HEAT TRANSFER TO MHD OSCILLATORY DUSTY VISCO-ELASTIC FLUID FLOW IN AN INCLINED CHANNEL FILLED WITH A POROUS MEDIUM

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***Abstract*— An analysis of the unsteady MHD oscillatory flow of an incompressible, conducting, optically thin, dusty visco-elastic fluid through an inclined channel filled with saturated porous medium in presence of heat sink parameter and thermal radiation has been considered. The governing nonlinear partial differential equations are non-dimensioalized and are solved analytically by using perturbation technique. The expressions for velocity field, temperature field, skin friction, heat flux in terms of Nusselt number, flow flux for fluid and dust particles are obtained within the channel and are discussed graphically for various values of flow parameters involved in the problem.**

***Keywords*— Visco-elastic, MHD, Dusty fluid, Radiation, Porous medium.**

I. INTRODUCTION

The analysis of magnetohydrodynamic flow for an electrically conducting visco-elastic fluid past a porous channel is of great theoretical interest because it has been applied in engineering, chemical technology and industry. Applications related to convective flows in porous media can be found in Nield and Bejan (1999), and Pop and Ingham (2001). Several investigators have analyzed the MHD flow in channel for Newtonian or non-Newtonian fluids in different physical situations. Some of them are Attia and Kotb (1966), Soundalgekar and Bhat (1971), Vajravelu (1988), Makinde, and Mhone (2005), Choudhury and Das (2012). Moreau (1900) presented a survey of MHD studies in the technological fields.

The problems of the flow and heat transfer of dusty fluids arise in many practical applications in the fields of fluidization, use of dust in gas cooling systems, centrifugal separation of matter from fluid, transport of liquid slurries in chemical industries, petroleum industry, purification of crude oil, electrostatic precipitation, polymer technology, fluid droplets in sprays, and powder technology. The equations of motion for binary mixture of fluid and dust particles were proposed by Saffman (1962). Michael and Miller (1966) analyzed the plane parallel flow of a dusty gas. Chamkha (1997) studied the unsteady flow of dusty gas in a channel due to an oscillating pressure gradient. Ghosh *et al.* (2000) made an initial value investigation of an incompressible visco-elastic (Rivlin-Ericksen) fluid with embedded small spherical inert particles between two infinite moving parallel plates in the presence of transverse magnetic field. Chamkha (2000) presented a closed-form transient solution for the problem of unsteady laminar hydromagnetic fluid-particle flow and heat transfer of a particulate suspension in an electrically conducting fluid through channels and circular pipes. Attia (2002) studied the unsteady flow and heat transfer of a dusty conducting fluid between two parallel plates with variable viscosity and electric conductivity. He assumed that the fluid is driven by a constant pressure gradient and an external uniform magnetic field is applied perpendicular to the plates. Again, Attia (2006) studied the problem of time varying hydromagnetic Couette flow with heat transfer of a dusty fluid in presence of uniform suction and injection considering Hall Effect. Gireesha *et al.* (2009) studied the pulsatile flow of an unsteady dusty fluid through rectangular channel. Makinde and Chinyoka (2010) investigated the MHD Transient flows of dusty fluid in a channel with variable physical properties with Navier slip condition. Prakash *et al.* (2012) investigated the effects of heat transfer in MHD flow of dusty visco-elastic stratified fluid through a porous channel by using perturbation technique. Saxena and Agarwal (2014) studied the problem of unsteady laminar flow of an incompressible conducting viscous dusty fluid between two infinitely non-conducting parallel plates bounded by porous medium. Prakash and Makinde (2015) studied the oscillatory Couette flow of dusty fluid in a porous channel in presence of magnetic field and radiative heat transfer. Madhura and Uma (2016) studied the flow of an unsteady incompressible electrically conducting fluid with uniform distribution of dust particles in a porous channel.

The objective of the present paper is to study the problem of an unsteady flow and heat transfer of an electrically conducting, incompressible, dusty visco-elastic fluid through an inclined channel filled with porous medium in presence of a transverse magnetic field and thermal radiation. The constitutive equations are taken as those proposed by Walter (1962), known as Walters' liquid B' for visco-elastic fluid.

II. FORMULATION OF THE PROBLEM

Consider an unsteady flow of an incompressible, electrically conducting, radiative, dusty visco-elastic fluid characterized by Walters liquid B' model flowing in an inclined channel filled with saturated porous medium under the influence of transverse magnetic field and heat sink parameter. Consider a Cartesian co-ordinate system (x,y) with x-axis along the center of the channel and y-axis perpendicular to it as shown in Fig. 1. To study the dusty fluid flow, the following assumptions are made

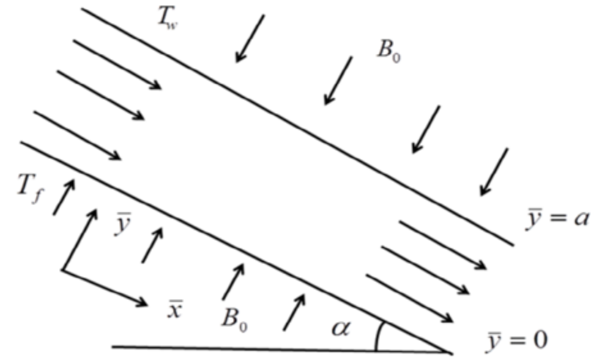

Fig. 1. Schematic diagram of the problem.

(i) The dust particles are assumed to be spherical in shape and uniformly distributed throughout the fluid and their number density N_0 is constant throughout the motion.

(ii) A uniform magnetic field B_0 is applied normal to the plane of the plates. The magnetic Reynolds number is assumed to be very small so that the induced magnetic field can be neglected.

(iii) The two channel plates are assumed to be electrically non-conducting and kept at two different temperatures T_f, which is fixed, for the lower plate and for the upper plate T_w, which is oscillating with time about a constant non-zero mean. The initial temperature of the fluid and dust particles is assumed to be T_0.

(iv) The viscous dissipation and the Joule dissipation are neglected in temperature distributions of fluid and dust particles.

Since the channel plates are infinitely long, the problem is essentially one dimensional with velocity of fluid, and dust particles are functions of y and t.

The flow of fluid is governed by the following momentum (Walter, 1962; Saffman, 1962) equation:

$$\frac{\partial \bar{u}}{\partial \bar{t}} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial \bar{x}} + \nu\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} - \frac{K_0}{\rho}\frac{\partial^3 \bar{u}}{\partial \bar{y}^2 \partial \bar{t}} + \frac{FN_0}{\rho}\left(\bar{u}_p - \bar{u}\right) - \frac{\nu}{K}\bar{u} - \frac{\sigma_e B_0^2}{\rho}\bar{u} + g\sin\alpha, \quad (1)$$

where ρ is the density of the fluid, $\bar{p}$ is the fluid pressure, $\bar{t}$ is the time, ν is the coefficient of kinematic viscosity of fluid, K_0 is the visco-elastic parameter, F $(=6\pi\rho\nu D)$ is the Stokes constant, D is the average radius of dust particles, $\bar{u}$ is the velocity of fluid, $\bar{u}_p$ is the velocity of dust particles, K is the porous medium permeability coefficient, σ_e is the conductivity of the fluid, g is the gravitational force and α is the inclination of the channel plates with the horizon.

The motion of the dust particles is governed by the Newton's second law of motion (Saffman, 1962):

$$m_p\frac{\partial \bar{u}_p}{\partial \bar{t}} = F\left(\bar{u} - \bar{u}_p\right), \quad (2)$$

where m_p is the average mass of the dust particles.

The energy equations describing the temperature distributions for fluid and dust particles are, respectively, given by

$$\frac{\partial \bar{T}}{\partial \bar{t}} = \frac{k}{\rho C_p}\frac{\partial^2 \bar{T}}{\partial \bar{y}^2} - \frac{1}{\rho C_p}\frac{\partial q_r}{\partial \bar{y}} - \frac{Q_0}{\rho C_p}\left(\bar{T} - \bar{T}_0\right) + \frac{\rho_p C_S}{\rho C_p \gamma_T}\left(\bar{T}_p - \bar{T}\right), \quad (3)$$

$$\frac{\partial \bar{T}_p}{\partial \bar{t}} = -\frac{\left(\bar{T}_p - \bar{T}\right)}{\gamma_T}, \quad (4)$$

where k is the thermal conductivity of the fluid, $\bar{T}$ is the fluid temperature, $\bar{T}_p$ is the temperature of dust particles, Q_0 is the heat absorption coefficient, q_r is the radiative heat flux, ρ_p is the mass of dust particles per unit volume of the fluid, γ_T $(=3\mathrm{Pr}\gamma_p C_s/(2C_p))$ is the temperature relaxation time, γ_p $(=2\rho_s D^2/(9\nu\rho))$ is the velocity relaxation time, Pr $(=\nu\rho C_p/k)$ is the Prandtl number, C_p is the specific heat capacity of the fluid at constant pressure, C_s is the specific heat capacity of the particles and ρ_s $(=3\rho_p/(4\pi D^3 N_0))$ is the material density of dust particles.

The corresponding initial and boundary conditions are

$$\begin{aligned}
&\bar{t} = 0 : \bar{u} = \bar{u}_p = 0, \bar{T} = T_0, \bar{T}_p = T_0 \quad \text{at} \quad \bar{y} = 0 \quad \text{and} \quad \bar{y} = a; \\
&\bar{t} > 0 : \bar{u} = 0, \bar{u}_p = 0, \bar{T} = T_f, \bar{T}_p = T_f \quad \text{at} \quad \bar{y} = 0; \\
&\bar{t} > 0 : \bar{u} = U_0\left(1 + \varepsilon e^{i\varpi\bar{t}}\right), \bar{u}_p = V_0\left(1 + \varepsilon e^{i\varpi\bar{t}}\right), \\
&\bar{T} = T_w = T_f + \varepsilon(T_f - T_0)e^{i\varpi\bar{t}} = \bar{T}_p \quad \text{at} \quad \bar{y} = a,
\end{aligned} \quad (5)$$

where ϖ is the frequency of oscillation.

Using Cogley *et al.* (1968), the radiative heat flux term q_r is given by

$$\frac{\partial q_r}{\partial \bar{y}} = 4\alpha^2\left(T_0 - \bar{T}\right), \quad (6)$$

where α is mean absorption radiation coefficient.

The following non-dimensional quantities are introduced

$$\begin{aligned}
&x = \bar{x}/a, y = \bar{y}/a, u = \bar{u}/U_0, u_p = \bar{u}_p/U_0, t = \bar{t}U_0/a, \\
&\theta = (\bar{T} - T_0)/(T_f - T_0), \theta_p = (\bar{T}_p - T_0)/(T_f - T_0), \\
&\mathrm{Re} = U_0 a/\nu, R = N_0 F a^2/(\rho\nu), M = \nu m_p/(Fa^2), \\
&\mathrm{Pr} = \nu\rho C_p/k, \omega = a\varpi/U_0, Da = K/a^2, Q = Q_0 a^2/k, \\
&N^2 = 4\alpha^2 a^2/k, p = a\bar{p}/(\nu\rho U_0), H^2 = a^2\sigma_e B_0^2/(\nu\rho), \\
&Gr = g\beta_T(T_f - T_0)a^2/(\nu U_0), s^2 = 1/(Da), \\
&K_1 = K_0/(\rho a^2), L_0 = a/(U_0\gamma_T)
\end{aligned} \quad (7)$$

Using (6) and (7), the Eqs. (1)-(4) can be written in non-dimensional form as

$$\mathrm{Re}\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} - K_1\mathrm{Re}\frac{\partial^3 u}{\partial y^2 \partial t} + R\left(u_p - u\right) - (s^2 + H^2)u + Gr\theta\sin\alpha, \quad (8)$$

$$\mathrm{Re}\,M\frac{\partial u_p}{\partial t} = u_p - u, \quad (9)$$

$$\mathrm{Re}\,\mathrm{Pr}\frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} + N^2\theta - Q\theta + \frac{2}{3}R\left(\theta_p - \theta\right), \quad (10)$$

$$\frac{\partial \theta_p}{\partial \bar{t}} = -L_0\left(\theta_p - \theta\right). \tag{11}$$

The corresponding initial and boundary conditions (5) in non-dimensional form are

$$\begin{gathered} t = 0 : u = u_p = 0, \theta = 0, \theta_p = 0 \text{ at } y = 0 \text{ and } y = 1; \\ t > 0 : u = 0, u_p = 0, \theta = 1, \theta_p = 1 \quad \text{at} \quad y = 0; \\ t > 0 : u = 1 + \varepsilon e^{i\omega \bar{t}}, u_p = 1 + \varepsilon e^{i\omega \bar{t}}, \\ \theta = \theta_p = 1 + \varepsilon e^{i\omega \bar{t}} \text{ at } y = a, \end{gathered} \tag{12}$$

where Re is the Reynolds number, K_1 is the visco-elastic parameter, R is the particle concentration parameter, s is the porous medium shape factor parameter, Da is the Darcy number, H is the Hartmann number, Gr is the Grashof number, M is the particle mass parameter, Pr is the Prandtl number, N is the radiation parameter, Q is the heat absorption parameter and L_0 is the temperature relaxation time parameter.

III. METHOD OF SOLUTION

To solve the Eqs. (8)-(11), we assume that the velocity and temperature are to be of the form

$$\begin{gathered} -\frac{\partial p}{\partial x} = A\left(1 + \varepsilon e^{i\omega \bar{t}}\right), \\ U(y,t) = U_0(y) + \varepsilon e^{i\omega \bar{t}} U_1(y) + O(\varepsilon^2), \end{gathered} \tag{13}$$

where U stands for u, u_p, θ and θ_p.

Using (13), the Eqs. (8)-(11) become

$$u_0'' - (s^2 + H^2 + R)u_0 = -A - Ru_{p0} - Gr\theta \sin\alpha, \tag{14}$$

$$\left(1 - i\omega K_1 \operatorname{Re}\right)u_1'' - (s^2 + H^2 + i \operatorname{Re}\omega)u_1 = -A - Ru_{p1}, \tag{15}$$

$$u_{p0} = u_0, \tag{16}$$

$$u_{p1} = \frac{u_1}{\left(1 + i\omega M \operatorname{Re}\right)}, \tag{17}$$

$$\theta_0'' - (N^2 - Q - \frac{2}{3}R)\theta_0 = -\frac{2}{3}R\theta_{p0}, \tag{18}$$

$$\theta_1'' - (N^2 - Q - \frac{2}{3}R - i\omega \operatorname{Re}\Pr)\theta_1 = -\frac{2}{3}R\theta_{p1}, \tag{19}$$

$$\theta_{p0} = \theta_0, \tag{20}$$

$$\theta_{p1} = \frac{L_0\theta_1}{L_0 + i\omega}, \tag{21}$$

where the primes denote derivative with respect to y.

The corresponding boundary conditions become

$$\begin{gathered} u_0 = 0, u_1 = 0, u_{p0} = 0, u_{p1} = 0, \theta_0 = 1, \theta_1 = 0, \\ \theta_{p0} = 1, \theta_{p1} = 0 \quad \text{at} \quad y = 0; \\ u_0 = 1, u_1 = 1, u_{p0} = 1, u_{p1} = 1, \theta_0 = 1, \theta_1 = 1, \\ \theta_{p0} = 1, \theta_{p1} = 1 \quad \text{at} \quad y = 1. \end{gathered} \tag{22}$$

The Eqs. (14)-(21) subject to boundary conditions (22) are solved and the solution for fluid velocity, dust particles velocity, fluid temperature, and dust particles temperature are obtained as follows:

$$\begin{gathered} u(y,t) = A_5 + A_6 e^{m_2 y} + A_7 e^{-m_2 y} + A_3 e^{m_1 y} + A_4 e^{-m_1 y} \\ + \varepsilon e^{i\omega \bar{t}}\left(A_{13} e^{m_4 y} + A_{14} e^{-m_4 y} + A_{10} e^{m_3 y} + A_{11} e^{-m_3 y} + A_{12}\right), \end{gathered} \tag{23}$$

$$\begin{gathered} u_p(y,t) = A_5 + A_6 e^{m_2 y} + A_7 e^{-m_2 y} + A_3 e^{m_1 y} + \\ A_4 e^{-m_1 y} + \frac{\varepsilon e^{i\omega \bar{t}}}{(1 + i\omega M \operatorname{Re})}(A_{13} e^{m_4 y} + A_{14} e^{-m_4 y} \\ + A_{10} e^{m_3 y} + A_{11} e^{-m_3 y} + A_{12}), \end{gathered} \tag{24}$$

$$\theta(y,t) = A_1 e^{m_1 y} + A_2 e^{-m_1 y} + \varepsilon e^{i\omega \bar{t}}\left(A_8 e^{m_3 y} + A_9 e^{-m_3 y}\right), \tag{25}$$

$$\begin{gathered} \theta_p(y,t) = A_1 e^{m_1 y} + A_2 e^{-m_1 y} + \frac{\varepsilon L_0 e^{i\omega \bar{t}}}{(L_0 + i\omega)}(A_8 e^{m_3 y} \\ + A_9 e^{-m_3 y}). \end{gathered} \tag{26}$$

The constants m's and A's are not presented here for the sake of brevity.

The skin friction at the plates for fluid (τ_f) and dust particles (τ_p) are given by

$$\tau_f = \left(\frac{1}{\operatorname{Re}}\frac{\partial u}{\partial y} - K_1\frac{\partial^2 u}{\partial y \partial t}\right)_{y=0,1}, \tag{27}$$

$$\tau_p = \left(\frac{1}{\operatorname{Re}}\frac{\partial u_p}{\partial y} - K_1\frac{\partial^2 u_p}{\partial y \partial t}\right)_{y=0,1}, \tag{28}$$

Nusselt number across the channel's wall for fluid (Nu_f) and dust particles (Nu_p) are given by

$$Nu_f = -\frac{a\left(k\dfrac{\partial \bar{T}}{\partial \bar{y}}\right)_{\bar{y}=a}}{k(T_f - T_0)} = -\left(\frac{\partial \theta}{\partial y}\right)_{y=1}, \tag{29}$$

$$Nu_p = -\frac{a\left(k\dfrac{\partial \bar{T}_p}{\partial \bar{y}}\right)_{\bar{y}=a}}{k(T_f - T_0)} = -\left(\frac{\partial \theta_p}{\partial y}\right)_{y=1}. \tag{30}$$

The flow flux for fluid (ϕ_f) and dust particles (ϕ_p) within the channel are given by

$$\phi_f = \int_0^1 u\,dy, \qquad \phi_p = \int_0^1 u_p\,dy. \tag{31}$$

IV. RESULT AND DISCUSSION

In order to study the behaviour of flow problems, the numerical computation of velocity profiles, temperature profiles, skin friction coefficient, Nusselt number and flow flux for fluid as well as dust particles are carried out by assigning the following parameter values for computations unless otherwise indicated in the figures:

Re=4, N=1, H=1, M=1, s=2, Gr=4, ε=0.1, A=0.5, R=0.4, ωt=2π, α=π/6, Q=0.5, Pr=5, K_1=0.2, L_0=0.4.

Figures 2a and 2b represents the effect of magnetic field on the fluid and dust particles velocity profiles, respectively, across the channel. It is seen that an increasing values of Hartmann number H from H=0 through H=1, 2 to 3 retards the fluid motion as well as the motion of dust particles. This is due to the fact that the introduction of a transverse magnetic field normal to the direction of flow creates the Lorentz force, which tends to resist the flow. Figures 3a and 3b represents the effect of porous medium shape factor parameter S on fluid and dust particles velocity profiles, respectively. It is observed that an increase in S decreases the fluid velocity and dust particles velocity. Physically, an in-

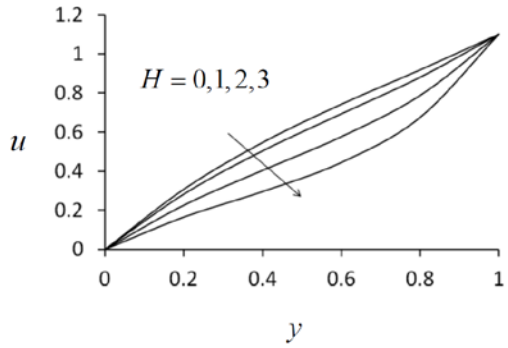

Fig. 2a. Effect of *H* on Fluid velocity.

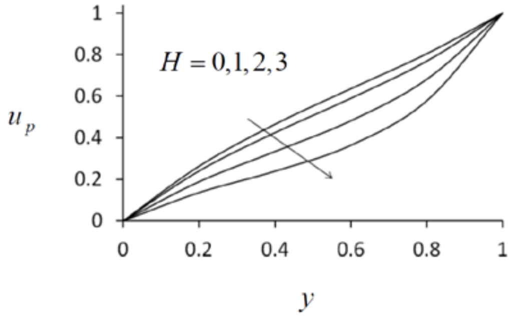

Fig. 2b. Effect of *H* on dust particles velocity.

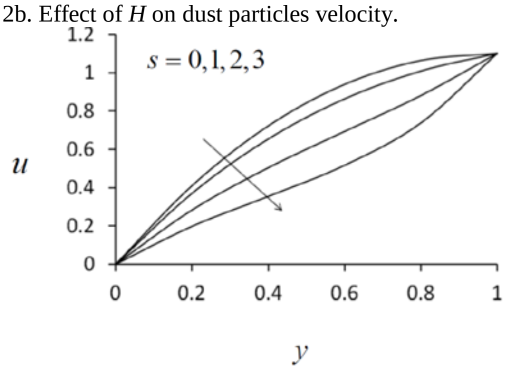

Fig. 3a. Effect of *s* on fluid velocity.

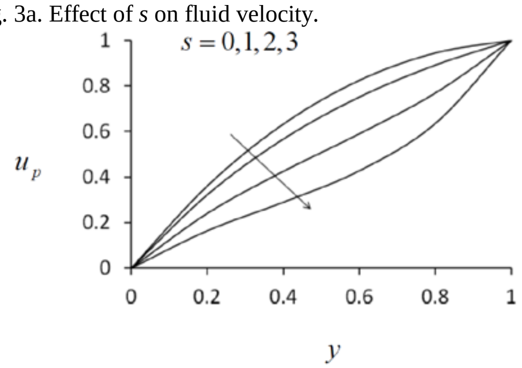

Fig. 3b. Effect of *s* on dust particles velocity.

crease in porous medium shape factor parameter leads to an increase in the resistance of the medium and thus decreases the fluid velocity and that of the dust particles velocity. Figures 4a and 4b depicts the influence of the heat sink parameter Q on fluid and dust particles velocity profiles, respectively. Velocity of the fluid and dust particles is decreasing for increasing of heat sink parameter. This is due to the fact that an increase in the heat sink i.e. absorption of heat decreases the buoyancy force and thus decreases in the velocity profiles for both fluid and dust particles. The above observations are in good agreement with the earlier results of Prakash *et al.* (2015) with $\alpha=\pi/2$, $u=u_p=0$ at $y=1$ in the case of Newtonian fluid.

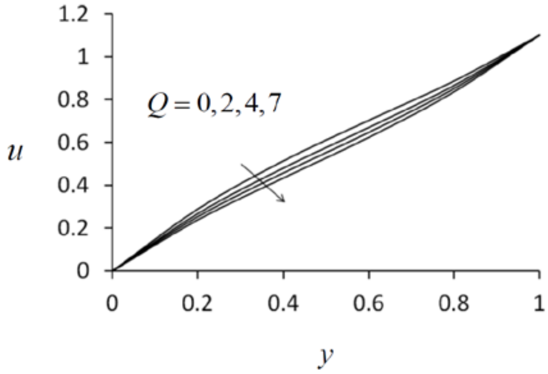

Fig. 4a. Effect of *Q* on fluid velocity.

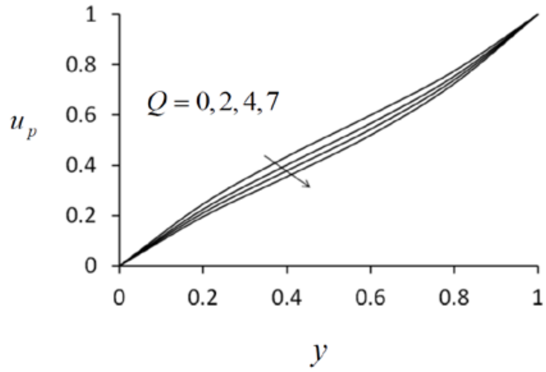

Fig. 4b. Effect of *Q* on dust particles velocity.

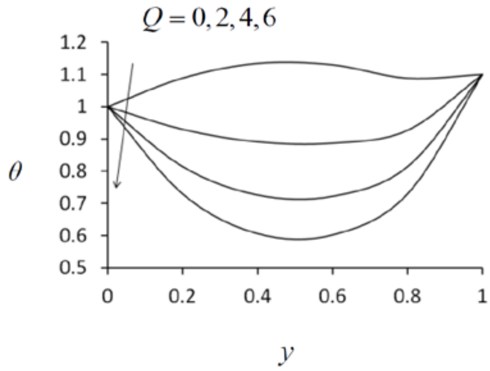

Fig. 5a. Effect of *Q* on fluid temperature.

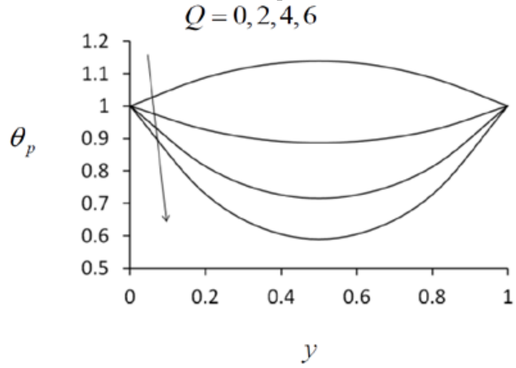

Fig.5b. Effect of *Q* on dust particles temperature.

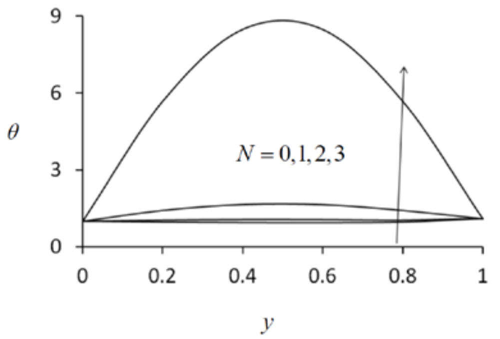

Fig. 6a. Effect of *N* on fluid temperature.

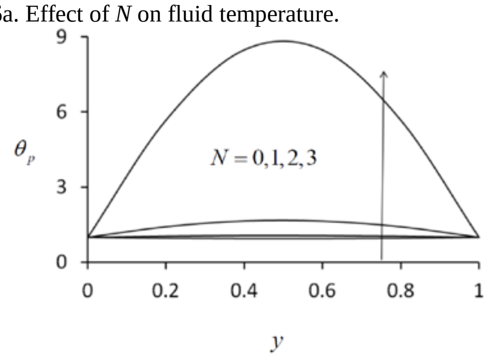

Fig. 6b. Effect of *N* on dust particles temperature.

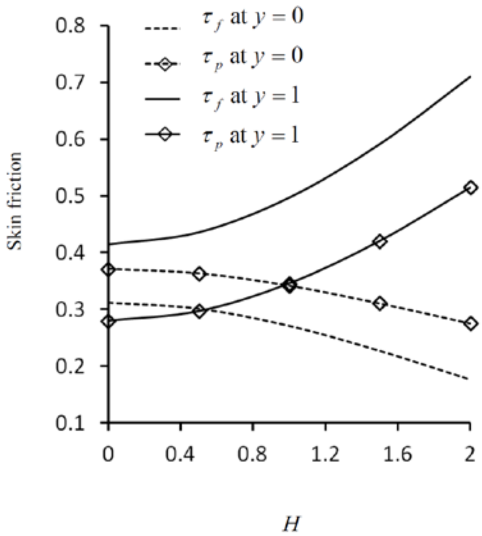

Fig. 7. Skin friction coefficient against *H*.

In general, from Figs. 2a-4a and Figs. 2b-4b, it is observed that the velocity of the fluid and the particles is low at the lower wall while it is high at the upper wall. Also, it is observed that the behaviours of fluid and dust particles velocity are same and the velocity of dust particles is less than the velocity of fluid.

Figures 5a and 5b illustrates the effect of the heat sink parameter on the temperature profiles of fluid and dust particles, respectively, across the channel. It is observed that the fluid and dust particles temperature decreases with an increase in the heat sink parameter. Further, minimum temperature of fluid and dust particles seems to occur within the centre line region of the channel (i.e. y=0.5) for Q = 2, 4, 6. But for Q = 0, for the maximum temperature seems to occur near y=0.5.

Figures 6a and 6b shows the effect of radiation parameter on the temperature profiles of fluid and dust particles, respectively. Increasing radiation parameter clearly enhances the temperature of both fluid and dust particles. Skin -friction coefficient of fluid and dust particles against Hartmann number is presented in Fig. 7. It is seen that as the Hartmann number increases, the values of skin-friction of the fluid as well as dust particles decrease for the lower wall y=0, whereas reverse effect is seen for the upper wall y=1. Nusselt number against the radiation parameter is presented in Fig. 8. It is seen that the Nusselt number for fluid and dust particles increases with an increase in radiation parameter. Figures 9a and 9b represent the flow flux for the fluid and the dust particles against Hartmann number and radiation parameter, respectively. It is seen that the flow flux decreases for increasing values of Hartmann number while increases for increasing values of radiation parameter. Also, it is observed that flow flux of dust particles is less than fluid.

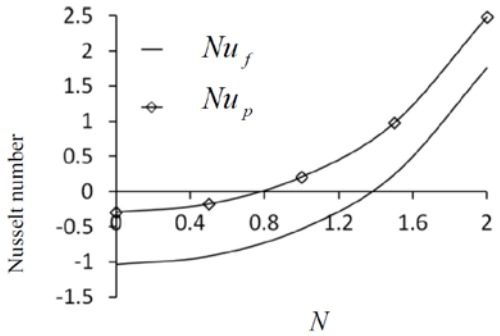

Fig. 8. Nusselt number against *N*.

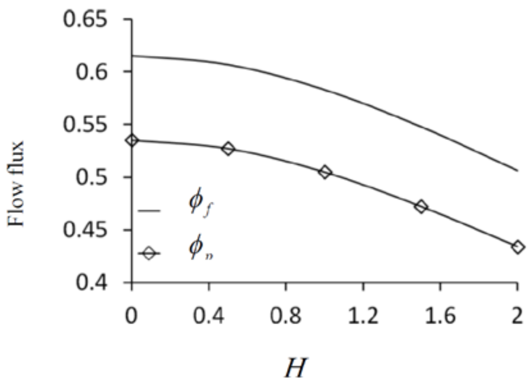

Fig. 9a. Flow flux against *H*.

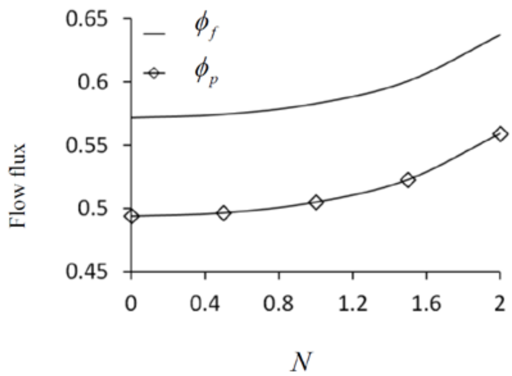

Fig. 9b. Flow flux against *N*.

V. CONCLUSIONS

The above study leads to the following conclusions:

1. The velocity profile for dust particles behaves the same as the fluid. Velocity of dust particles is less than velocity of fluid.
2. Velocity for both fluid and dust particles decreases with the increasing values of Hartmann number, porous medium shape factor parameter, heat sink parameter.
3. Increasing values of radiation parameter increases the temperature of both fluid and dust particles, but reverse trend is seen in case of heat sink parameter.
4. At the lower wall, the skin friction coefficient for both fluid and dust particles decreases with the increasing values of Hartmann number, but reverse trend is seen at the upper wall. Increasing values of radiation parameter increases the Nusselt number for both fluid and dust particles.
5. Flow flux of the dust particles is less than fluid.

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