HYDROMAGNETIC OLDROYD FLUID FLOW PAST A FLAT SURFACE WITH DENSITY AND ELECTRICAL CONDUCTIVITY STRATIFICATION

D. DEY

Department of Mathematics, Dibrugarh University, Dibrugarh-786004, India
Email: debasish41092@gmail.com

***Abstract*— An usteady stratified flow of visco-elastic fluid past an inifnite flat surafce has been studied in presence of applied magnetic field. The visco-elastic fluid flow is characterized by Oldroyd fluid model contaning the rheological parameters: retardation and relaxation parameters. A magnetic field of strength B_0 is applied along the transverse direction to surface. Fluidic properties like density, viscosity and electrical conductivity are assumed to vary along the axis of gravity. The governing equations of motion are solved analytically using separation of variable method. Results are discussed graphically and in tabular form for various values of flow parameters involved in the problem.**

***Keywords*— Oldroyd fluid, stratification, retardation and relaxation time, shearing stress, Deborah numbers.**

I. INTRODUCTION

Oldroyd fluid Model has attracted many researchers because of its two rheological parameters, characterize the relaxation and retardation of governing visco-elastic fluid flow. Oldroyd (1950, 1958) formulated a consitititive model for visco-elastic model and is named as Oldroyd fluid model. The consitutive equation of Odroyd fluid model is

$$\sigma_{ij} = -p\delta_{ij} + \tau_{ij}, \tag{1}$$

$$\left(1+\lambda_1 \frac{d}{dt}\right)\tau_{ij} = 2\mu\left(1+\lambda_2 \frac{d}{dt}\right)e_{ij}, \tag{2}$$

where σ_{ij} is stress tesnor, $-p$ be the pressure, δ_{ij} be Kronecker delta, τ_{ij} be viscous stress tensor, μ be the coefficient of viscosity, $e_{ij} = 1/2(v_{i,j}+v_{j,i})$ be the deformation rate tensor. In this model, the parameters λ_1 and λ_2 characterize relaxation and retardation times of viscoelastic material.

Rajagopal and Bhatnagar (1995) have found the exact solution of simple flows of Oldroyd-B fluid. Exact solution of hydro-magnetic flow of an Oldroyd fluid in a channel has been given by Ray *et al.* (1999). Hayat *et al.* (2001) have studied some simple flow problems of Oldroyd fluid and Hayat *et al.* (2004) also considered the hydromagnetic Couette flow of Oldroyd-B fluid in a rotating system. Unsteady hydromagnetic flow of Oldroyd fluid in presence of Hall effects has been studied by Asghar *et al.* (2003). Jamil and Khan (2011) have investigated the behaviour of Oldroyd fluid Couette flow in an annulus. MHD flow of Oldroyd fluid through a porous channel has been analysed by Hayat *et al.* (2012). Ghosh (2013) has discussed unsteady hydromagnetic flow of Oldroyd fluid through porous channel with oscillating walls. Behaviour of unsteady visco-elastic fluid flow in a channel bounded by two infinite rigid non-conducting plates induced by tooth pulses has been studied by Ghosh (2015). Dey (2016) has studied the flow problem of hydromagnetic Oldroyd fluid in a horizontal channel with volume fraction of dust particles and energy dissipation. Shakeel *et al.* (2016) have analysed the Oldroyd fluid flow with slip at the boundary using Laplace Transform technique. Hayat *et al.* (2016) have investigated the nature of boundary layer flow guided by Oldroyd-B fluid model past a stretching surface using Homotopy analysis method.

Various visco-elastic fluids such as molten plastics, pulps, lubricants and emulsions, Bitumen, polymers, polymeric melts such as Nylon etc. have special importance because of their density variation. This leads to the concept of stratification in fluid flow (Chakraborty, 2011). Rayleigh (1882) investigated the problem of incompressible heavy fluid flow with variable density. Flow of stratified fluid with variable viscosity past a porous bed has been investigated by Channabasappa and Ranagan (1976). Gupta and Sharma (1978) have studied the problem of stratified fluid flow between a porous bed and moving impermeable plate. Agarwal *et al.* (2012) have investigated hydro-magnetic free convective flow with heat and mass transfer past a vertical porous plate in presence of stratification. Sharma *et al.* (2013) have analyzed free convective stratified fluid flow with thermal diffusion effects. Dusty stratified fluid flow governed by Rivlin-Ericksen fluid model with heat transfer through an inclined porous medium has been studied by Chakraborty (2001). Prakash *et al.* (2012) solved the problem of dusty stratified fluid flow through an inclined porous medium using Walters liquid model.

Application of transverse magnetic field generates a Lorentz force $\vec{J}\times\vec{B}$, $\vec{J}$ is current density and $\vec{B}$ is applied magnetic field) in the fluid motion. Using Gauss's law of magnetism ($\nabla\cdot\vec{B}=0$), the magnitude of applied magnetic field is obtained as a constant B_0 (say). The hydro-magnetic fluid flows are encountered in many geophysical, astrophysical and engineering problems. These flows have a key role in the fields of aeronautics, stellar and planetary magnetospheres. In the design of heat exchangers, pumps, thermal protection, in space vehicle propulsion, control and re-entry and in creating novel power-generating systems, magneto-hydro-dynamics concepts are used. MHD is also used in the purification of molten metals from non-metallic inclusions (Shehzad *et al.*, 2016)

The objective of the present paper is to study the effects of stratification of various fluidic properties like density, electrical conductivity and viscosity on hydromagnetic Oldroyd fluid flow past a flat surface.

II. MATHEMATICAL FORMULATION

An unsteady flow of hydromagnetic Oldroyd fluid past an infinite flat surface has been considered in presence of stratification along the axis of gravity. A magnetic field of strength B_0 is applied along transverse direction to the surface. Let x'-axis be taken along the length of the surface and y'-axis be taken perpendicular to it. Various fluidic properties like density, viscosity and electrical conductivity are assumed to vary along the axis of y and are considered as

$$\rho=\rho_0 e^{-n\left(\frac{y'}{U_0/v}\right)},\ \mu=\mu_0 e^{-n\left(\frac{y'}{U_0/v}\right)},\ \sigma=\sigma_0 e^{-n\left(\frac{y'}{U_0/v}\right)}$$

where n is the stratification parameter. For stable stratification, $n > 0$. ρ_0, μ_0 and σ_0 are the density, viscosity and electrical conductivity of fluid at the surface, U_0 the velocity of fluid at the surface on time, $t = 0$, v the kinematic viscosity. As the length of surface is infinite in length, so all fluidic properties except the pressure are independent with respect to x. So the velocity profile is a function of two variables y and t only (Ahmed and Das, 1992; Ahmed *et al.*, 2012; and Chaudhary and Jha, 2008).

The continuity equation for stratified flow is given by

$$\frac{\partial(\rho v')}{\partial y'}=0, \tag{3}$$

where, v' be the component of velocity along y' axis. Substituting the expression of ρ and integrating, we get

$$v'=A_1 e^{\rho_0 n\left(\frac{y'v}{U_0}\right)} \tag{4}$$

In absence of suction ($v'=0$ at $y'=0$) at the plate, we get $A_1=0$ and so, $v'=0$.

Equations of motion for the governing fluid flow are

$$\rho\frac{\partial u'}{\partial t'}=-\frac{\partial p}{\partial x'}+\frac{\partial \tau'}{\partial y'}-\sigma B_0^2 u' \tag{5}$$

$$0=-\frac{\partial p}{\partial y'} \tag{6}$$

Constitutive equation:

$$\left(1+\lambda_1\frac{\partial}{\partial t'}\right)\tau'=\mu\left(1+\lambda_2\frac{\partial}{\partial t'}\right)\frac{\partial u'}{\partial y'} \tag{7}$$

Using τ' from the constitutive Eq. (7) to equation of motion (5) are

$$\rho_0 e^{-n\frac{y'}{h}}\frac{\partial u}{\partial t'}+\lambda_1\rho_0 e^{-n\frac{y'}{h}}\frac{\partial^2 u'}{\partial t'^2}=-\left(1+\lambda_1\frac{\partial}{\partial t'}\right)\frac{\partial p'}{\partial t'}+\frac{\partial\left\{\mu_0 e^{-n\frac{y'}{h}}\left(1+\lambda_2\frac{\partial}{\partial t'}\right)\frac{\partial u'}{\partial y'}\right\}}{\partial y'}-\sigma_0 e^{-n\frac{y'}{h}}B_0^2\left(1+\lambda_1\frac{\partial}{\partial t'}\right)u' \tag{8}$$

The boundary conditions of the problem are

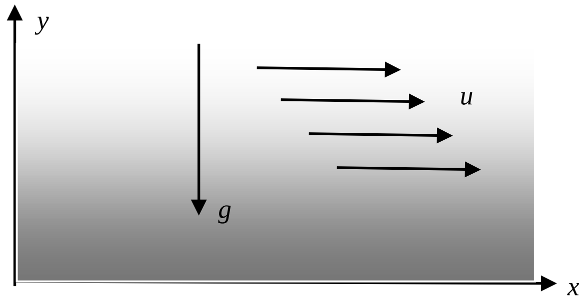

Figure 1: Geometry of the problem

$$u'=U_0 e^{-nt'\frac{U_0^2}{v}} \quad \text{at} \quad y'=0 \qquad u'=0 \quad \text{at} \quad y'\to\infty \tag{9}$$

where, u' is the component of velocity along x' axis and t' the time. It indicates, at time t'=0, the fluid is moving with a constant velocity U_0 at the surface and fluid velocity is decreasing with the time on the solid surface. At a large distance from the surface, fluid flow remains undisturbed. The velocity of fluid at the surface decays with time. Using the boundary condition at free stream case, the pressure gradient term in (8) may be neglected.

III. METHOD OF SOLUTIONS

Let us define the following non- dimensional parameters

$$y=\frac{y'U_0}{v},u=\frac{u'}{U_0},t=\frac{t'U_0^2}{v},a=\lambda_1\frac{U_0^2}{v}, \quad b=\lambda_2\frac{U_0^2}{v},M=\frac{\sigma B_0^2 v}{\rho_0 U_0^2} \tag{10}$$

where y is dimensionless displacement variable, u is dimensionless velcoity, t is dimensionless time, a and b are Deborah numbers, M be the magnetic parameter.

Introducing the non-simensional parameters in (8), we get the following differential equation in dimensionless form:

$$\frac{\partial u}{\partial t}+a\frac{\partial^2 u}{\partial t^2}=-n\left(\frac{\partial u}{\partial y}+b\frac{\partial^2 u}{\partial t^2}\right)+\frac{\partial^2 u}{\partial y^2}+b\frac{\partial^2 u}{\partial y^2\partial t}-M\left(u+a\frac{\partial u}{\partial t}\right) \tag{11}$$

Relevant boundary conditions of the problem are

$$u=e^{-nt} \quad \text{at} \quad y=0 \qquad u=0 \quad \text{at} \quad y\to\infty \tag{12}$$

To solve the Eq. (11), we use separation of variable method, where the solution is assumed to be product of functions of two independent variables namely y and t. Subject to the boundary conditions, expression of velocity in the neighbourhood of surface is considered as (Chakraborty, 2001; and Hayat *et al*., 2012)

$$u=f(y)e^{-nt}. \tag{13}$$

Using (13) in the equation (11), we get the following ordinary differential equations as

$$(1-bn)f''+(bn^2-n)f'+(n-an^2-M+an)f=0 \tag{14}$$

and its solution is

$$f(y)=C_1 e^{-A_1 y}+C_2 e^{A_2 y}$$

where

$$A_1 = \frac{\left(bn^2 - n\right) + \sqrt{\left(bn^2 - n\right)^2 - 4\left(1 - bn\right)\left(n - an^2 - M + an\right)}}{2\left(bn^2 - n\right)}$$

$$A_2 = \frac{-\left(bn^2 - n\right) + \sqrt{\left(bn^2 - n\right)^2 - 4\left(1 - bn\right)\left(n - an^2 - M + an\right)}}{2\left(bn^2 - n\right)}$$

IV. RESULTS AND DISCUSSIONS

The velocity of fluid flow in the neighbourhood of the surface is given by

$$u = \left(C_1 e^{-A_1 y} + C_2 e^{A_2 y}\right) e^{-nt}. \tag{15}$$

The dimensionless shearing stress is given by

$$a \frac{d}{dt} Sh + Sh = -A_1 \left(1 - bn\right) e^{-A_5 y - nt} \tag{16}$$

where $Sh = \dfrac{\tau'}{\rho U_0^2}$.

Using $y \to \infty$: $Sh=0$ gives

$$Sh = \frac{A_1 a \left(1 - bn\right)}{\left(1 - an\right)\left(e^{-nt - A_5 y}\right)} \tag{17}$$

where $C_1=1$, $C_2=0$, $A_5= A_1$.

A. Effects of Deborah numbers

Effects of Deborah numbers (relaxation and retardation time parameters) on velocity profile and shearing stress are represented by Figs. 2 to 6. Figures 2 to 4 notify the behaviour of velocity of fluid motion for various values of Deborah numbers along with other flow parameters. The relaxation parameter (a) has a positive impact on velocity of fluid motion as its higher values increase the energy of governing fluid system and as a result velocity increases (Fig. 2). Another Deborah number (b retardation parameter) might reduce systems enrgy and as a consequence a decreasing trend in velocity is noticed (Fig. 3). Also, it can be concluded that the maximum effect of Deborah numbers in noticed in the neighbourhood of the surface. Figure 4 shows that pattern of various fluid motions, the set (a=0, b=0) characterizes the motion of Newtonian fluid, (a=0, $b\neq$0), ($a\neq$0, b=0) and ($a\neq$0, $b\neq$0) characterize the motion of fluid governed by Second grade fluid model, Maxwell fluid model and Oldroyd fluid model respectively. In absence of relaxation time parameter, motion governed by Second grade fluid model experiences the slowest velocity. In the similar way, the presence of relaxation parameter enhances the velocity of fluid motion governed by Maxwell fluid model. The figures also state that velocity decreases rapidly in the neighbourhood of the surface and then it converges to zero in the region $y > 3$.

Figures 5 to 6 indicate the influences of visco-elastic parameters on shearing stress formed at the surface. Shearing stress is drawn against time and it is seen that shearing stress declines with the increasing values of time. Relaxation parameter might lessen the effect of friction and hence skin friction at the plate also experiences declining trend. But the growth of retardation increases the resistance of governing fluid motion and so, the shearing stress at the plate increases with the increase of retardation parameter.

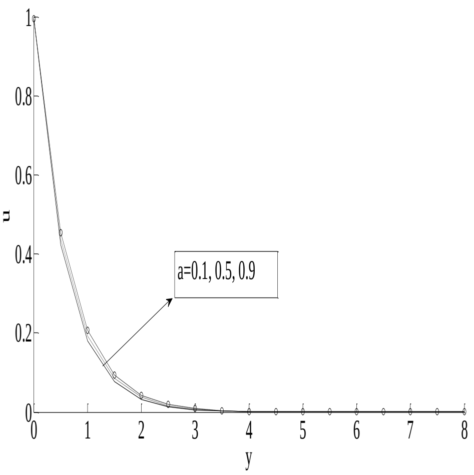

Figure 2: Velocity profile against y for b=0.6, M=2, n=0.5, t=0.01.

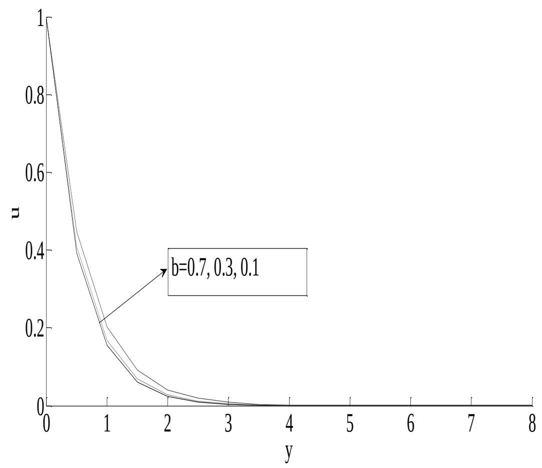

Figure 3: Velocity profile against y for a=0.5, M=2, n=0.5, t=0.01.

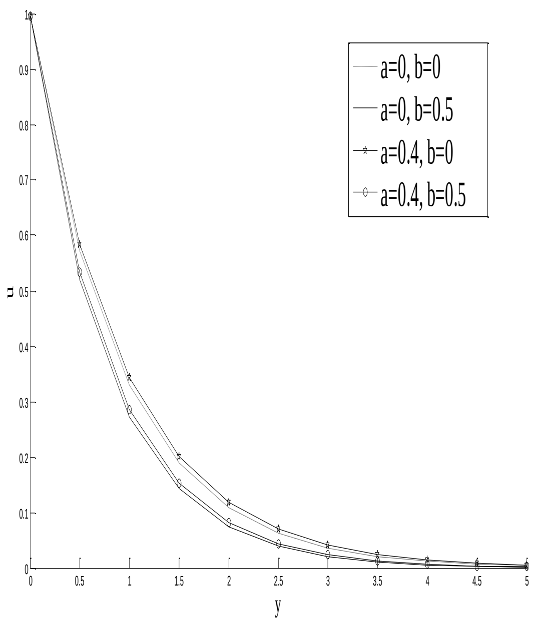

Figure 4: Velocity profile against y for M=2, n=0.5, t=0.01.

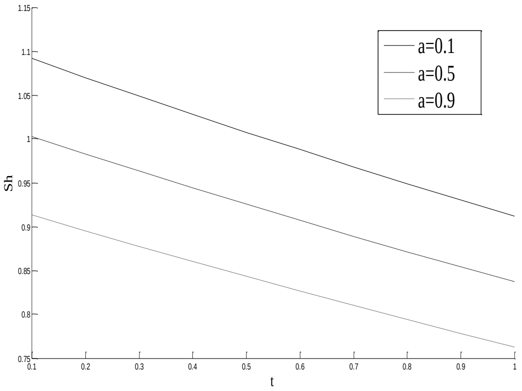

Figure 5: Shearing stress at the surface against *t* for *b*=0.6, *M*=2, *n*=0.2.

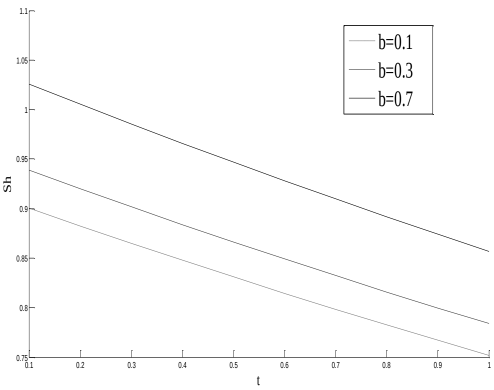

Figure 6: Shearing stress at the surface against *t* for *a*=0.5, *M*=2, *n*=0.2.

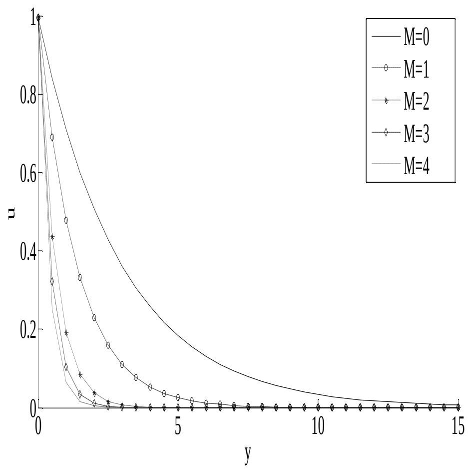

Figure 7: Velocity profile against *y* for *a*=0.5, *b*=0.6, *n*=0.5, *t*=0.01.

B. Effects of Magnetic parameter:

Application of transverse magnetic field on the fluid flow creates a Lorentz force and its effect is exhibited through the non-dimensional parameter *M*. The Magnetic parameter *M* has a retarding effect on the fluid motion as it controls the governing Oldroyd fluid flow. Increasing values of Magnetic parameter makes it thicker and as a consequence velocity slows down (Fig. 7).

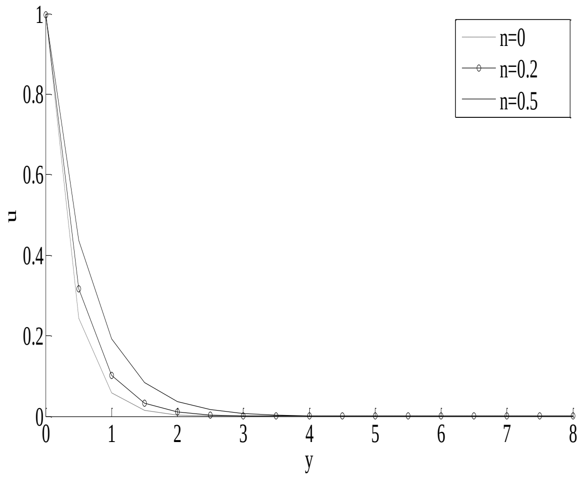

Figure 8: Velocity profile against y for *a*=0.5, *b*= 0.6, *M*=2 on *t*=0.01

Table 1: Shearing Stress against *t* for *a*=0.5, *b*=0.6, *M*=2

	n=0	*n*=0.2	*n*=0.3	*n*=0.4
t =0.1	1.4142	1.0025	1.0059	1.0114
t =0.2	1.4142	0.9826	0.9762	0.9717
t =0.3	1.4142	0.9632	0.9474	0.9336
t =0.4	1.4142	0.9441	0.9194	0.8970
t =0.5	1.4142	0.9254	0.8922	0.8618
t =0.6	1.4142	0.9071	0.8658	0.8280
t =0.7	1.4142	0.8891	0.8402	0.7956
t =0.8	1.4142	0.8715	0.8154	0.7644
t =0.9	1.4142	0.8543	0.7913	0.7344
t =1	1.4142	0.8373	0.7679	0.7056

C. Effect of stratification parameter:

The effects of stratification are exhibited through the parameter *n* and shown in Fig. 8 and Table 1. Velocity profile for various values of displacement variable (*y*) and stratification parameter (*n*) has been shown in Fig. 8. Increasing values of stratification reduces the thickness of Oldroyd fluid and hence velocity of corresponding fluid motion increases. Maximum discrepency is noticed in the neighbourhood of the surface. Shearing stress against the time (*t*) has been given in Tabular form (Table. 1). Skin friction at the surface experiences a decreasing pattern during the growth of stratification parameter. It also concludes that during *n*=0 (for non-stratified fluid), shearing stress remains unchanged with respect to time

V. CONCLUSIONS

The relaxation parameter (*a*) has a positive impact on velocity of fluid motion.

Maximum effects of retardation and relaxation parameters are noticed in the neighbourhood of the surface.

Growth of stratification decreases the power of skin friction at the plate.

Relaxation parameter lessens the effect of skin friction but an oppsoite pattern is noticed during the growth of retardation parameter.

REFERENCES

Agrawal, V.P., J.K. Agrawal and N.K. Varshney, "Effect of stratified viscous fluid on MHD free convection flow with heat and mass transfer past a vertical porous plate," *Ultra Scientist*, **24**, 139-146 (2012).

Ahmed, N. and U.N. Das, "Convective MHD oscillatory flow past a uniformly moving infinite vertical plate," *Defence Science Journal*, **42**, 53-57 (1992).

Ahmed, N., D. Sarma and H. Deka, "MHD mixed convection and mass transfer from an infinite vertical porous plate with chemical reaction in presence of a heat source," *Applied Mathematical Sciences*, **6**, 1011–1020 (2012).

Asghar, S., S. Parveen, S. Hanif, A.M. Siddique and T. Hayat, "Hall effects on the unsteady hydromagnetic flows of an Oldroyd-B fluid," *Int. J. Engg. Sci.*, **41**, 609-619 (2003).

Channabasappa, M.N. and G. Ranagan, "Flow of a viscous stratified fluid of variable viscosity past a porous bed," *Proc. Indian Acad. Sci.*, **83**, 145-155 (1976).

Chakraborty, S., "MHD flow and heat transfer of a dusty visco-elastic stratified fluid down an inclined channel in porous medium under variable viscosity,· *Theoretical and Applied Mechanics*, **26**, 1-14 (2001).

Chakraborty, S., "A study of viscous stratified MHD flow of dusty visco-elastic fluid under longitudinal pressure gradient," *Oriental Journal of Physics*, **3**, 59-68 (2011).

Chaudhary¸ R.C. and A.K. Jha, "Heat and mass transfer in elastico-viscous fluid past an impulsively started infinite vertical plate with hall effect," *Latin American Applied Research*, **38**, 17-26 (2008)

Dey, D., "Dusty hydromagnetic Oldroyd fluid flow in a horizontal channel with volume fraction and energy dissipation," *International Journal of Heat Technology*, **34**, 415-422 (2016).

Ghosh, S.K., "Unsteady hydro-magnetic flow of an Oldroyd fluid through a porous channel with oscillating walls," *Journal of Physical Sciences*, **17**, 155-167 (2013).

Ghosh, S., "On hydromagnetic channel flow of an Oldroyd-B fluid induced by tooth pulses in a rotating system," *Journal of Applied and Computational Mathematics*, **4**, http://dx.doi.org/10.4172/ 2168-9679.1000211 (2015).

Gupta, S.P. and G.C. Sharma, "Stratified viscous flow of variable viscosity between a porous bed and moving impermeable plate," *Indian J. Pure and Appl. Math.*, **9**, 290-297 (1978).

Hayat, T., A.M. Siddiqui and S. Asghar, "Some simple flows of an Oldroyd-B fluid," *International Journal of Engineering Science*, **39**, 135-147 (2001).

Hayat, T., S. Nadeem and S. Asghar, "Hydromagnetic couette flow of an Oldroyd-B fluid in a rotating system," *International Journal of Engineering Science*, **42**, 65–78 (2004).

Hayat, T., S.A. Shehzad, M. Mustafa and A.A. Hendi, "MHD flow of an Oldroyd-B fluid through a porous channel," *International Journal of Chemical Reactor Engineering*, **10**, DOI 10.1515/1542-6580.2655 (2012).

Hayat, T., M. Imtiaz and A. Alsaedi, "Boundary layer flow of Oldroyd-B fluid by exponentially stretching Sheet," *Applied Mathematics and Mechanics*, **37**, 573-582 (2016).

Jamil, M. and N.A. Khan, "Axial couette flow of an Oldroyd –B fluid in an annulus," *Theoretical & Applied Mechanics Letters*, **2**, 012001 (2011).

Oldroyd, J.G., "On the formulation of rheological Equations of state," *Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences*, **200**, 523–541 (1950).

Oldroyd, J.G., "Non-Newtonian effects in steady motion of some idealized elastic-viscous liquids," *Proc. Roy. Soc. Lond*., **A245**, 278-297 (1958).

Prakash, O., D. Kumar and Y.K. Dwivedi, "Heat transfer in MHD flow of dusty visco-elastic (Walters' liquid model-B) stratified fluid in porous medium under variable viscosity," *Pramana-Journal of Physics*, **79,** 1457-1470 (2012).

Rajagopal, K.R. and R.K. Bhatnagar, "Exact solutions for some simple flows of an Oldroyd-B fluid," *Acta Mechanica*, **113**, 233-239 (1995).

Ray, R.N., A. Samad and T.K. Chaudhury, "An exact periodic solution of a hydro-magnetic flow of an Oldroyd fluid in a channel," *ASME J. Appl. Mech.*, **66**, 974-977 (1999).

Rayleigh, L., "Investigation of the character of the equilibrium of an incompressible heavy fluid of variable density," *Proc. London Math. Soc.*, **14**, 170-177 (1882).

Shakeel, A., S. Ahmad, H. Khan, N.A. Shah and S.U. Haq, "Flows with slip of Oldroyd-B fluids over a moving plate," *Advances in Mathematical Physics*, http://dx.doi.org/10.1155/2016/8619634 (2016).

Sharma, A.K., G.K. Dubey and N.K. Varshney, "Thermal diffusion effect on MHD free convection flow of stratified viscous fluid with heat and mass transfer," *Advances in Applied Science Research*, **4,** 221-229 (2013)

Shehzad, S.A., T. Hayat and A. Alsaedi, "Three dimensional MHD flow of a casson fluid in porous medium with heat generation," *Journal of Applied Fluid Mechanics*, **9**, 215-223 (2016).

Received: September 23, 2015.

Sent to Subject Editor: December 30, 2015.

Accepted: October 31, 2016.

Recommended by Subject Editor: Gianfranco Caruso