaying form. The partial differential equations are transformed into non-similar equations and solved by Keller box method (KBM). Compared with the previously published articles, the results are considered to be very consistent. Numerical results for the local Nusselt number with the four parameters: 1) the blowing/suction parameter ξ, 2) the viscosity-variation parameter θr, 3) internal heat source coefficient A^*, 4) the non-linear temperature parameter δ_1 are expressed in tables. Through the Taguchi method to predict the best point of the maximum of local Nusselt number is $\xi(-4)$, $\theta r(-2)$, $A^*(0)$, $\delta_1(5)$.**

***Keywords*–– Taguchi experimental method, Variable viscosity, Internal heat generation, Non-linear Boussinesq, Vertical permeable plate.**

I. INTRODUCTION

The heat transfer of free convection in saturated porous media has many important applications in nature and engineering. Examples include geothermal flow, nuclear waste storage, electronic heat transfer systems, building insulation, powder metallurgy, groundwater contamination, osmotic cooling and chemical industry separation procedures.

Cheng and Minkowycz (1977) examined free convection about a vertical flat plate embedded in a porous medium with application to heat transfer from a dike. Minkowycz and Cheng (1982) solved the uniform lateral mass flux on natural convection over a vertical plate in porous media by using the local non-similarity (LNS) method. The uniform blowing/suction effect on free convection past a vertical plate and cone with uniform wall temperature (UWT) in porous media was solved by Yih (1997) utilizing Keller box method (KBM).

The effect of the internal heat generation on the changes of the pure heat transfer has been the focus of a great deal of research. The application of internal heat generation contains the storage of radioactive materials and reactors safety analyses. The free convection boundary-layer over a vertical permeable flat plate in a porous medium with internal heat generation was solved by Postelnicu *et al.* (2000). Mohamed (2007) reported that the effect of lateral mass flux on the natural convection boundary layers induced by a heated vertical plate embedded in a saturated porous medium with internal heat generation.

The study of the variable viscosity in porous media has been widely discussed. Lai and Kulacki (1990) considered the effect of variable viscosity on convective heat transfer along a vertical surface in a saturated porous medium, and fluid viscosity has been assumed to be an inverse function of temperature. The non-linear Boussinesq approximation has been the subject of much research. Vajravelu *et al.* (2011) used the Darcy model and the non-Boussinesq approximation to explore free convection boundary layer flow past a vertical surface in a porous medium with temperature-dependent properties. Yih (2017) studied non-linear Boussinesq and internal heat generation effects on free convection of non-Newtonian fluids over a convectively heated vertical plate in porous media. The above researchers are only concentrated upon the numerical simulation.

However, the Taguchi method to find the maximum values simulation of the research is still very few. Chou and Wu (1999) studied application of Taguchi non-linear dynamic quality characteristics to fuzzy control system. Ho *et al*. (2009) reported adaptive network-based fuzzy inference system for prediction of surface roughness in end milling process using hybrid Taguchi-genetic learning algorithm. Investigation into the effect of cutting fluid concentration on the machinability of Ti-6Al-4V using vegetable oil-based cutting fluids utilized the L27 fractional factorial Taguchi array was presented by Salah *et al*. (2017).

In this exploring, the partial differential equations are transformed into non-similar equations and solved in the Keller box method (KBM) proposed by Cebeci and Bradshaw (1984). The numerical simulation of four different parameters is carried out to obtain the maximum value of the local Nusselt number and find the best parameter ratio. The study is divided into two phases: the first phase of the data is compared with the previously published article, the results are considered very consistent. The variable viscosity and internal heat generation effects on the local Nusselt number are presented in tabular form. The second stage uses the Taguchi method by Chou (2012) to optimize the four parameters of the local Nusselt number. The larger the local Nusselt number, the more amount of heat is taken away from the plate.

II. METHODS

A. Mathematical Equations

Part 1: consider the influence of the variable viscosity and internal heat generation effects on heat transfer by free convection flow over a vertical permeable flat plate embedded in a saturated porous medium with non-linear Boussinesq. The concept map is shown in Fig. 1.

This article is an extension of Vajravelu *et al.* (2011) and Yih and Payne (2017).

The origin of the coordinate system is placed at the leading edge of the vertical flat plate, where x and y are Cartesian coordinates measuring distance along and normal to the surface of vertical flat plate, respectively. All the fluid properties are assumed to be constant, except for the viscosity μ of fluid and the density variation in the buoyancy term. The viscosity depends on the temperature T in the following form (Lai and Kulacki, 1990 and Vajravelu *et al.*, 2011):

$$\frac{1}{\mu} = \frac{1}{\mu_\infty}[1 + \gamma(T - T_\infty)] = a(T - T_r) \quad (1)$$

where γ is a viscosity-variation constant and μ_∞ is the viscosity of the ambient fluid with the following relation

$$a = \frac{\gamma}{\mu_\infty} \quad \text{and} \quad T_r = T_\infty = \frac{1}{\gamma} \quad (2)$$

Both a and T_r are constants and their values depend on the reference state and the thermal property of the fluid, i.e., γ. In general, $a > 0$ ($\gamma > 0$) for liquid and $a < 0$ ($\gamma < 0$) for gas. The viscosity of the fluid usually reduces with increasing temperature while it enhances for gas.

To further show the appropriateness of Eq. (1), correlations between viscosity and temperature for air and water are given below because these two are the most common working fluids found in engineering applications.

For air

$$\frac{1}{\mu} = -123.2(T - 742.6) \quad (3)$$

$$T_\infty = 293\text{K } (20°\text{C})$$

and for water

$$\frac{1}{\mu} = 29.83(T - 258.6) \quad (4)$$

$$T_\infty = 288\text{K } (15°\text{C})$$

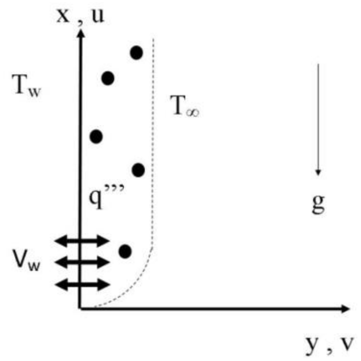

Figure 1: Flow model and physical coordinate system.

The data used for these correlations are taken from Weast (1990). While Eq. (3) is good to within 1.2% from 278K (5°C) to 373K (100°C), Eq. (4) is good to within 5.8% from 283K (10°C) to 373K (100°C). The reference temperatures thus selected for the correlations are very practical in most applications.

Introducing the boundary layer approximation and non-Boussinesq approximation, the governing equa-tions and the boundary conditions based on the Darcy law can be written as follows:

Continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (5)$$

Momentum (Darcy) equation:

$$u = -\frac{K}{\mu}\left(\frac{\partial p}{\partial y} + \rho g\right) \quad (6)$$

$$v = -\frac{K}{\mu} \cdot \frac{\partial p}{\partial y} \quad (7)$$

Energy equation:

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left(\frac{\partial^2 T}{\partial y^2}\right) + \frac{q'''}{\rho C_p} = 0 \quad (8)$$

Non-Boussinesq approximation:

$$\rho = \rho_\infty[1 - \beta_{T0}(T - T_\infty) - \beta_{T1}(T - T_\infty)^2] \quad (9)$$

Boundary conditions:

$$y = 0: v = V_w,\ T = T_w \quad (10)$$

$$y \to \infty: u = 0,\ T = T_w \quad (11)$$

Here, u and v are the Darcian velocities in the x- and y-directions; μ, p and ρ are the variable viscosity, the pressure and the density of the fluid, respectively; K is the permeability of the porous medium; g is the gravitational acceleration; T is the volume-averaged temperature; α is the equivalent thermal diffusivity; q''' is the internal heat generation rate per unit volume; C_p is the specific heat at constant pressure; β_{T0}, β_{T1} are the thermal expansion coefficients of the first and second orders, respectively; V_w is the uniform blowing/suction velocity.

The stream function ψ is defined by

$$u = \frac{\partial \psi}{\partial y} \quad , \quad v = -\frac{\partial \psi}{\partial x} \quad (12)$$

Thus, the continuity equation is automatically satisfied.

We note the governing Eqs. (6) and (7). If we do the cross-differentiation $\partial(\mu \cdot u)/\partial y - \partial(\mu \cdot v)/\partial x$, then the pressure terms in Eqs. (6) and (7) can be removed. Next step, with the help of the Eq. (9) and the boundary layer approximation ($\partial/\partial x \ll \partial/\partial y$), then we can obtain

$$\frac{\partial}{\partial y}(\mu \cdot u) = \rho_\infty g K\left[\beta_{T0}\frac{\partial T}{\partial y} + 2\beta_{T1}(T - T_\infty)\frac{\partial T}{\partial y}\right] \quad (13)$$

Integrating Eq. (13) once and via Eqs. (1) and (11), then we get

$$u = [1 + \gamma(T - T_\infty)]\frac{\rho_\infty g K}{\mu_\infty}$$

$$[\beta_{T0}(T - T_\infty) - \beta_{T1}(T - T_\infty)^2] \quad (14)$$

Using the following dimensionless non-similarity variables:

$$\xi = \frac{2V_w \cdot x}{\alpha \cdot Ra_x^{1/2}} \quad (15a)$$

$$\eta = \frac{y}{x}Ra_x^{1/2} \quad (15b)$$

$$f(\xi, \eta) = \frac{\psi}{\alpha \cdot Ra_x^{1/2}} \quad (15c)$$

$$\theta(\xi, \eta) = \frac{T - T_\infty}{T_w - T_\infty} \quad (15d)$$

$$Ra_x = \frac{\rho_\infty g \beta_{T0} K (T_W - T_\infty)}{\mu_\infty} \cdot \frac{x}{\alpha} \quad (15e)$$

The internal heat generation rate per unit volume q''' is modeled conforming to the following equation (Postelnicu *et al.*, 2000; Mohamed, 2007; Grosan and Pop, 2001; Huang, 2016):

$$q''' = A^* \frac{k(T_W - T_\infty)}{x^2} \cdot Ra_x \cdot e^{-\eta} \quad (16)$$

Here, k is the equivalent thermal conductivity. A^* is the internal heat generation coefficient. Note that when $A^* = 0$ corresponds to the case 1: no internal heat gen-eration (assign NIHG); while for $A^* > 0$ corresponds to the case 2: with internal heat generation (WIHG).

Substituting Eqs. (15) and (16) into Eqs. (14), (8) to (11), we obtain

$$f' = \left[1 - \frac{\theta}{\theta_r}\right] \cdot \left[\theta + \frac{\delta_1}{2} \cdot \theta^2\right] \quad (17)$$

$$\theta'' + \frac{1}{2} \cdot f\theta' + A^* \cdot e^{-\eta} = \frac{\xi}{2}\left(f' \cdot \frac{\partial \theta}{\partial \xi} - \theta' \cdot \frac{\partial f}{\partial \xi}\right) \quad (18)$$

The boundary conditions are defined as follows:

$$\eta = 0: f = -\xi/2\,,\ \theta = 1 \quad (19)$$

$$\eta \to \infty: \qquad \theta = 0 \quad (20)$$

Furthermore, in terms of the new variables, the Darcian velocities in x- and y- directions are, respectively, given by

$$u = \frac{\alpha Ra_x}{x} f' \quad (21)$$

$$v = -\frac{\alpha Ra_x^{1/2}}{x}\left[\frac{1}{2} f + \frac{1}{2}\left(\xi \frac{\partial f}{\partial \xi} - \eta f'\right)\right] \quad (22)$$

where primes denote differentiation with respect to η. ξ defined in Eq. (15a) is the surface blowing/suction parameter; Eq. (19) can be obtained by integrating Eq. (22) versus ξ once and by setting $\eta = 0$ (at the surface, $y = 0$, then $\eta = 0$), and with the help of Eq. (10). On the one hand, for the case of blowing, $V_w > 0$ and hence $\xi > 0$. On the other hand, for the case of suction, $V_w < 0$, and hence, $\xi < 0$.

Via the Eqs. (15d) and (2), the viscosity-variation parameter θ_r is a constant and is defined by

$$\theta_r = \frac{T_r - T_\infty}{T_W - T_\infty} = -\frac{1}{\gamma(T_W - T_\infty)} \quad (23)$$

For Eq. (1), it is value referring that for $\gamma \to 0$, i.e. $\mu = \mu_\infty$ (constant viscosity) then $\theta_r \to \infty$.

The non-linear temperature parameter δ_1 is defined as follows:

$$\delta_1 = \frac{2\,\beta_{T1}(T_W - T_\infty)}{\beta_{T0}} \quad (24)$$

It is also important to note that θ_r is negative for liquid and positive for gas.

The result of practical interest in many applications is the surface heat transfer rate. The surface heat transfer rate is expressed in terms of the local Nusselt number Nu_x divided by $Ra_x^{1/2}$, which is basically defined as following

$$\frac{Nu_x}{Ra_x^{1/2}} = -\theta'(\xi, 0) \quad (25)$$

The system of Eqs. (17) to (20) is solved by using Keller box method of Cebeci and Bradshaw (1984). First, the partial differential equation is first converted to a system of three first-order equation. Then, these first-order equations are expressed in finite difference form and solved by the iterative scheme along with their boundary conditions. This method provides a better convergence rate and reduces the numerical calculation times.

The initial size of the calculated grid is $\Delta\eta_1 = 0.01$, the variable grid increment parameter is set to 1.01, the value of η_∞ is between 1 and 15, and $\Delta\xi = 0.1$. When the error values of θ'_w become less than 10^{-5}, the iteration process is stopped and the final temperature distribution is given.

B. Taguchi method

Part 2: in this work, Taguchi method is used to simulate three different values of four parameters, A: the blowing/suction parameter $\xi(-4{,}0{,}4)$, B: the viscosity-variation parameter $\theta_r(-2{,}0{,}2)$, C: internal heat source coefficient $A^*(0{,}0.5{,}1)$, D: the non-linear temperature parameter $\delta_1(0{,}1{,}2)$.

If arranged in full factor method to set the parameter, it will take quite a large computation time. Therefore, this experiment selected Taguchi method for numerical optimization design by numerical simulation can reduce from 81 times to 9 times. This study was chosen L9(3^4) orthogonal table.

The four factors (A, B, C, D, 4 columns) have three levels (1, 2, 3), the number of simulations is 9 times. The levels of these four factors appear the same, and this is a balance. This method will be used to establish the factor-level and preliminary data combinations as shown in Table 1. Select the Taguchi L9(3^4) orthogonal table via Table 1 as shown in Table 2.

For this purpose, the signal to noise ratio (S/N ratio) measured with dB is used. Minimizing quality characteristics is equivalent to maximizing the S/N ratio, defined by

The Signal / Noise:

$$\frac{S}{N} = -10 \cdot \log_{10} \frac{1}{n} \sum_{i=1}^{n} \left(\frac{1}{Y_i^2}\right) \quad (26)$$

where Y_i denote local Nusselt number in Table 5.

Table 1: Factor - Level table

			Level		
			1	2	3
Factor	blowing/suction parameter ξ	A	-4	0	4
	viscosity-variation parameter θ_r	B	-2	∞	2
	internal heat source coefficient A^*	C	0	0.5	1
	non-linear temper-ature parameter δ_1	D	0	1	5

Table 2: Orthogonal table

No. \ Row	1	2	3	4
1	1	1	1	1
2	1	2	2	2
3	1	3	3	3
4	2	1	2	3
5	2	2	3	1
6	2	3	1	2
7	3	1	3	2
8	3	2	1	3
9	3	3	2	1

Table 3: Comparison of the values of $-\theta'(\xi,0)$ for various values of ξ with $A^*=0$, $\theta_r=\infty$, $\delta_1=0$.

	$-\theta'(\xi,0)$		
ξ	Minkowycz and Cheng (1982)	Yih (1997)	Present results
-10	5.0000	4.9999	4.9999
-8	—	3.9999	4.0000
-6	—	2.9999	3.0000
-4	2.0020	2.0015	2.0015
-2	1.0680	1.0725	1.0726
0	0.4438	0.4437	0.4437
2	0.1423	0.1416	0.1411
4	0.0335	0.0333	0.0329
6	0.0043	0.0055	0.0055
8	0.0002	0.0006	0.0006
10	—	0.0001	0.0000

Table 4: Comparison of the values of $-\theta'(\xi,0)$ for various values of A^* and δ_1 with $\theta_r=\infty$, $\xi=0$.

		$-\theta'(\xi,0)$		
A^*	δ_1	Postelnicu *et al.* (2000)	Yih and Payne (2017)	Present results
0	0	0.4441	0.4437	0.4437
0	1	—	0.5180	0.5180
0	2	—	0.5827	0.5827
0.5	0	—	—	0.1089
0.5	1	—	0.2005	0.2005
0.5	2	—	—	0.2767
1	0	−0.2152	−0.2153	−0.2153
1	1	—	−0.1036	−0.1036
1	2	—	−0.0146	−0.0146

Table 5: The orthogonal table of numerical (local Nusselt number) and the signal / noise ratio (S/N)

	local Nusselt number	signal / noise ratio (S/N)
$\xi(-4)$, $\theta_r(-2)$, $A^*(0)$, $\delta_1(0)$	2.0058	6.0458
$\xi(-4)$, $\theta_r(\infty)$, $A^*(0.5)$, $\delta_1(1)$	1.4748	3.3747
$\xi(-4)$, $\theta_r(2)$, $A^*(1)$, $\delta_1(5)$	1.0192	0.1652
$\xi(0)$, $\theta_r(-2)$, $A^*(0.5)$, $\delta_1(5)$	0.6167	-4.1985
$\xi(0)$, $\theta_r(\infty)$, $A^*(1)$, $\delta_1(0)$	-0.2153	-13.3391
$\xi(0)$, $\theta_r(2)$, $A^*(0)$, $\delta_1(1)$	0.4104	-7.7359
$\xi(4)$, $\theta_r(-2)$, $A^*(1)$, $\delta_1(1)$	-0.2285	-12.8223
$\xi(4)$, $\theta_r(\infty)$, $A^*(0)$, $\delta_1(5)$	0.2155	-13.3311
$\xi(4)$, $\theta_r(2)$, $A^*(0.5)$, $\delta_1(0)$	-0.1703	-15.3757
average value	0.5698	-6.3574

C. Results and Discussion

In order to verify the accuracy of our current method, we compared our results with those of Minkowycz and Cheng (1982), Yih (1997), Postelnicu *et al.* (2000) and Yih and Payne (2017). The results of comparing these ar-ticles are very consistent. The negative values in Ta-ble 4 mean heat transfer from porous media to the ver-tical flat plate.

The orthogonal table of numerical (local Nusselt number) and the signal/ noise ratio (S/N) is shown in Ta-ble 5. Orthogonal table of simulation results by Taguchi analysis results is listed in Table 6. From the analog data it is can be obtained that the A factor is the level 1, B factor is the level of 1, C factor is the level 1, D factor is the level 2 or 3.

In order to verify the result obtained by Taguchi method is the best. Using $\xi(-4)$, $\theta_r(-2)$, $A^*(0)$, $\delta_1(1$ or 5) to compare with the highest value in the orthogonal table $\xi(-4)$, $\theta_r(-2)$, $A^*(0)$, $\delta_1(0)$. The results show for the case of $\xi(-4)$, $\theta_r(-2)$, $A^*(0)$, $\delta_1(5)$, the local Nusselt number is 2.4569. For $\xi(-4)$, $\theta_r(-2)$, $A^*(0)$, $\delta_1(0)$, the local Nusselt number is 2.0058. It is known from the above verification by Taguchi method, for the case of $\xi(-4)$, $\theta_r(-2)$, $A^*(0)$, $\delta_1(5)$, the best data is ob-tained. The values of $-\theta'(\xi,0)$ for ξ, θ_r, A^*, δ_1 are shown in Table 7.

Table 6: The orthogonal table of simulation results

Factor	signal / noise ratio (S/N)			
Level	A	B	C	D
1	3.1952	-3.6583	-5.0071	-6.8748
2	-8.4245	-7.7652	-5.3999	-5.7278
3	-13.8430	-7.6488	-8.6654	-5.7881
Delta	17.0382	4.1069	3.6583	12.6026
Ranking	1	1	1	2 or 3
Level	Mean Value			
1	1.4999	0.7980	0.8772	0.5401
2	0.2706	0.4917	0.6404	0.5522
3	-0.0611	0.4198	0.1918	0.6171
Delta	1.5610	0.3782	0.6854	0.0770

Table 7: Values of $-\theta'(\xi,0)$ for various values ξ, θ_r, A^*, δ_1.

ξ	θ_r	A^*	δ_1	$-\theta'(\xi,0)$
-4	-2	0	0	2.0058
0	-2	0	0	0.5180
4	-2	0	0	0.0660
-4	-2	0	0	2.0058
-4	∞	0	0	2.0015
-4	2	0	0	2.0001
-4	-2	0	0	2.0058
-4	-2	0.5	0	1.4748
-4	-2	1	0	0.9607
-4	-2	0	0	2.0058
-4	-2	0	1	2.0175
-4	-2	0	5	2.4569

III. CONCLUSIONS

The first phase of the conclusion is as follows:

1. As the blowing/suction parameter ξ increases from -4 (suction) to 4 (blowing), the local Nusselt number de-creases.
2. The local Nesselt number enhances (reduces) for the viscosity-variation parameter $\theta_r=-2$: liquid ($\theta_r=2$: gas).
3. When the internal heat source coefficient A^* in-creases from 0 (without hot source) to 1 (with heat source), the local Nusselt number is diminished.
4. As the non-linear temperature parameter δ_1 en-hances from 0 (linear) to 5 (non-linear), the local Nusselt number increases.

The second phase is to invoke Taguchi experimental method to replace the traditional single factor experi-mental method. All parameter optimization design is to find the maximum local Nusselt number. The local Nusselt number is largest representing that the most heat energy is taken away from the plate by the fluid. The number of experiments can be greatly reduced from 81 times to 9 times, effectively saving the computer simula-tion time. Computer numerical simulation shows that the maximum value of local Nusselt number is 2.4569 for the case of $\xi(-4)$, $\theta_r(-2)$, $A^*(0)$ and $\delta_1(5)$.

NOMENCLATURE

A^* internal heat generation coefficient
a constant
C_p specific heat at constant pressure
f dimensionless stream function
g gravitational acceleration
K permeability of the porous medium
k equivalent thermal conductivity
Nu_x local Nusselt number
p pressure
q''' internal heat generation rate per unit volume
Ra_x local Rayleigh number
T temperature
u Darcy velocity in the x-direction
v Darcy velocity in the y-direction
V_w uniform blowing/suction velocity
x streamwise coordinate
y transverse coordinate

Greek symbols

α equivalent thermal diffusivity
β_{T0} first order coefficient of thermal expansion
β_{T1} second order coefficient of thermal expansion
γ viscosity-variation constant
δ_1 non-linear temperature parameter
η pseudo-similarity variable
θ dimensionless temperature
θ_r viscosity-variation parameter
μ variable viscosity
ξ blowing/suction parameter
ρ density
ψ stream function

Subscripts

w condition at the wall
∞ ambient

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Received October 25, 2018
Sent to Subject Editor August 20, 2019
Accepted June 29, 2020
Recommended by Subject Editor Fabio Giannetti