ON THE FINITE ELEMENT APPROXIMATION FOR DIFFUSIVE AND MIGRATORY MOVEMENT OF A POPULATION INFECTED BY THE FOOT-AND-MOUTH DISEASE VIRUS

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Abstract— **The foot-and-mouth disease is a contagious virus frequently present in cattle and chicken and swine raising that produces a hard loss of animal weight and can contaminate the farm pasture and its all environment. Considering that, many a researcher has been working to establish biomathematical models in attempt to simulate the dissemination of this pathology. This article aims to build finite element approximations for the spatial diffusive and migratory movement of a population infected by the foot-and-mouth virus in a healthy beef zone subjected to low and high migratory fluxes. The biomathematical model adopted gives rise to a spatial boundary-value problem, which includes diffusive and migratory and mortality/natality effects. The numerical simulations performed have attested the instability of the Galerkin method when subject to very high migratory fluxes and deathrates. As a remedy for the Galerkin numerical pathologies, a stabilized finite element formulation - the so-called Gradient-Galerkin/Least-Squares method - formulation was employed.**

Keywords— **Foot-and-mouth disease, spatial populational model, finite element approximations, stabilized method.**

I. INTRODUCTION

The foot-and-mouth disease is a contagious virus that plays an important role in the international food commercialization. This kind of disease, frequently present in cattle and chicken and swine raising, produces a hard loss of animal weight and can contaminate the farm pasture and all its environment. (For practical purposes, this sickness may be treated as a fatal one, besides being highly contagious.) Then, a single cattle fever can render unusable stables, transportation trucks, feeders and even the slaughterhouse installations, spreading in this way the disease to a large number of healthy animals. Taking into account the severe economic consequences of this viral fever, various efficient vaccines have been already developed, and all countries have implemented official programs for foot-and-mouth disease control (Obiaga *et al.*, 1979). In parallel, all beef producer has to exhibit an official sanity certificate for every animal commercialization.

To deal with this complex tableau, we have, in South America, an official association of the World Healthy Organization (WHO) named *Pan-American Healthy Organization (PAHO)*. When the PAHO was founded, there was an appalling situation in South America once that this virus infected almost all livestock. First, the efforts of the Organization were dedicated to establish a profound epidemiological study in order to classify all types of existing viruses. Next, the Center set out an extensive vaccination program aiming the whole eradication of the foot-and-mouth disease in our continent. Furthermore, the PAHO has an additional educational program to disseminate the idea of the importance of foot-and-mouth disease control among producer and population of the involved countries. Decades have been passed since its foundation and, nowadays, this disease is nearly controlled in South America. Countries like Chile, Uruguay and Argentine are already free from foot-and-mouth disease. But, in the other hand, there are other ones, among them we can list Brazil, Peru, Colombia and Venezuela, where the control is still in progress.

In Brazil, the states of Rio Grande do Sul, Santa Catarina and Paraná - in southern region, São Paulo and Minas Gerais - in southeastern, Mato Grosso and Goiás - middle-eastern, have a leading participation in the national beef production, with intense internal fluxes of animals within these regions. Unfortunately, the foot-and-mouth disease still occurs in some of these