A numerical study on MHD flow of a Casson fluid passing through a vertical cone with suction velocity and thermo-diffusive effects

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ABSTRACT A mathematical model is developed for an unsteady MHD flow of a Casson fluid passing through a vertical cone. The present study examines the thermo-diffusive effects for a free convective flow and the role of suction on the fluid passing through a vertical cone. The effect of chemical reaction has been scrutinized. The radiative heat flux in the energy equation is described by the Rosseland approximation. The non-dimensional governing equation of the fluid problem are discretized by finite difference method to a system of algebraic equations and solved numerically subject to the relevant boundary conditions. Numerical solution of velocity, temperature, and concentration along with the influence of different parameters are obtained and their nature is described graphically. The results of the study indicate a considerable effect of parameters on velocity, temperature, concentration. Casson parameter has considerable influence on rate of flow and it has its impact on the skin friction. The rate of heat and mass transfer has also been influenced by the existing parameters.

KEYWORDS MHD; Unsteady; Casson; Soret; Suction

1. Introduction

Magnetohydrodynamics (MHD) is the discipline which includes the study of behaviour of electrically conducting fluids. This study has been able to attract numerous researchers and engineers for the last several decades because of its fascination and importance in various technological devices. MHD convection flow has significant applications in different fields of chemical engineering, astrophysics, geophysics, etc.

In modern technology and engineering, the characteristics of fluid flow are not expicable with the newtonian fluid model, so in most of the fluid problem we consider non-Newtonian fluid. The analysis of non-Newtonian fluid flow through a different medium has made influential impact on many researchers. Along with the convective fluid flow there has also been some attentions in the study of heat and mass transfer for a fluid flow problem with various boundary conditions. Heat transfer for a non-Newtonian flow is significant in many industries for manufacturing processes. According to different rheological properties of non-Newtonian fluids existing in nature, there is no specific constitutive relation between stress and rate of strain for observation of non-Newtonian fluids.

Casson fluid is a simple non-Newtonian fluid generally considered for rheological effects of viscoelastic fluids.

It was initially found out by Casson (1959). Later, Bird et al. (1983) analyzed that Casson fluid model shows high viscosity shear thinning properties and yield stress below which no flow occurs. In addition to all these characteristics, it estimates practically the rheological properties of other liquids such as honey and juice etc.

So, the study of Casson fluid has found wide applications in different pharmaceutical, chemical and cosmetic industries for production of various chemicals, cosmetics, syrup, cleanser etc.

With all this application of Casson fluid many researchers have widely extended their research on convective fluid flow passing through in different medium. Kodi and Mopuri (2022) analyzed the characteristics of Casson fluid flow with Soret-aligned magnetic field in an inclined plate with suction velocity. They made comparisons of the previously performed work by Choudhury and Ahmed (2018). Talukdar and Nath (2018) made an analytical study on transient MHD Casson fluid flow through an inclined moving plate.

The analysis of fluid flow through different physical configurations and different medium has recently been able to draw the attention of many researchers. In the intervening years, MHD flow of fluid with chemical reaction has received considerable attention due to its significant importance of heat and mass transfer involving in chemical reaction. Along with it the radiative transfer of heat has its wide applications for the design of nuclear plants and various devices for missiles, satellites, etc.

Keeping in view with all these significant applications, Takhar et al. (2003) investigated the unsteady mixed convective flow through a rotating vertical cone with the influence of magnetic field and time-dependent angular velocity. Their results indicate the significant effects of magnetic field on velocity and temperature and the effect of the decreasing angular velocity on the temperature and the velocity fields is not directly proportional to the increasing angular velocity. Dutta and Sharma (2019) made a numerical analysis for a convective flow passing through a vertical cone under the influence of

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thermal radiation and reaction by chemical. Sharma and Kumawat (2021) numerically studied the MHD convective blood flow through the stretching surface under the combined influence of ohmic heating, thermal conductivity, and temperature-dependent viscosity. Considering a vertical stretching surface Sharma et al. (2022b) has examine the flow of Jeffrey fluid with the effect of exponential space and thermal-dependent heat sources. Kumawat et al. (2021) mathematically analyzed the flow of two-phase blood through a stenosed curved artery with the influence of hematocrit and temperature-dependent viscosity. Sharma et al. (2023a) has numerically illustrated the effect of Arrhenius activation and micro-organisms through an inclined stretching sheet with the effect of porosity, thermo-phoresis, and Brownian motions. Along with the effect of thermophoresis and Brownian motion Sharma et al. (2023b) has extended the study of MHD mixed convective flow through a stretching surface with chemical reaction and thermal radiations.

Chamkha and Rashad (2014) numerically analyzed the effect of Soret and Dufour on an unsteady mixed convective flow over a rotating vertical cone with the influence of chemical reaction and magnetic field. Vijayaragavan et al. (2021) analytically studied the heat and mass transfer effect of MHD Casson fluid flow past through an inclined plate with the effect of Dufour number and reaction by chemical. Here perturbation method is used to find the expression for fluid velocity, concentration, and temperature. Venkateswarlu and Satya Narayana (2016) investigated the convective Casson fluid flow with Soret and Dufour effects and under the influence of variable thermal conductivity. They have considered electrically conducting fluid flow which is induced by Stretching surface. Their results indicate the significant impact of thermal conductivity parameter on the rate of heat and mass transfer. Seth et al. (2018) studied the MHD convective flow of a Casson fluid near a vertical oscillating plate in a non-Darcy porous medium. The non-dimensional governing equations of the transient MHD flow problem are solved numerically by implicit finite difference technique of Crank-Nicolson type. Sharma and Gandhi (2022) have numerically analyzed the heat and mass transfer of viscous incompressible fluid embedded in a Darcy-Forchheimer porous medium and passing through a vertical stretching surface. Sharma et al. (2022c) investigated the effect of magnetic field and porosity on an unsteady oscillatory Couette flow. Couette flow is a flow that occurs on the fluctuation of temperature and species concentration with time around a non-zero constant.

Kim (2000) analyzed the unsteady convective laminar flow past through a semi-infinite moving vertical plate in porous medium. There exists a uniform magnetic field that acts perpendicular to the porous medium that absorbs the fluid with a time varying suction velocity. Ahmed (2016) analytically studied the MHD convective mass transfer transient flow through an infinite vertical porous plate under the influence of thermal radiation and with suction velocity.

Entropy generation for a MHD flow have recently been able to grap the attention for different researchers. Many researchers have contributed their work on entropy generation. Recently Sharma et al. (2022a) has described the study of entropy generation of convective fluid flow over a stretching surface. Khanduri and Sharma (2022) describe the entropy of a MHD flow subject to thermal conductive and temperature- dependent viscosity.

The present study of MHD free convective flow through a vertical cone surface is expected to be used in various engineering phenomena and will have significant role in manufacturing process. The chemical reaction in a flow generally changes the physical state of matter, such as melting of ice to water and water evaporating to vapour. So, it plays and important role in the process of heat and mass transfer. The effect of radiation can be seen in various space technology and in design of nuclear power plants. Thermo-diffusion is process of mass transfer caused due to temperature gradient. It has its wide applications in geothermal systems, heat insulations and drying technology, etc. Hence from the above properties of different parameters and its wide applications in various fields has motivated to do the present analysis. The novel aspects of the present study are:

- To examine the thermos-diffusion effect on a free convective flow through a vertical cone.
- To study the combined effect of thermal radiations and chemical reactions on the rate heat and mass transfer.
- As suction plays an important role to prevent the boundary layer separations. We address the suction velocity to analyse its influence on the fluid properties.
- The equations that govern the flow problem are discretize by finite difference methods and solved numerically in MATLAB.

2. Mathematical modelling

We consider a one-dimensional free convective unsteady flow of an incompressible Casson fluid through a vertical cone. The fluid is viscous and electrically conducting with the influence of reaction by chemical and with a non-uniform concentration and temperature existing on the surface of the fluid. The viscous dissipation effects and pressure gradient are negligible along the boundary layer. On the surface of the cone a uniform transverse magnetic field is applied normally. All the fluid properties except the density is constant (Fig. 1).

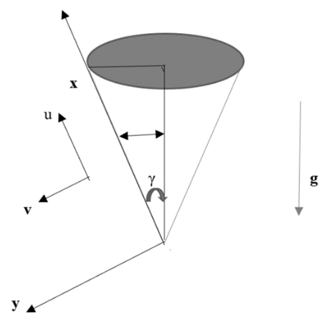

Figure 1: Schematic diagram.

We consider a coordinate system where x-axis is the along the surface of the cone from the apex and the y-axis is the perpendicular distance outside. The temperature T_∞ and concentration C_∞ on the surrounding fluid and surface of the cone are assumed to be same.

At a time, $t > 0$ the temperature of the cone surface increases to $T_\infty + ax_n$ and concentration near the cone surface increases to $C_\infty + bx_m$.

With respect to the above assumptions and under Boussinesq approximation and as specified by Dutta and Sharma (2019), the leading equations and the corresponding boundary conditions for the fluid problem are:

$$\frac{\partial (rv)}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y} = \nu\left(1+\frac{1}{\alpha}\right)\frac{\partial^2 y}{\partial x^2} + g\beta\left(T_w - T_\infty\right)\cos\gamma + \\ + \beta^*\left(C_w - C_\infty\right)\cos\gamma - \frac{\sigma}{\rho}B_0^2 u \tag{2}$$

$$\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho C_p}\frac{\partial q_r}{\partial y} \tag{3}$$

$$\frac{\partial C}{\partial t} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} - K\left(C - C_\infty\right) + D_1\left(\frac{\partial^2 T}{\partial y^2}\right) \tag{4}$$

The initial and the boundary conditions are:

$$u = 0,\ T_w(x) = T_\infty + ax_n,\ C_w(x) = C_\infty + bx_m \text{ at } y = 0 \tag{5}$$

$$u \to 0,\ T \to 0,\ C \to 0 \text{ as } y \to \infty \tag{6}$$

The equation of continuity (Eq. 1) leads that v is either a constant or a function of time, so we assume

$$v = -v_0\left(1 + A\varepsilon e^{nt'}\right) \tag{7}$$

where v_0 is a non-zero positive constant, which is the scale of suction velocity, A is the positive constant and ε and $A\varepsilon$ are less then unity. The suction velocity is exponentially varying with time.

The radiative term $\frac{\partial q_r}{\partial y}$ present in the energy Eq. 3 can be simplified by using Rosseland approximation as given below:

$$q_r = -\frac{4\sigma_s}{3k_e}\frac{\partial T^4}{\partial y} \tag{8}$$

where Stefan-Boltzmann constant is σ_s and the mean absorption coefficient is k_e.

The temperature difference within the flow is assumed to be sufficiently small, so T^4 can be expressed as a linear function. By using Taylor's series, we expand T^4 about the free stream temperature. Hence neglecting the higher order terms the result of the approximation transforms into the form:

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \tag{9}$$

Using Eq. 8 and 9, we have

$$\frac{\partial q_r}{\partial y} = -\frac{16\sigma_s T_\infty^3}{3k_e}\frac{\partial^2 T}{\partial y^2} \tag{10}$$

Then Eq. 3 takes the form:

$$\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{16\sigma_s T_\infty^3}{3k_e\rho C_p}\frac{\partial^2 T}{\partial y_2} \tag{11}$$

We use the following non-dimensional quantities:

$$y' = \frac{y}{L}\left(Gr_L\right)^{\frac{1}{4}},\ r' = \frac{r}{L},\ \text{where } r = x\sin\phi$$
$$u' = \frac{uL}{\nu}\left(Gr_L\right)^{-\frac{1}{2}},\ v' = \frac{vL}{\nu}\left(Gr_L\right)^{-\frac{1}{4}},\ t' = \frac{\nu t}{L^2}\left(Gr_L\right)^{\frac{1}{2}}$$
$$\theta = \left(\frac{T - T_\infty}{T_w - T_\infty}\right),\ \phi = \left(\frac{C - C_\infty}{C_w - C_\infty}\right),\ Pr = \frac{\nu}{\alpha},\ Sc = \frac{\nu}{D}$$
$$Gr_L = \frac{g\beta\left(T_w - T_\infty\right)L^3\cos\gamma}{\nu^2},\ M = \frac{\sigma\left(\beta_0\right)^2 L^2}{\nu}\left(Gr_L\right)^{-\frac{1}{2}}$$
$$Gr_C = \frac{g\beta_C\left(C_w - C_\infty\right)L^3\cos\gamma}{\nu^2},\ Ra = \frac{16\sigma_s T_\infty^3}{3k_e\rho\alpha C_p},\ N = \frac{Gr_C}{Gr_L}$$
$$k' = \frac{KL^2}{\nu}\left(Gr_L\right)^{-\frac{1}{2}},\ Sr = \frac{D_1\left(T_w - T_\infty\right)}{\nu\left(C_w - C_\infty\right)},\ D_1 = \frac{D_M K_T}{T_m}$$

Equations 1, 2, 11 and 4 reduces to the following non-dimensional form:

$$\frac{\partial\left(r'v'\right)}{\partial y'} = 0 \tag{12}$$

$$\frac{\partial u'}{\partial t'} - v_0\left(1 + A\varepsilon e^{nt'}\right)\frac{\partial u'}{\partial y'} = \left(1 + \frac{1}{\alpha}\right)\frac{\partial^2 u'}{\partial y'^2} + \\ + \left(\theta + N\phi\right) - Mu' \tag{13}$$

$$\frac{\partial\theta}{\partial t'} - v_0\left(1 + A\varepsilon e^{nt'}\right)\frac{\partial\theta}{\partial y'} = \frac{1}{Pr}\left(+1\frac{4}{3Ra}\right)\frac{\partial^2\theta}{\partial y'^2} \tag{14}$$

$$\frac{\partial\phi}{\partial t'} - v_0\left(1 + A\varepsilon e^{nt'}\right)\frac{\partial\phi}{\partial y'} = \frac{1}{Sc}\frac{\partial^2\phi}{\partial y'^2} - Kr'\phi + Sr\frac{\partial^2\theta}{\partial y'^2} \tag{15}$$

The corresponding non-dimensional boundary conditions are:

$$u' = 0,\ \theta = x'^n,\ \phi = x'^m \text{ at } y' = 0 \tag{16}$$

$$u' \to 0,\ \theta \to 0,\ \phi \to 0 \text{ as } y' \to \infty \tag{17}$$

The non-dimensional skin friction (τ), Nusselt number (Nu) and Sherwood number (Sh) is given by:

$$\tau = -\left(\frac{\partial u'}{\partial y'}\right)_{y'=0},\ Nu = -x'\left(\frac{\partial\theta}{\partial y'}\right)_{y'=0}, \\ Sh = -x'\left(\frac{\partial\phi}{\partial y'}\right)_{y'=0} \tag{18}$$

3. Solution of the problem

The above transformed Eq. 12-15 are coupled non-linear partial differential equations. As the analytical or exact solutions seems to be not feasible so we use Finite Difference Method (FDM) method to solve the differential equations. This method is comparatively precise, effective and has better stability characteristics. Here we discretized the governing PDE about the point (i,j) and reduce it to a system of algebraic equations. The following equations are the reduced form of finite difference approximation by truncating the higher order terms:

$$\left.\frac{\partial u}{\partial t}\right|_{i,j} = \frac{u_{i+1,j} - u_{i,j}}{\Delta t}, \quad \left.\frac{\partial u}{\partial y}\right|_{i,j} = \frac{u_{i,j+1} - u_{i,j}}{\Delta y}$$

$$\left.\frac{\partial^2 u}{\partial y^2}\right|_{i,j} = \frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{(\Delta y)^2}$$

The equation above is used to discretize the PDE and then implement a numerical method to solve. The equivalent finite difference scheme for Eq. 13 to 15 is as follows:

$$\frac{u_{i+1,j} - u_{i,j}}{\Delta t} - v_0\left(1 + A\varepsilon e^{nt'}\right)\frac{u_{i,j+1} - u_{i,j}}{\Delta y} = \left(1 + \frac{1}{\alpha}\right)\frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{(\Delta y)^2} + \theta_{i,j} + N\phi_{i,j} - Mu_{i,j} \tag{19}$$

$$\frac{\theta_{i+1,j} - \theta_{i,j}}{\Delta t} - v_0\left(1 + A\varepsilon e^{nt'}\right)\frac{\theta_{i,j+1} - \theta_{i,j}}{\Delta y} = \frac{1}{Pr}\left(+1\frac{4}{3Ra}\right)\frac{\theta_{i,j+1} - 2\theta_{i,j} + \theta_{i,j-1}}{(\Delta y)^2} \tag{20}$$

$$\frac{\phi_{i+1,j} - \phi_{i,j}}{\Delta t} - v_0\left(1 + A\varepsilon e^{nt'}\right)\frac{\phi_{i,j+1} - \phi_{i,j}}{\Delta y} = \frac{1}{Sc}\frac{\phi_{i,j+1} - 2\phi_{i,j} + \phi_{i,j-1}}{(\Delta y)^2} - Kr'\phi + Sr\frac{\theta_{i,j+1} - 2\theta_{i,j} + \theta_{i,j-1}}{(\Delta y)^2} \tag{21}$$

where Δt is the dimensionless time-step and Δy is the dimensionless finite difference grid size in the y-direction. Here i is the grid point in the time variable t and j designates the grid point along the y-directions.

The mesh size is considered as $\Delta y = 0.1$ with time step $\Delta t = 0.14$. The order of accuracy of this method is $o\left[(\Delta t), (\Delta y)^2\right]$. The scheme is unconditionally stable and so this method is compatible.

4. Result and discussion

An unsteady MHD fluid flow under the influence of different parameters like chemical reaction, thermal radiation and Soret effect along with suction velocity through a vertical cone has been investigated. The non-dimensional governing equations has been discretized to a system of algebraic equations. This system of algebraic equations together with the boundary conditions is solved numerically by Gauss-Seidel iterative method. The numerical results along with the influence of the parameters have been displayed through figures and tables. Throughout our discussion we have chosen arbitrarily some of the values for the parameters such as Soret number (Sr), radiation parameter ($Ra = 5$), chemical reaction parameter ($Kr = 1$) and Casson parameter ($\alpha = 0.5$). We have considered the value of Prandtl number Pr to be 0.7 which corresponds to air in a range where the hydrodynamic and the thermal boundary layer are approximately equal. The value of the Schmidt number is taken as $Sc = 0.22$ which corresponds to Hydrogen diffused in air.

The validation of the present work with the previously published work by Dutta and Sharma (2019) on MHD has been compared graphically through the Fig. 2, 4 and 10. This present study is an extension of the work done by Dutta and Sharma (2019) by considering a Casson fluid with the influence thermos-diffusive effect along with suction velocity.

Figures 2-11 illustrate the influence of embedded flow parameters on concentration, temperature, and velocity. The variation of temperature with the addition of radiation parameter and Prandtl number are pictorially described in Fig. 2 and 3. In both the figures the temperature profile is maximum at the plate, and it tends to zero as $y \to \infty$. It is clear from Fig. 2 that temperature distribution falls as the radiation parameter rise, that is for increasing value of radiation parameter the thickness of thermal boundary layer decreases. Prandtl number is a parameter that explains

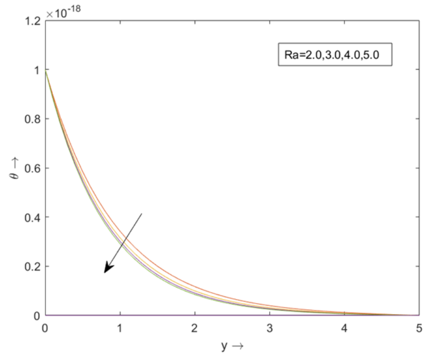

Figure 2: Radiation parameter influence on temperature profile.

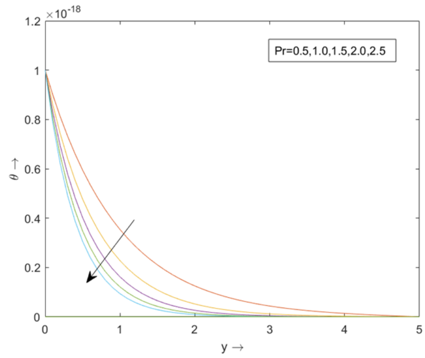

Figure 3: Prandtl number influence on temperature profile.

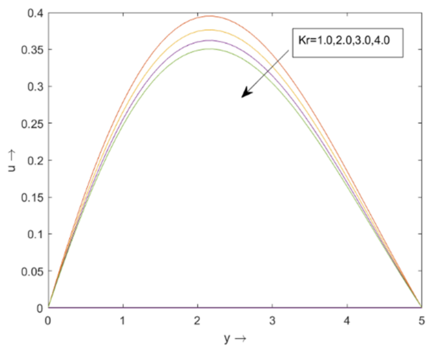

Figure 4: Chemical reaction parameter influence on velocity profile.

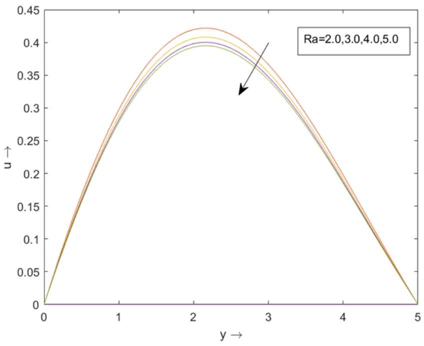

Figure 5: Radiation parameter influence on velocity profile.

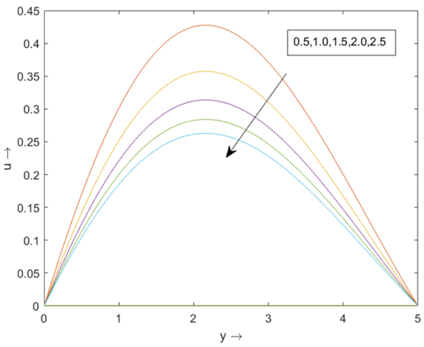

Figure 6: Prandtl number influence on velocity profile.

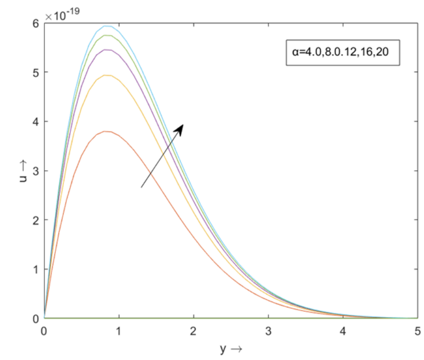

Figure 7: Casson parameter influence on velocity profile.

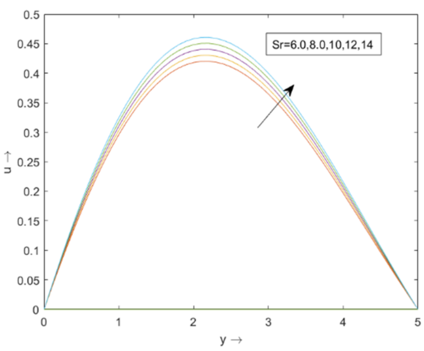

Figure 8: Soret number influence on velocity profile.

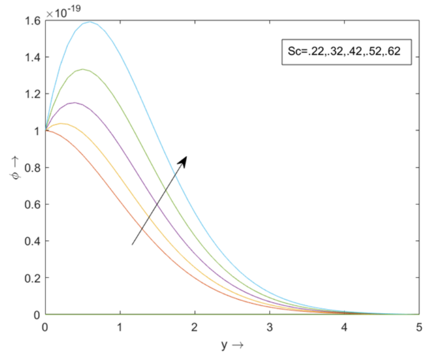

Figure 9: Schmidt number influence on concentration profile.

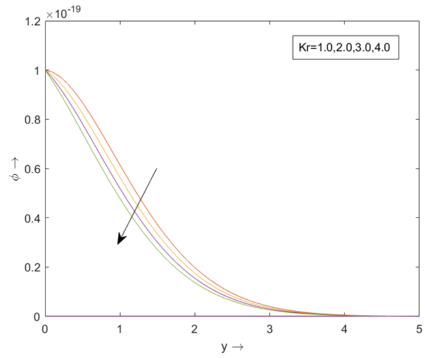

Figure 10: Chemical reaction parameter influence on concentration profile.

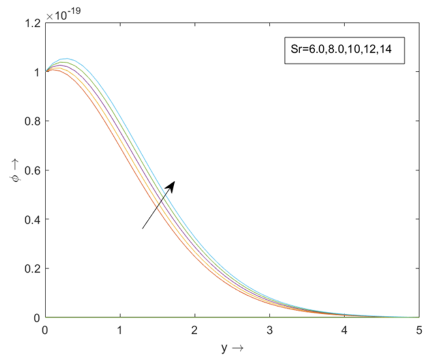

Figure 11: Soret influence on concentration profile.

the relation between molecular diffusivity of heat and the molecular diffusivity of momentum. It measures the relative value of momentum and heat in the velocity and thermal boundary layer. Figure 3 shows that temperature decreases with the increase in Prandtl number which leads to decrease in the thickness of thermal boundary layer. Figs. 4-8 illustrates the effect of different parameters on velocity. With the addition of chemical reaction, the thickness of velocity boundary layer changes. As the Fig. 4 goes, increase in chemical reaction, the rate of velocity of the fluid is reduced, that is the thickness of velocity boundary layer decreases.

Figure 5 illustrates the influence of thermal radiation on velocity profile. It is observed that increase in radiation parameter, the thickness of the velocity boundary layer decreases. The velocity profile in Fig. 6 displays that the fluid flow is decelerated with the rise in Prandtl number. This is due to the reason that fluids with higher

Table 1: Nusselt number with change in Ra.

Ra	Pr	Nusselt number
2	0.7	-1.0967×10^{-18}
3	0.7	-1.1537×10^{-18}
4	0.7	-1.1883×10^{-18}
5	0.7	-1.2115×10^{-18}
6	0.7	-1.2282×10^{-18}

Table 2: Nusselt number with change in Pr.

Ra	Pr	Nusselt number
5	0.5	-1.0738×10^{-18}
5	1	-1.3981×10^{-18}
5	1.5	-1.6675×10^{-18}
5	2	-1.8980×10^{-18}
5	2.5	-2.1000×10^{-18}

Table 3: Sherwood number with change in Sc.

Sc	Kr	Sr	Ra	Pr	Sherwood number
0.22	1	1	5	0.7	-0.00839×10^{-19}
0.32	1	1	5	0.7	0.02762×10^{-19}
0.42	1	1	5	0.7	0.07209×10^{-19}
0.52	1	1	5	0.7	0.1276×10^{-19}
0.62	1	1	5	0.7	0.1976×10^{-19}

Table 4: Sherwood number with change in Kr.

Sc	Kr	Sr	Ra	Pr	Sherwood number
0.22	2	1	5	0.7	-0.0839×10^{-20}
0.22	3	1	5	0.7	-0.2160×10^{-20}
0.22	4	1	5	0.7	-0.3393×10^{-20}
0.22	5	1	5	0.7	-0.4546×10^{-20}
0.22	6	1	5	0.7	-0.5627×10^{-20}

Table 5: Sherwood number with change in Sr.

Sc	Kr	Sr	Ra	Pr	Sherwood number
0.22	1	6	5	0.7	0.0748×10^{-20}
0.22	1	8	5	0.7	0.1383×10^{-20}
0.22	1	10	5	0.7	0.2018×10^{-20}
0.22	1	12	5	0.7	0.2654×10^{-20}
0.22	1	14	5	0.7	0.3289×10^{-20}

Table 6: Sherwood number with change in Ra.

Sc	Kr	Sr	Ra	Pr	Sherwood number
0.22	1	1	2	0.7	-0.854×10^{-21}
0.22	1	1	3	0.7	0.847×10^{-21}
0.22	1	1	4	0.7	0.842×10^{-21}
0.22	1	1	5	0.7	0.839×10^{-21}
0.22	1	1	6	0.7	0.837×10^{-21}

Table 7: Sherwood number with change in Pr.

Sc	Kr	Sr	Ra	Pr	Sherwood number
0.22	1	1	5	0.5	-0.857×10^{-21}
0.22	1	1	5	1	-0.814×10^{-21}
0.22	1	1	5	1.5	-0.772×10^{-21}
0.22	1	1	5	2	-0.732×10^{-21}
0.22	1	1	5	2.5	-0.695×10^{-21}

Table 8: Skin friction with change in Ra.

Ra	Pr	Kr	Sr	Sc	α	Skin friction
2	0.7	1	1	0.22	0.5	0.4012×10^{-18}
3	0.7	1	1	0.22	0.5	0.3945×10^{-18}
4	0.7	1	1	0.22	0.5	0.3905×10^{-18}
5	0.7	1	1	0.22	0.5	0.3878×10^{-18}
6	0.7	1	1	0.22	0.5	0.3859×10^{-18}

Table 9: Skin friction with change in Pr.

Ra	Pr	Kr	Sr	Sc	α	Skin friction
5	0.5	1	1	0.22	0.5	0.4038×10^{-18}
5	1	1	1	0.22	0.5	0.3666×10^{-18}
5	1.5	1	1	0.22	0.5	0.3373×10^{-18}
5	2	1	1	0.22	0.5	0.3138×10^{-18}
5	2.5	1	1	0.22	0.5	0.2944×10^{-18}

Table 10: Skin friction with change in Kr.

Ra	Pr	Kr	Sr	Sc	α	Skin friction
5	0.7	1	1	0.22	0.5	0.3878×10^{-18}
5	0.7	2	1	0.22	0.5	0.3849×10^{-18}
5	0.7	3	1	0.22	0.5	0.3822×10^{-18}
5	0.7	4	1	0.22	0.5	0.3796×10^{-18}
5	0.7	5	1	0.22	0.5	0.3772×10^{-18}

Prandtl number have high viscosity and its movement is slow. Casson fluid is a fluid with high viscosity shear thinning properties and yield stress, so the effect of Casson parameter on velocity is expected. Figure 7 shows that the rate of motion increases significantly with the increase in Casson fluid parameter. The influence of different values of Soret number on velocity profile is shown in Fig. 8. Soret number is generally considered as the effect of the mass transfer which is the resultant of temperature gradient. It is observed that the velocity profile increases proportionately with increasing Soret number.

Figures 9, 10 and 11 illustrates the influences of Schmidt number, chemical reaction and Soret effect on concentration profile of the fluid problem. Schmidt number defines the ratio of momentum to the mass diffusivity. Figure 9 shows the increasing value of Schmidt number in-

Table 11: Skin friction with change in Sr.

Ra	Pr	Kr	Sr	Sc	α	Skin friction
5	0.7	1	6	0.22	0.5	0.3961×10^{-18}
5	0.7	1	8	0.22	0.5	0.3994×10^{-18}
5	0.7	1	10	0.22	0.5	0.4027×10^{-18}
5	0.7	1	12	0.22	0.5	0.4060×10^{-18}
5	0.7	1	14	0.22	0.5	0.4093×10^{-18}

Table 12: Skin friction with change in Sc.

Ra	Pr	Kr	Sr	Sc	α	Skin friction
5	0.7	1	1	0.22	0.5	0.3878×10^{-18}
5	0.7	1	1	0.32	0.5	0.3948×10^{-18}
5	0.7	1	1	0.42	0.5	0.4030×10^{-18}
5	0.7	1	1	0.52	0.5	0.4127×10^{-18}
5	0.7	1	1	0.62	0.5	0.4243×10^{-18}

Table 13: Skin friction with change in α.

Ra	Pr	Kr	Sr	Sc	α	Skin friction
5	0.7	1	1	0.22	4	0.0954×10^{-17}
5	0.7	1	1	0.22	8	0.1223×10^{-17}
5	0.7	1	1	0.22	12	0.1345×10^{-17}
5	0.7	1	1	0.22	16	0.1414×10^{-17}
5	0.7	1	1	0.22	20	0.1458×10^{-17}

creases the thickness of the concentration boundary layer whereas on addition of chemical reaction the concentration boundary layer decreases in thickness. Fig. 10 depicts the effect of Soret number on concentration distribution. As the figure goes that the variation of concentration is directly proportional to the increasing value of Sr number.

Nusselt number is a non-dimensional heat transfer coefficient. It is a coefficient that is used to determine the conduction or convection of heat transfer. Table 1 and 2 exhibits the influence of radiation parameter Ra and Prandtl number Pr on rate of heat transfer. It is observed that the rate of heat transfer is slowed down as the radiation parameter increases and similarly as the Prandtl number increases. Sherwood number is the rate of mass transfer for a fluid problem. Tables 3, 5, 6 and 7 shows the influence of various parameter like Schmidt number, Chemical reaction, Soret number, radiation parameter and Prandtl number. The influence of Schmidt number, Soret number, radiation parameter and Prandtl number is directly proportional to the increase in Sherwood number and by Table 4 it is observed that the influence of chemical reaction is inversely proportional to the decrease in Sherwood number. That is as the chemical reaction rises the rate of mass transfer is reduced. The variations of Skin friction with influence of various parameters are displayed in Tables 8, 9, 10, 11, 12 and 13. The displayed table shows that the effect of radiation parameter, Prandtl number, chemical reaction is inversely proportional to the decrease in skin friction whereas for higher values of Sr number, Sc number and α parameter, the skin friction increases, i.e., the surface drag force is enhanced.

The variations of Skin friction with influence of various parameters are displayed in Tables 8, 9, 10, 11, 12 and 13. The displayed table shows that the effect of radiation parameter, Prandtl number, chemical reaction is inversely proportional to the decrease in skin friction whereas for higher values of Soret number, Schmidt number and Casson parameter, the skin friction increases, i.e., the surface drag force is enhanced.

5. Conclusion

This present work illustrates the study of free convective unsteady flow of an incompressible Casson fluid through a vertical cone with the role of suction in the boundary layer. The fluid is considered to viscous and electrically conducting with the influence of different parameters like reaction by chemical, thermal radiation, and thermos-diffusive effect. The governing equations are non-dimensionalized to a system of PDE's and then solve numerically. The graphical representations of the fluid properties under the influence of embedded parameters shows considerable changes. The tabular illustration of rate of heat and mass transfer and the surface drag force shows significant influence of the parameters. The present study of fluid through a vertical cone is frequently used in many engineering phenomena. The suction velocity plays an important role in artificially controlling the behavior of the boundary layer and hence it has wide application in industries and aeronautical engineering. The results of the present study can be summarized as bellow:

- The temperature distribution in the thermal boundary layer falls with the application of radiation parameter and Prandtl number.
- The rise in Prandtl number, radiation parameter and chemical reaction parameter leads to the fall in velocity profile.
- The velocity profile increases with the increase in Casson parameter and Soret number.
- Concentration profile raised with rise in Schmidt number and Soret number and lowers with rise in chemical reaction parameter.
- The rate of heat transfer for the fluid problem is decreased by the radiation parameter and Pr number.
- The rate of mass transfer is decreased by the chemical reaction parameter.
- With the application of Casson parameter the viscous drag force increases.

Nomenclature

r = radius of the cone, (unit: m)
ν = kinematic viscosity (unit:m^2 s^{-1}))
α = Casson parameter
γ = angle of inclination (unit: radian)
β = thermal expansion coefficient
β^* = mass expansion coefficient
g = acceleration due to gravity (unit: m s^{-2})
C = species concentration
C_∞ = fluid concentration far away from the wall

T_w = temperature of the fluid (unit: K)
T_∞ = fluid temperature far away from the wall (unit: K)
σ = electrical conductivity (unit: s m^{-1})
ρ = fluid density (unit: kg m^{-3})
B_0 = magnetic field (unit: Tesla)
C_p = specific heat at constant pressure (unit: J kg^{-1} K^{-1})
D_m = mass diffusivity (unit: m^2 s^{-1})
Kr = chemical reaction rate constant
K_T = thermal diffusion ratio
T_m = mean fluid temperature (unit: K)
q_r = radiative heat flux (unit : W m^2)
u = dimensionless velocity
θ = nondimensional temperature
ϕ = nondimensional concentration
Gr = Grashof number for heat transfer
Gm =Grashof number for mass transfer
Pr = Prandtl number
Sc = Schmidt number
M = Hartmann number
R = radiation parameter
Kr = chemical reaction parameter
Sr = Soret number
τ = skin friction
Nu = Nusselt number
Sh = Sherwood number

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