Optimal profile for a fin of constant cross section with modelation of the thermophysical properties

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ABSTRACT The dependence of the temperature and composition on thermal conductivity of steel and its influence in the optimal profile for a fin of constant cross section to be used in air cooled condenser is presented. In Air Cooled Condensers (ACC), air is used as refrigerant, which forces the need for fins to improve the heat transfer process leading to appreciable increments in the costs of the installation. Therefore, developing better fins profile will allow optimizing the operation, maximizing the heat transfer and minimizing the manufacturing cost of the fins. To this objective, in the present paper, three geometric parameters, thickness, height and surface of heat transfer of the fin, these parameters influence directly the optimal obtained profile for conditions of maximum heat flux and optimal Biot number. The study was accomplished for fins of constant cross section. In a future work, fins of variable cross sections will be considered.

KEYWORDS Optimal profile; Fins; Constant section; Optimal length; Optimal thickness; Thermal conductivity

1. Introduction

A way to intensify heat transfer is accomplished by increasing the area of heat exchange with fins. Fins are used when the coefficient of heat transfer is low, which happens when the fluid is a gas, and mainly in conditions of natural convection. Fins are located on the surface, being the most usual a tube in which the number of fins is variable, with a typical separation around 1 to 6 mm (Camaraza-Medina, 2021; Zadhoush et al., 2017; Song et al., 2017).

When the fluid moves around the fins it is confined and it is displaced by means of fluid machinery, we have to take into account the necessary energy to hold the average heat transfer coefficient through the fins, trying that the extracted thermal energy is larger with respect to the used energy in displacing the fluid (Das and Kundu, 2018).

This situation suggests that a study of methods and cost of manufacturing, maintenance and shape/design of the fins in order. Perhaps the optimal fin shape/design does not exactly lead to the optimal thermal conditions. Thus, it will be essential to analyze this, plus its manufacturing process, to choose a configuration ideal for its industrial use in an ACC.

Therefore, it is required to obtain the geometrical optimal elements for a fin of constant cross section, that allows incrementing the coefficient of heat transfer and that simultaneously decrease the manufacturing costs (Kumar et al., 2017).

In the development of the optimal profile of fins, is important obtaining an optimal value of the amount of heat transfer for a determined surface Ω, per tube length. The optimal thickness fulfills additionally that the derivative of the heat flux with respect to the thickness of the fin is equal to zero; therefore, the more rational is developing a mathematical model that enables determining the ideal fin profile to get optimal conditions of heat transfer and manufacturing costs. For this purpose, in this paper developing an optimal fins profile of cross section is sought, for a practical use in ACC equipment (Huang et al., 2018; Jin et al., 2018).

The optimization proposed in this paper considers several physical elements of the fin: thickness, height and area of the cross section of the fins.

2. Methods and validation

2.1 Introductory elements on heat transfer in fins of constant profile

In stationary regime, heat is transmitted through the fins and then, it is transferred by convection. The energy dissipated is proportional to its surface area. A fin of constant cross section has a length a and a rectangular cross section of height l. The temperature of the environment fluid is T_F and the heat transfer coefficient α has a constant value (see Fig. 1 for more details) (Webb, 1980; Caruso et al., 2014; Cattani et al., 2017).

From Fig. 1, the balance of thermal flux in an elementary volume located in the position x, is equal to the sum of the heat transferred by convection through the elementary volume and the heat conducted out of the control volume $x+\Delta x$ Mathematically, it can be written as:

$$Q_X - \left(Q_X - \frac{\partial Q_X}{\partial x}\right) - Q_C = 0\,;\ \frac{\partial Q_X}{\partial x}\Delta x + Q_C = 0 \quad (1)$$

Applying Fourier's law to Eq. 1

$$Q_X - \lambda S\left(\frac{\partial Q_X}{\partial x}\right)_X \rightarrow \frac{\partial Q_X}{\partial x} = -\lambda S\left(\frac{\partial^2 T}{\partial x^2}\right)_X \quad (2)$$

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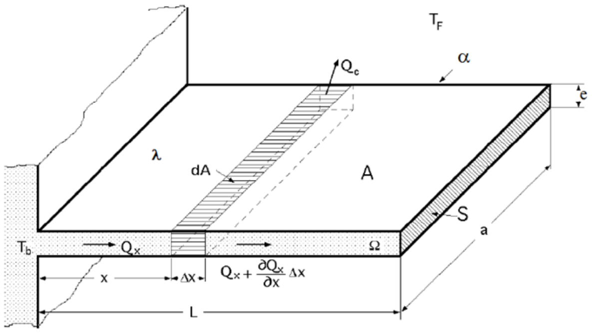

Figure 1: Effect of biochar and P applications on dry weight.

Newton's law of cooling allows considering the dissipation of heat for convection from the fin. An energy balance in the control volume gives (Pis'mennyi et al., 2016):

$$Q_C = \alpha dA\left(T_X - T_F\right) - \alpha P\Delta X\left(T_X - T_F\right)_C = 0 \tag{3}$$

In Eq. 2 and 3, S and P are the area and the perimeter of the cross section, respectively. The differential equation for the temperature distribution is then:

$$-\lambda S\left(\frac{\partial^2 T}{\partial x^2}\right)_X \Delta C + \alpha P\Delta X\left(T_X - T_F\right) = 0 \tag{4}$$

Introducing the following definitions:

$$\phi\left(\xi\right) = \frac{T_x - T_F}{T_B - T_F} \rightarrow T_X = T_F + \phi\left(\xi\right)\cdot\left(T_B - T_F\right) \tag{5}$$

Substituting Eq. 5 into Eq. 4:

$$\begin{cases} \dfrac{dt}{dx} = \left(T_B - T_F\right)\dfrac{d\phi\left(\xi\right)}{d\xi}\cdot\dfrac{d\xi)}{dx} = \dfrac{\left(T_B - T_F\right)}{L}\dfrac{d\phi\left(\xi\right)}{d\xi} \\ \left|\xi = \dfrac{x}{L}\,;\,\dfrac{d\xi)}{dx} = \dfrac{1}{L}\right| \\ \dfrac{d^2 t}{dx^2} = \dfrac{\left(T_B - T_F\right)}{L}\dfrac{d^2\phi\left(\xi\right)}{d\xi^2}\cdot\dfrac{d\xi)}{dx} = \dfrac{\left(T_B - T_F\right)}{L}\dfrac{d^2\phi\left(\xi\right)}{d\xi^2} \end{cases} \tag{6}$$

Simplifying Eq. 6:

$$\frac{d^2\phi\left(\xi\right)}{d\xi^2} - \frac{\alpha L^2 P}{\lambda S}\phi\left(\xi\right) \tag{7}$$

Thus, the temperature distribution can be expressed then in a dimensionless form in terms of the Biot number. Considering that the perimeter P multiplied by the length of the fin L is equal to the total area of the lateral surface (Şahinoğlu and Rafighi, 2021):

$$A = PL \tag{8}$$

From the Eq. 8 it is obtained that:

$$PL^2/S = AL/S = L^* \tag{9}$$

Equation 9 includes length dimensions, therefore, L^* can be considered as the characteristic length of the fin. The dimensionless Biot number is obtained as:

$$Bi = \alpha L^2 P/\left(\lambda S\right) = \alpha L^*/\lambda \tag{10}$$

Thus, the differential equation for the temperature distribution in dimensionless form (corresponding to a constant cross section fin), in terms of the Biot number, is given by Lieth et al. (2021):

$$\frac{d^2\phi\left(\xi\right)}{d\xi^2} - Bi\phi = 0 \tag{11}$$

The general solution of Eq. 11 is given by the following expression:

$$\phi\left(\xi\right) = C_1 e^{-\sqrt{Bi}\cdot\xi} + C_2 e^{\sqrt{Bi}\cdot\xi} \tag{12}$$

The values of the integration constants C_1 and C_2 are determined from the boundary conditions of the problem. In order to establish these boundary conditions, it is initially considered that the temperature at the base of the fin is known, that is, $x = 0$, $T_{x=0} = T_B$. Therefore, the first condition of contour is given by (Saha et al., 2019, 2020):

$$\begin{cases} x = 0\,;\,\xi = 0 \\ \phi\left(0\right) = \dfrac{T_B - T_F}{T_B - T_F} = 1\,;\,C_1 + C_2 = 1 \end{cases} \tag{13}$$

The condition given in Eq. 13 is common to fins of constant cross section. The second boundary condition takes various ways according to the thermal state at the other end of the fin, which are:

Very long fin: The temperature of its free extreme is equal to the temperature of the fluid that surrounds the fin.

$$\begin{cases} T_{X\rightarrow\infty} = T_F\,;\,\xi = x/L = 1 \\ \phi\left(1\right) = \dfrac{T_B - T_F}{T_B - T_F} = 0\,;\,C_1 e^{-\sqrt{Bi}} + C_2 e^{\sqrt{Bi}} \end{cases} \tag{14}$$

Considering that L is very large and that the Biot number is proportional to L^2, then:

$$0 + C_2 e^{\sqrt{Bi}} = 0\,;\,C_2 = 0\,;\,C_1 = 1 \tag{15}$$

Thus, the temperature distribution is given by (Razelos, 2003):

$$\begin{aligned} \phi\left(\xi\right) &= \frac{T_\xi - T_F}{T_B - T_F} = e^{-\sqrt{Bi\xi}} \\ T_\xi &= T_F + \left(T_B - T_F\right)\cdot e^{-\sqrt{Bi\xi}} \end{aligned} \tag{16}$$

In order to determine the heat dissipated, it is considered that the heat transferred by convection is equal to the heat that is added by conduction to the fin at its base ($x = 0$), therefore:

$$\begin{aligned} Q = -\lambda S\left(\frac{\partial T}{\partial x}\right)_{x=0} &= \frac{-\lambda S}{L}\left(T_B - T_F\right)\left(\frac{\partial\phi\left(\xi\right)}{\partial\xi}\right)_{\xi=0} \\ &= \frac{-\lambda S}{L}\left(T_B - T_F\right)\cdot\sqrt{Bi}\cdot\left(C_1 - C_2\right) = \left|\begin{array}{c} C_1 = 0 \\ C_2 = 1 \end{array}\right| \\ &= \frac{-\lambda S}{L}\left(T_B - T_F\right)\cdot\sqrt{Bi} \end{aligned} \tag{17}$$

Fin with convection at its free end: In this type of fin, the boundary conditions at its free end for $\xi = 1$, are (Razelos, 2003):

$$\begin{cases} -\lambda \left.\dfrac{\partial T}{\partial x}\right|_{x=L} = \alpha\left(T-T_F\right)_{x=L} = \alpha\phi\left(1\right)\left(T-T_F\right) \\ -\lambda \left.\dfrac{\partial T}{\partial x}\right|_{x=L} = \dfrac{-\lambda\left(T_B-T_F\right)}{L}\left.\dfrac{d\phi}{dx}\right|_{\xi=1} \end{cases} \tag{18}$$

With this condition and having a constant temperature at the base, the solution of the governing differential equation is:

$$\frac{-\alpha L}{\lambda}\left(C_1 e^{-\sqrt{Bi}} + C_2 e^{\sqrt{Bi}}\right) = -\sqrt{Bi}C_1 e^{-\sqrt{Bi}} + \sqrt{Bi}C_2 e^{\sqrt{Bi}} \tag{19}$$

Solving the integration constant C_1 in Eq. 19

$$C_1 = C_2 \frac{e^{\sqrt{Bi}}\cdot\left(\sqrt{Bi}+\frac{\alpha L}{\lambda}\right)}{e^{-\sqrt{Bi}}\cdot\left(\frac{\alpha L}{\lambda}-\sqrt{Bi}\right)} = 0 \tag{20}$$

From Eq. 13 $C_1 + C_2 = 1$, then:

$$C_1 = \frac{e^{\sqrt{Bi}}\cdot\left(\sqrt{Bi}+\frac{\alpha L}{\lambda}\right)}{\sqrt{Bi}\cdot\left(e^{\sqrt{Bi}}+e^{-\sqrt{Bi}}\right)+\frac{\alpha L}{\lambda}\cdot\left(e^{\sqrt{Bi}}-e^{-\sqrt{Bi}}\right)} = \frac{0.5\cdot e^{\sqrt{Bi}}\cdot\left(\sqrt{Bi}+\frac{\alpha L}{\lambda}\right)}{\sqrt{Bi}\cdot\cosh\left(\sqrt{Bi}\right)+\frac{\alpha L}{\lambda}\cdot\sinh\left(\sqrt{Bi}\right)} \tag{21}$$

$$C_2 = \frac{e^{-\sqrt{Bi}}\cdot\left(\sqrt{Bi}-\frac{\alpha L}{\lambda}\right)}{\sqrt{Bi}\cdot\left(e^{\sqrt{Bi}}+e^{-\sqrt{Bi}}\right)+\frac{\alpha L}{\lambda}\cdot\left(e^{\sqrt{Bi}}-e^{-\sqrt{Bi}}\right)} = \frac{0.5\cdot e^{-\sqrt{Bi}}\cdot\left(\sqrt{Bi}-\frac{\alpha L}{\lambda}\right)}{\sqrt{Bi}\cdot\cosh\left(\sqrt{Bi}\right)+\frac{\alpha L}{\lambda}\cdot\sinh\left(\sqrt{Bi}\right)} \tag{22}$$

The temperature distribution is given by (Razelos, 2003):

$$\phi\left(\xi\right) = \frac{T_\xi - T_F}{T_B - T_F} = C_1 e^{\xi-\sqrt{Bi}} + C_2 e^{\xi\sqrt{Bi}} \tag{23}$$

Combining Eq. 21 and 22 with Eq. 23:

$$\phi\left(\xi\right) = \frac{T_\xi - T_F}{T_B - T_F} = \frac{e^{\sqrt{Bi}}\cdot\left(\sqrt{Bi}+\frac{\alpha L}{\lambda}\right)e^{-\xi\sqrt{Bi}} + e^{-\sqrt{Bi}}\cdot\left(\sqrt{Bi}-\frac{\alpha L}{\lambda}\right)e^{\xi\sqrt{Bi}}}{2\left(\sqrt{Bi}\cdot\cosh\left(\sqrt{Bi}\right)+\frac{\alpha L}{\lambda}\cdot\sinh\left(\sqrt{Bi}\right)\right)} \tag{24}$$

Arranging Eq. 24:

$$\phi\left(\xi\right) = \frac{\cosh\left(\left(1-\xi\right)\sqrt{Bi}\right)+\frac{\alpha L}{\lambda}\cdot\sinh\left(\left(1-\xi\right)\sqrt{Bi}\right)}{\sqrt{Bi}\cdot\cosh\left(\sqrt{Bi}\right)+\frac{\alpha L}{\lambda}\cdot\sinh\left(\sqrt{Bi}\right)} \tag{25}$$

In Eq. 25:

$$Bi = \frac{\alpha P L^2}{\lambda S} \rightarrow \frac{\alpha L}{\lambda} = \frac{S\cdot Bi}{PL} \tag{26}$$

Substituting Eq. 26 into Eq. 25:

$$\phi\left(\xi\right) = \frac{\cosh\left(\left(1-\xi\right)\sqrt{Bi}\right)+\frac{S\sqrt{Bi}}{P\lambda}\cdot\sinh\left(\left(1-\xi\right)\sqrt{Bi}\right)}{\cosh\left(\sqrt{Bi}\right)+\frac{S\sqrt{Bi}}{PL}\cdot\sinh\left(\sqrt{Bi}\right)} \tag{27}$$

The heat flux is given by:

$$Q = \frac{-\lambda S}{L}\left(T_B-T_F\right)\left.\frac{\partial\phi}{\partial\xi}\right|_{\xi=0} = \frac{\lambda S}{L}\left(T_B-T_F\right)\sqrt{Bi}\cdot\left(C_1-C_2\right) \tag{28}$$

Substituting Eq. 21 and 22 into Eq. 28:

$$Q = \chi\cdot\frac{e^{\sqrt{Bi}}\cdot\left(\sqrt{Bi}+\frac{\alpha L}{\lambda}\right)-e^{-\sqrt{Bi}}\cdot\left(\sqrt{Bi}-\frac{\alpha L}{\lambda}\right)}{\sqrt{Bi}\cdot\cosh\left(\sqrt{Bi}\right)+\frac{\alpha L}{\lambda}\cdot\sinh\left(\sqrt{Bi}\right)} \qquad \chi = \lambda S\left(T_B-T_F\right)\sqrt{Bi}/L \tag{29}$$

Arranging Eq. 29 (Chen et al., 2019):

$$Q = \chi\cdot\frac{\sqrt{Bi}\cdot\sinh\left(\sqrt{Bi}\right)+\frac{\alpha L}{\lambda}\cdot\cosh\left(\sqrt{Bi}\right)}{\sqrt{Bi}\cdot\cosh\left(\sqrt{Bi}\right)+\frac{\alpha L}{\lambda}\cdot\sinh\left(\sqrt{Bi}\right)} = \chi\frac{\tanh\left(\sqrt{Bi}\right)+\frac{\alpha L}{\lambda\sqrt{Bi}}}{1+\frac{\alpha L}{\lambda\sqrt{Bi}}\cdot\tanh\left(\sqrt{Bi}\right)} \tag{30}$$

Simplifying in terms of the Biot number

$$Q = \frac{\lambda S}{L}\cdot\left(T_B-T_F\right)\cdot\sqrt{Bi}\frac{\tanh\left(\sqrt{Bi}\right)+\frac{S\sqrt{Bi}}{PL}}{1+\frac{S\sqrt{Bi}}{PL}\cdot\tanh\left(\sqrt{Bi}\right)}$$
$$Bi = \frac{\alpha P L^2}{\lambda S} = \frac{\alpha 2 a L^2}{a\lambda\cdot e} = \frac{\alpha 2 L^2}{\lambda\cdot e} = -m^2L^2 \qquad \sqrt{Bi} = mL\ ;\ m = \sqrt{2\alpha/\lambda e} \tag{31}$$

Simplifying Eq. 31:

$$Q = m\lambda S\cdot\left(T_B-T_F\right)\cdot\frac{\tanh\left(mL\right)+\frac{\alpha}{\lambda m}}{1+\frac{\alpha}{\lambda m}\cdot\tanh\left(mL\right)} \qquad m = \sqrt{2\alpha/\lambda e} \tag{32}$$

2.2 Elements on the application of the fins of constant cross section

If it is considered that the derivative of the heat flux with respect to the thickness of the fin is equal to zero, $dQ/dL = 0$, substituting this condition in Eq. 32:

$$\frac{dQ}{de} = \Upsilon\cdot\frac{R\cdot\Psi-\left[\tanh\left(mL\right)+\frac{\alpha}{m\lambda}\right]\cdot\frac{\alpha^2\cdot\Psi}{m\lambda}}{\left(1+R\right)^2}$$
$$\Upsilon = m\lambda S\cdot\left(T_B-T_F\right)\ ;\ R = 1+\frac{\alpha}{m\lambda}\cdot\tanh\left(mL\right) \qquad \Psi = m/\left(\cosh^2\left(mL\right)\right) \tag{33}$$

Grouping variables in Eq. 33

$$1+\frac{\alpha}{m\lambda}\cdot\tanh\left(mL\right) = \left[\tanh\left(mL\right)+\frac{\alpha}{m\lambda}\right]\cdot\frac{\alpha}{m\lambda} \qquad 1 = \left(\frac{\alpha}{m\lambda}\right)^2 = \frac{\alpha\cdot e}{2\lambda} \tag{34}$$

Table 1: Influence on the heat transfer by the uses of fins.

Condition	Influence
$(\alpha e/2\lambda) > 1$	Fins produces an insulating or refrigerating effect
$(\alpha e/2\lambda) = 1$	Fins do not produce any effects
$(\alpha e/2\lambda) < 1$	Fins produce an increase of the heat flux

The condition comes true for any value of L and suggests the technical conditions to have in account to lay fins on a surface, (see Table 1).

For heat transfer to be effective, the conditions given in Eq. 35 must be fulfilled.

$$\frac{\alpha e}{2\lambda} \leq 0.2 \; ; \; \frac{\alpha S}{P\lambda} \leq 0.2 \; ; \; \frac{P}{S} = \frac{2\,(a+e)}{a \cdot e} = \frac{2^2}{e} \tag{35}$$

If the selected fin does not fulfill the conditions given in Eq. 35, then is not convenient its use in air cooled condenser equipment (Camaraza-Medina, 2021).

3. Development of the optimal profile for a fin

3.1 Mathematical considerations in the optimal profile development

In the development of the optimal profile of fins, it is very important obtaining an optimal value of the amount of heat transfer for a determined surface Ω, per tube length.

The optimal thickness fulfills additionally that the derivative of the heat flux with respect to the thickness of the fin is equal to zero, therefore:

$$\begin{aligned} Q &= \lambda S \cdot \frac{(T_B - T_F)}{L} \cdot \sqrt{Bi} \tanh\left(\sqrt{Bi}\right) = \\ &= m\lambda S \cdot (T_B - T_F) \cdot \tanh(mL) \\ m &= \sqrt{2\alpha/(\lambda \cdot e)} \end{aligned} \tag{36}$$

Grouping terms in Eq. 36:

$$Q = \begin{cases} \lambda S \cdot \dfrac{(T_B - T_F)}{L} \cdot \sqrt{\frac{2\alpha \cdot Bi}{\lambda \cdot e}} \tanh\left(\sqrt{\frac{2\alpha L}{\lambda \cdot e}}\right) \\ \left| \begin{array}{l} S = a \cdot e \; ; \; a = 1 \\ S = e \; ; \; \Omega = L \cdot e \end{array} \right| \\ (T_B - T_F) \cdot \sqrt{2\alpha\lambda \cdot e} \cdot \tanh\left(\sqrt{\frac{2\alpha\Omega}{\lambda \cdot e}}\right) \end{cases} \tag{37}$$

If the fin has a fixed weight, or it has been selected previously, then, the surface Ω is constant and the Eq. 37 suggests the variation of the thermal flux in terms of the fin thickness e.

Deriving the heat obtained in Eq. 37 with respect to the thickness of the fin, and equating the derivative to zero then:

$$\begin{aligned} \frac{dQ}{de} &= (T_B - T_F) \cdot \left[\frac{2\alpha\lambda}{2\sqrt{2\alpha\lambda \cdot e}}\right] \tanh\left(\sqrt{\frac{2\alpha \cdot \Omega}{\lambda \cdot e^3}}\right) - \\ &- \frac{\sqrt{2\alpha\lambda \cdot e}}{\cosh^2\left(\sqrt{\frac{2\alpha \cdot \Omega}{\lambda \cdot e^3}}\right)} \cdot \frac{\Omega}{2 \cdot \sqrt{\frac{2\alpha}{\lambda \cdot e^3}}} \cdot \frac{6\alpha}{\lambda \cdot e^4} \end{aligned} \tag{38}$$

Applying properties of trigonometric functions:

$$\tanh\left(\sqrt{\frac{2\alpha \cdot \Omega}{\lambda \cdot e^3}}\right) = 3 \cdot \sqrt{\frac{2\alpha \cdot \Omega}{\lambda \cdot e^3}} \cdot \operatorname{sech}^2\left(\sqrt{\frac{2\alpha \cdot \Omega}{\lambda \cdot e^3}}\right) \tag{39}$$

As $\sqrt{\frac{2\alpha}{\lambda \cdot e^3}}\Omega = \sqrt{Bi}$, then substituting in Eq. 39

$$\tanh\left(\sqrt{Bi}\right) = 3\sqrt{Bi} \cdot \operatorname{sech}^2\left(\sqrt{Bi}\right) \tag{40}$$

Solving Equation 40, the optimal value of the Biot number is obtained, $Bi \approx 2.0142$. Therefore, the optimal thickness will be (Camaraza-Medina et al., 2018):

$$\begin{aligned} &\left(\begin{array}{l} m^2 = \dfrac{2\alpha}{\lambda \cdot e} \\ m^2 = \dfrac{Bi}{L^2} = \dfrac{Bi \cdot e^2}{\Omega^2} \end{array} \right) \rightarrow \frac{2\alpha}{\lambda e^3} = \frac{Bi \cdot e^2}{\Omega^2} \\ &e_{Opt} = \sqrt[3]{\frac{2\alpha \cdot \Omega^2}{\lambda \cdot Bi_{Opt}}} = 0.9976 \cdot \sqrt[3]{\frac{\alpha \cdot \Omega^2}{\lambda}} \end{aligned} \tag{41}$$

And the optimal length will be:

$$L_{Opt} = \frac{\Omega}{e_{Opt}} = \frac{\Omega}{0.9976 \cdot \sqrt[3]{\frac{\alpha \cdot \Omega^2}{\lambda}}} = 1.0023 \cdot \sqrt[3]{\frac{\lambda \cdot \Omega}{\alpha}} \tag{42}$$

In general, the physical constant and the operating conditions of the fin are known, (among them, α, λ, Q, $(T_B - T_F)$) therefore, it is possible to obtain another formulation for the optimal dimensions, in terms of these parameters and of the dimensionless optimal number of Biot. This formulation is provided by (Camaraza-Medina, 2021):

$$Q = (T_B - T_F) \cdot \sqrt{2\alpha\lambda \cdot e} \cdot \tanh\left(\sqrt{Bi_{Opt}}\right) \tag{43}$$

Solving for the thickness e in Equation 43, then the optimal thickness for the maximum amount of heat transfer can be determined, described mathematically by means of the following relation:

$$\begin{aligned} e_{Opt} &= \left(\frac{Q}{T_B - T_F}\right)^2 \cdot \frac{0.5}{\alpha\lambda \cdot \tanh^2\left(\sqrt{Bi}\right)} = \\ &= \frac{0.63202}{\alpha \cdot \lambda} \cdot \left(\frac{Q}{T_B - T_F}\right)^2 \end{aligned} \tag{44}$$

Equating Eqs. 44 and 41 the proposed equation for the design of fins with constant thickness is obtained, leading to a minimal use of material (Wen and Ho, 2009; Camaraza-Medina, 2021; Camaraza-Medina et al., 2022):

$$e_{Opt} = \frac{0.63202}{\alpha \cdot \lambda} \cdot \left(\frac{Q}{T_B - T_F}\right)^2 = 0.9976 \cdot \sqrt[3]{\frac{\alpha \cdot \Omega^2}{\lambda}} \tag{45}$$

Eq. 45 contains two equalities:

$$e_{Opt} = \frac{0.63202}{\alpha \cdot \lambda} \cdot \left(\frac{Q}{T_B - T_F}\right)^2 \tag{46}$$

$$e_{Opt} = 0.9976 \cdot \sqrt[3]{\alpha \cdot \Omega^2/\lambda} \tag{47}$$

Table 2: Validity range for Eq. 50 to 59.

Parameter	validity range
AISI-SAE	1008 ; 1030 ; 1045 ; 1078 ; 1095 ; 1145 ; 1345 ; 1524 ; 2330 ; 2515 ; 301 ; 302 ; 304 ; 310 ; 316 ; 347 ; 4028 ; 405 ; 410 ; 4130 ; 4140 ; 420 ; 430 ; 4320 ; 4626 ; 5132 ; 5140 ; 6150 ; 8115 ; 8617 ; 8650 ; 8822
T (°C)	0 °C to 8×10^2 °C
Composition (%)	$0.05 \leq C \leq 1$; $0.3 \leq Mn \leq 2$; $0.015 \leq P \leq 0.04$; $0.02 \leq S \leq 0.1$; $0.12 \leq Si \leq 1.5$; $0.2 \leq Ni \leq 22$; $0.2 \leq Cr \leq 26$; $0.08 \leq Mo \leq 3$; $0.1 \leq V \leq 0.15$

Equating Eq. 46 and 47 the optimal fin surface Ω can be obtained:

$$\Omega_{Opt} = \frac{0.5048}{\alpha^2 \lambda} \cdot \left(\frac{Q}{T_B - T_F}\right)^3 \tag{48}$$

In Eq. 42 it is shown that for fins of constant thickness it comes true that $L_{Opt} = \omega / e_{Opt}$, therefore, combining Eqs. 48 and 43 the value of the optimal length of the fin is obtained:

$$L_{Opt} = \frac{0.7979Q}{\alpha\,(T_B - T_F)} \tag{49}$$

In Eq. 48, the thermal conductivity can be computed as (Roque-Villalonga and Camaraza-Medina, 2022; Camaraza-Medina et al., 2022):

$$Y_1 = -0.002 \cdot (R_1)^2 + 0.023 \cdot R_1 - 0.043 \tag{50}$$

$$Y_2 = -8.705 \cdot R_1 + 55.93 \tag{51}$$

$$Y_3 = -0.002 \cdot (R_2)^2 + 0.0092 \cdot R_2 - 0.0098 \tag{52}$$

$$Y_4 = -4.812 \cdot (R_2)^2 + 9.96 \cdot R_2 - 3.386 \tag{53}$$

$$Y_5 = 2454 \cdot (R_2)^2 - 8331 \cdot R_2 + 16080 \tag{54}$$

$$\overline{\lambda}_{(1)} = Y_1 \cdot T + Y_2 \tag{55}$$

$$\overline{\lambda}_{(2)} = Y_3 \cdot T^2 + Y_4 \cdot T + Y_5 \tag{56}$$

$$\overline{\lambda}_{(est)} = \overline{\lambda}_{(1)} \cdot \overline{\lambda}_{(2)} / 10^4 \tag{57}$$

$$R_1 = \sqrt{C + Cr} \tag{58}$$

$$R_2 = \sqrt[4]{Mn + S + P + Si + Ni + Mo + V} \tag{59}$$

In Eq. 50 to 59:
C: Elementary composition of the steel (Carbon), %
Cr: Elementary composition of the steel (Chrome), %
$\overline{\lambda}_{(1)}$: Thermal conductivity for first iteration.
$\overline{\lambda}_{(2)}$: Thermal conductivity for second iteration.
$\overline{\lambda}_{(est)}$: Thermal conductivity calculated, W/(m·K)
Mn: Composition of the steel (Manganese), %
Mo: Composition of the steel (Molybdenum), %
Ni: Composition of the steel (Nickel), %
P: Composition of the steel (Phosphor), %
R_1: Composition factor 1, defined in Eq. 58
R_2: Composition factor 2, defined in Eq. 59
S: Composition of the steel (Sulphur), %
Si: Composition of the steel (Silicon), %
T: Temperature, °C
V: Composition of the steel (Vanadium), %
Y_1: Adjust parameter for the first iteration.
Y_2: Adjust parameter for the first iteration.
Y_3: Adjust parameter for the second iteration.
Y_4: Adjust parameter for the second iteration.
Y_5: Adjust parameter for the second iteration.

In Table 2 the validity range for Eq. 50 to 59 is detailed, while the Fig. 2 provides a graphical representation of these equations.

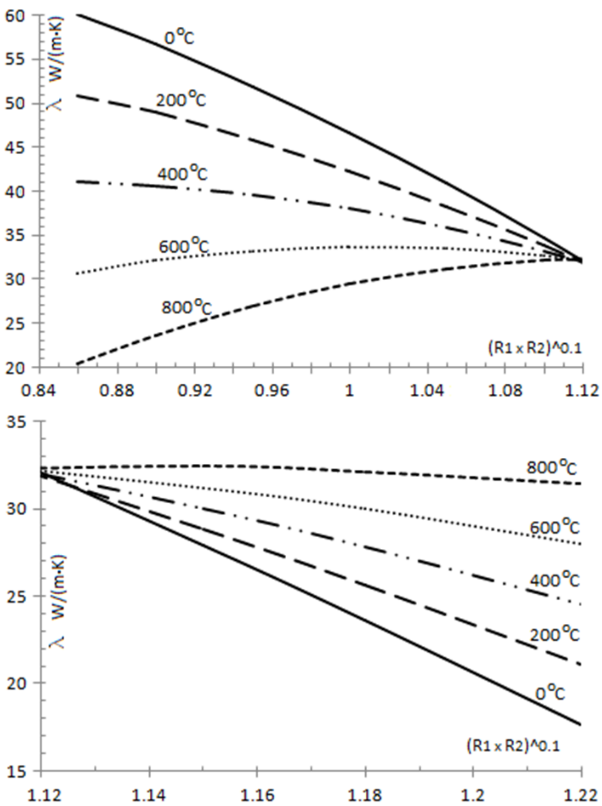

Figure 2: Graphical representation of the Eq. 50 to 59.

4. Conclusions

In the present paper, the influence of the temperature and composition on steel thermal conductivity, and its influence in the optimal profile for a fin of constant cross section to be used in air cooled condenser is presented.

Additionally, three geometrical parameters elements to accomplish the study, (thickness, height and surface of heat transfer) of fins of constant cross section, that have directly influence on the optimal fin profile for conditions of maximum heat flux and optimal Biot number, were considered.

The optimal parameters of thickness, length and heat transfer surface of a fin are inversely proportional to the gradient of temperatures in the fin, to the thermal conductivity of the material and to the average heat transfer coefficient. The convective heat transfer coefficient has the strongest influence.

The heat flux has influence right into the increment of the optimal required dimensions, showing in the three cases a potential of the same-order behavior than the gradient of temperatures.

A way to establish optimal parameters for a minimal cost can be attained by manipulating temperature gradients, simultaneously with using materials with high values of thermal conductivity.

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