SOME NONLINEAR MECHANICAL PROBLEMS SOLVED WITH ANALYTICAL SOLUTIONS

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Abstract— **In this paper the analytical solution of nonlinear ordinary differential systems is addressed. Some of the problems are classical in the related literature and exhibit chaotic behavior in certain ranges of the involved parameters despite being simple-looking deterministic systems. The solutions are approached by means of the old technique of power series to solve ordinary differential equations. The independent variable is time in all the illustrations and elementary recurrence algorithms are obtained. This is an alternative to the standard numerical techniques and ensures the theoretical exactness of the response. Several examples are included and trajectories diagrams, phase plots, etc. are shown. The desired numerical precision is attained using time steps several times larger than the usual ones. The availability of an analytical solution may be an additional tool within a standard qualitative analysis. The solution of higher order problems and governed by partial differential equations is under study.**

Keywords— **ordinary differential equations, nonlinear equations, power series.**

I. INTRODUCTION

Power series is an old technique to solve ordinary differential equations (ODE's). A wide open literature is available on the subject. Simmons, 1972, Coddington, 1989, Kreyszig, 1999 may be useful as references in this methodology. The efficiency of this standard technique in solving linear ODE's with variable coefficients is well known. Also an extension known as Frobenious method allows to tackle differential equations with coefficients that are not analytic. Numerical tools such as time integration schemes (e.g. Runge-Kutta, Newmark method, central difference, see for instance Bathe, 1995) are commonly employed to solve nonlinear differential problems. The authors have addressed similar problems with a variational method named WEM (Rosales and Filipich, 2000, 2002). The authors have applied power series numerical tools in various problems (Filipich and Rosales, 2001a, 2002).

A method to solve nonlinear differential problems governed by ordinary equations (ODEs) is herein employed. The solution is found with an analytical solution using algebraic series. A previous manipulation of the equations leads to very convenient recurrence algorithms which ensure the exactness of the solution as well as the computational efficiency of the method. The approach is straightforward and is illustrated with several problems, i.e. a) projectile motion; b) N bodies with gravitational attraction; c) Lorenz equations; d) Duffing equations and, e) a strong nonlinear oscillator. In all the cases the results are given in plots (state variables vs. time, phase plots, Poincaré maps). Neither divergence nor numerical damping was found in any case. The availability of an analytical solution may be also a helpful tool in the qualitative analysis of nonlinear equations.

In this section the general algebra of the approach is stated. The examples will be presented in the following sections Let us consider an analytical function $x = x(\tau)$ in $[0,1]$. We will denote its expansion in power series as (with $N \to \infty$, theoretically)

$$[x] = \sum_{k=0}^{N} a_{1k}\ \tau^k \tag{1}$$

and for powers m

$$[x^m] = \sum_{k=0}^{N} a_{mk}\ \tau^k \tag{2}$$

In order to fulfill an *algebraic consistence (A.C.) condition* the following relationships have to be satisfied

$$[x^m] = \left[x^{m-1}\right][x] \tag{3}$$

After replacing the series expressions in each factor of this equation, one obtains the next recurrence formula