THE SET OF FEASIBLE CONTROL VECTORS IN A TRAFFIC CONTROL PROBLEM

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***Abstract*— Signal Groups of a compatibility graph is used to solve conflict between the traffic streams and hence can used as a solution for traffic control problem at an intersection. The compatibility relation of the signal groups is defined and the control vectors i.e. the phases of the compatibility graph are used to control the flow of traffic. The set of feasible control vectors U_f is determined and phasing of signal group is done by choosing the controls from the set of feasible control U_f.**

***Keywords*—Compatibility Graph, Control Vector, Signal Group, Traffic Control.**

I. INTRODUCTION

Traffic jams are severe problems affecting cities around the world and they are only getting worse as the population and the number of vehicles continues to increase significantly while the area available for building new roads does not. Annually many citizens waste countless hours struck in traffic and the amount of time spent is expected to increase. One of the most cost effective measures to deal with the problem is urban traffic signal control. The lights of the city could be used to redirect traffic such that vehicles are moved away from heavily congested roads to the less congested ones, thereby reducing the amount of time wasted in traffic congestion. The problem then is figuring out how to generate appropriate signal timing patterns, as most simple patterns are not sufficient to optimally redirect traffic and reduce congestion. (Gazis, 1970)

Vehicular traffic control at road crossing has always been a matter of concern for administrations. Several attempts have been made to design efficient automated system to solve the problem, the most present day systems use predetermined timing circuits to operate traffic signals. However these systems are inefficient because they do not operate according to the current volume of traffic at the crossing.

Control of traffic signals for efficient movement of traffic on urban streets constitutes a challenging part of an urban traffic control system. (Stoffers, 1968) Traffic signal control varies in complexity, from simple system that use historic data to determine fixed time plans, to adaptive signal control, which optimizes timing plans for a network of signals according to traffic conditions in real time. Some of the common traffic signal operations for controlling the traffic flow are cycle time adjustment, split adjustment, where split is defined as the fraction of cycle time that is allocated to each phase for a set of traffic movements. Traffic control at an intersection is to optimize the system performance measure that is to adopt performance measure that will evaluate the efficiency and effectiveness of the road network thus improving the transportation system performance.

A. Artificial Intelligence Techniques

Artificial Intelligence (AI) techniques aim at imitating aspects of human intelligence and thinking while solving a problem by introducing human intelligence (to perceive a situation, to reason about it, and to act accordingly) into computer programs. AI techniques are mainly used in decision support systems, and one way to classify them is as follows:

- Case-based reasoning,
- Fuzzy logic,
- Rule-based systems,
- Artificial neural networks,
- Multi-agent systems

Case-based reasoning, as the name suggests, solves a problem using the knowledge that was gained from previously experienced similar situations (cases). In this way, this technique learns the way a new problem is solved, tests the proposed solution using simulation methods, and stores the new solution in a database. A disadvantage of this approach is that it might not be clear what should be done for a case that is not yet present in the case base. However, new cases could be added on-line to deal with this problem.

Fuzzy logic systems, like humans, can handle situations where the available information about the system is vague or imprecise. To deal with such situations, fuzzy sets are used to qualify the variables of the system in a non-quantitative way. Fuzzy sets are characterised using membership functions (e.g. Gaussian, triangle, or normal) that take a value between 0 and 1, and that indicate to what degree a given element belongs to the set (e.g., a speed could be 60% "high" and 40% "medium"). The membership degrees can then be used to combine various rules and to derive conclusions.

Rule-based systems solve a problem using "if-then" rules. These rules are constructed using expert knowledge and stored in an inference engine. The inference engine has an internal memory that stores rules and information about the problem, a pattern matcher, and a rule applier. The pattern matcher searches through the memory to decide which rules are suitable for the problem, and next the rule applier chooses the rule to apply. These systems are suited to solve problems where experts can make confident decisions. However, this system works only with already created rules and in its basic implementation it does not involve learning.

Artificial neural networks try to mimic the way in which the human brain processes information. These systems are useful in solving nonlinear problems where the rules or the algorithm to find solutions are difficult to derive. The basic processing unit of a neural network is called neuron or node. Each node fires a new signal when it receives a sufficiently high input signal from the other connected nodes. These nodes are organised in layers (an input layer, an output layer, and a number of hidden layers) and are interconnected by links or synapses, each associated with weights. A disadvantage is that artificial neural networks are non-informative models, and do not provide an explanation for the outcomes or for any failure that may occur in the process.

An agent is an entity that can perceive its environment through sensors and act upon its environment through actuators in such a way that the performance criteria are met. Multi-agent systems consist of a network of agents that are interacting among themselves to achieve specified goals. A high-level agent communication language is used by the agents for communication and negotiation purposes. Multi-agent systems can be applied to model complex systems, but their dynamic nature and the interactions between agents may give rise to conflicting goals or resource allocation problems.

Of the AI methods discussed above case-based reasoning and fuzzy logic are most often described in literature for traffic control purposes. In practice, rule-based systems are also used very often for traffic management and control (Baskar *et al.*, 2011).

Traffic control covers all measures, in respect to predetermined objectives, aimed at distributing and controlling traffic flows in space and time, in order to avoid disturbances such as traffic congestion or to reduce their impacts (Gazis, 2002).

B. Intelligent Vehicles and Traffic Control. Intelligent Vehicles (IV)

We can divide IV application areas into three categories depending on the level of support provided to the driver:

- Advisory systems use a human machine interface (HMI) (e.g., optic or acoustic) to provide an advisory or warning to the drivers. Some examples include blind spot warning, parking assistance, lane departure warning systems, and drowsy driver monitoring.
- Semi-autonomous systems can take partial control of vehicle manoeuvres and often use haptic (meaning “based on the sense of touch”) measures to assist the driver and can take partial control of vehicle manoeuvres. Examples are an intelligent speed adaptation system using an active accelerator pedal that exerts a counterforce at speeds over the speed limit, or a forward collision mitigation or avoidance system that first warns the driver via seat vibrations and next starts to brake in case the driver does not react to the warning.
- Fully autonomous systems take complete control of vehicle operations and eliminate the driver from the control loop. Examples include fully automated adaptive cruise control and anti-lock braking systems.

C. IV-based traffic management

The currently implemented traffic control and management systems are mainly using intelligence in the roadside infrastructure for controlling and managing the traffic system, However, such a system does not make use of the significant benefits offered by the intelligence — including the additional control, sensing, and communication capabilities — provided by IVs. So one way to improve performance and safety of the current transportation systems is by applying automation and advanced intelligent control methods to roadside infrastructure and vehicles (Baskar *et al.*, 2011).

II. COMPATIBILITY RELATION OF SIGNAL GROUP

In determining the control (signal plan) for an intersection, essential information relates to the feasibility of the simultaneous giving the right-of-way to different signal groups, and to determination of signal group pairs, belonging to a complete set of signal groups, such that the components of the pair are not allowed to move simultaneously through the intersection. (Guberinic *et al.*, 2008).

The compatibility relation is defined as follows: Two signal groups are compatible if each traffic stream belonging to one signal group is compatible with each traffic stream belonging to the other signal group.

The compatibility relation $C_g \subset D \times D$ can be defined by the following expression where D is the complete set of signal groups.

$$C_\downarrow g = \{(D_\downarrow p, D_\downarrow q)/(\sigma_\downarrow i, D_\downarrow p) \wedge (\sigma_\downarrow j, D_\downarrow q)\} \Rightarrow (\sigma_\downarrow i, \sigma_\downarrow j) \in C,$$

where D_p, D_q are signal groups and C is the compatibility relation of the traffic streams.

The graph $G_g=(D, C_g)$ is the graph of signal group compatibility of a complete set of signal group D. The graph G_g is a non oriented graph without any loop and multiple edges.

III. CONTROL VECTORS: PHASES

In any signal plan there exists some interval in which no components changes its value. A signal plan can be described by a sequence of control vectors, together with their durations. Each control vector represents the control in the same interval in which no component changes its value.

Traffic control on a signalized intersection during a cycle is defined by the vector time function

$$U(.) = [u_1(.), u_2(.), \cdots, u_p(.), \cdots, u_p(.)]^T.$$

Components of this function are functions of time, in the interval $[0,c]$, where c is the cycle time. A common name for vector time function $U(.)$ is signal plan.

The information contained in the signal group compatibility relation, C_g and the compatibility graph of signal groups, G_g can be used for determination of the set of feasible control vectors (phases).

Relation C_g contains information on pair of signal groups that can simultaneously get the right-of-way. The control vector- phase in the interval k is

$$U^k = \left[u_1^k, u_2^k, \cdots, u_p^k, \cdots, u_p^k\right]^T.$$

where $u^k_p \in \{0,1\}$, $(k \in K, p \in P)$.

Introducing the notation $B \in \{0,1\}$, the expression can be written as $u^k \in B^p$, $(k \in K)$, where $B^p = B \times B \times \ldots \times B$ (p groups), i.e., u^k is a vector whose components are equal to 0 and 1. (Guberinic *et al.*, 2008).

Two components u^k_p and u^k_q of this vector can have value 1 only if their respective signal group, D_p and D_q are compatible i.e.

$$\left(u_p^k \cdot u_q^k = 1\right) \Rightarrow \left(D_p, D_q\right) \in C_g.$$

However, one phase can give the right-of-way to more than two signal groups, or only to one signal group or no signal group. It is necessary of course, that the signal groups getting the simultaneous right-of-way are mutually compatible.

For determining all feasible phases, it is necessary to form all subsets of the chosen complete set of signal group, D_a with the property that each pair of signal groups from one subset belongs to the compatibility graph C_g. The subsets having this property are infact cliques of the graph (Chartrand, 1977; Deo, 2002).

$$G_g = \left(D_a, C_q\right) = \left(D_a, \Gamma_q\right).$$

where Γ_g is the mapping Γ_g: $D_a \rightarrow P(D_a)$, $P(D_a)$ is the partitive set of the set D_a.

In order to determine the set of feasible phases, it is necessary to find all the cliques of the graph G_g, and assign to each clique the phase that gives the right-of-way to signal groups that are members of the clique. In the set of feasible phases one more element shall be added: the phase whose components all have value 0 (all signal indications are red.)

Each phase

$$U^k = \left[u_1^k, u_2^k, \cdots, u_p^k, \cdots, u_p^k\right]^T$$

in the interval k belongs to the set of feasible phases:

$$u_f^k \in U_f.$$

If the sequence $\boldsymbol{u} = [u^1, u^2, \ldots, u^k, \ldots, u^K]$ represents the structure of a feasible signal plan, then the elements of the sequence (i.e. phases) in each interval $k \in K$, have to be chosen from the set of feasible phases U_f..

IV. AN EXAMPLE

Let us consider a traffic control problem at a signalized intersection with six streams. The intersection presented in the Fig. 1 bellow has six streams.

$$\tau = \{\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \sigma_6\}$$

The set of all cliques, D of the graph G_g i.e. the set of all signal groups are:

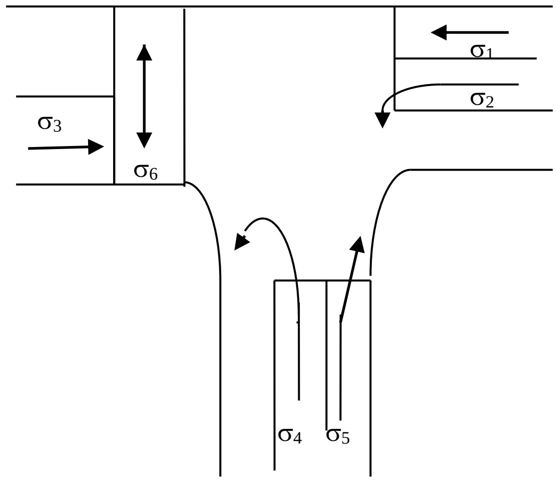

Fig. 1: A traffic intersection with six streams.

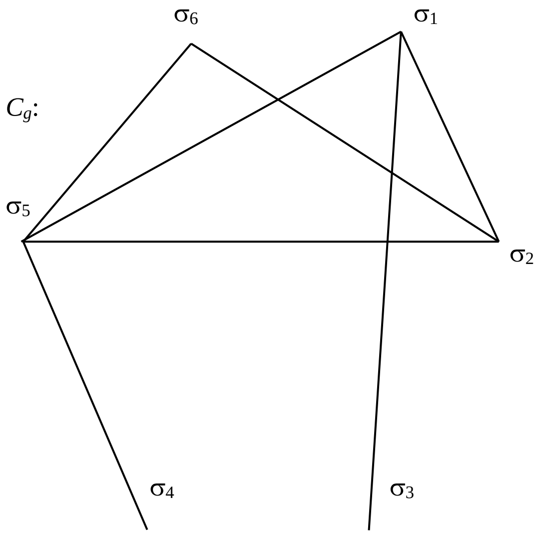

Fig. 2: Compatibility Graph of the intersection.

$$D = \{\{\sigma_\downarrow(1)\}, \{\sigma_\downarrow(2)\}, \{\sigma_\downarrow(3)\}, \{\sigma_\downarrow(4)\}, \{\sigma_\downarrow(5)\}, \{\sigma_\downarrow(6)\}, \{\sigma_\downarrow(1), \sigma_\downarrow(2)\}, \{\sigma_\downarrow(1), \sigma_\downarrow(3)\}, \{\sigma_\downarrow(1), \sigma_\downarrow(5)\}, \{\sigma_\downarrow(2), \sigma_\downarrow(5)\}, \{\sigma_\downarrow(4), \sigma_\downarrow(5)\}, \{\sigma_\downarrow(1), \sigma_\downarrow(2), \sigma_\downarrow(5)\}\}$$

$$D = \{D_\downarrow 1, D_\downarrow 2, D_\downarrow 3, D_\downarrow 4, D_\downarrow 5, D_\downarrow 6, D_\downarrow 7, D_\downarrow 8, D_\downarrow 9, D_\downarrow 10, D_\downarrow 11, D_\downarrow 12\}$$

The compatibility graph representing the above intersection is shown in Fig. 2 bellow denoted by C_g (Baruah and Baruah, 2012)

Let us consider the complete set of signal group D_a= {D_1, D_2, D_3, D_4, D_1, D_5}, where D_1= {σ2}, D_2= {σ4}, D_3= {σ5}, D_4= {σ6}, D_5= {σ1, σ3} for which the set of feasible control vector is to be determined.

Considering the signal groups of the complete set of signal groups D_a as the nodes the graph of the signal group compatibility is denoted by G_g.

The set of all cliques of the signal group compatibility graph G_g are :

$$\overline{D}_a = \{\{D_1\}, \{D_2\}, \{D_3\}, \{D_4\}, \{D_5\}, \{D_1, D_3\}, \{D_1, D_4\}, \{D_2, D_3\}, \{D_3, D_4\}, \{D_1, D_3, D_4\}\}$$

The phases that correspond to these cliques are:

$u(1)=(1,0,0,0,0)^T$, $u(2)=(0,1,0,0,0)^T$, $u(3)=(0,0,1,0,0)^T$,

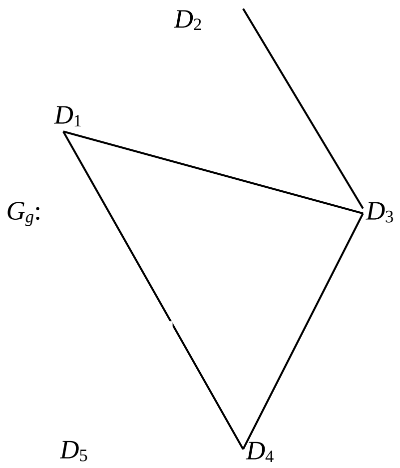

Fig. 3: Signal group compatibility graph G_g

$u(4)=(0,0,0,1,0)^T$, $u(5)=(0,0,0,0,1)^T$, $u(6)=(1,0,1,0,0)^T$, $u(7)=(1,0,0,1,0)^T$, $u(8)=(0,1,1,0,0)^T$, $u(9)=(0,0,1,1,0)^T$, $u(10)=(1,0,1,1,0)^T$,

The set of feasible phases is:

$U_f=\{(1,0,0,0,0)^T, (0,1,0,0,0)^T, (0,0,1,0,0)^T, (0,0,0,1,0)^T, (0,0,0,0,1)^T, (1,0,1,0,0)^T, (1,0,0,1,0)^T, (0,1,1,0,0)^T, (0,0,1,1,0)^T, (1,0,1,1,0)^T\}$.

V. SOLUTION OF THE PROBLEM

Solutions of traffic control problems are today based on application of modern mathematical optimization methods and digital computers. For efficient application, it is necessary to give exact formulations of their solution. According to the general system theory, a control problem can be formulated as a satisfaction problem or as an optimization problem.

The optimal control problem is the problem of choosing control $u(.)$ from the set of feasible controls $U_f \subset U$, which gives the optimal value of the chosen optimization criterion. Therefore it is necessary to:

i) Define the set whose elements are the controls that can be chosen, i.e. define the set of feasible control U_f.
ii) Define the measure that can be used for comparing effects of any two controls and for the choice of the best control because the set U_f is unordered.

Here in the present problem, our main objective is to minimize the waiting time of the traffic participants and to control the flow of traffic in such a way that most of the traffic streams are moving during the cycle. If we consider the cycle time to be 60 seconds, it is not possible to allow all the phases i.e. the signal group of the set of feasible phases (control vector) to move through the cycle. Therefore we can choose a set of phases or control corresponding to the signal group of the set feasible phases U_f so that the flow of traffic can be controlled by the selected set of phases. The entire traffic can be controlled by allowing the signal group of the particular set.

Also from the compatibility graph of the signal group G_g, it is significant that the signal group of the graph G_g is enough to control the flow of traffic at that particular intersection i.e. the set of signal group $\{\{D_2, D_3\}, \{D_1, D_3, D_4,\}, \{D_5\}\}$

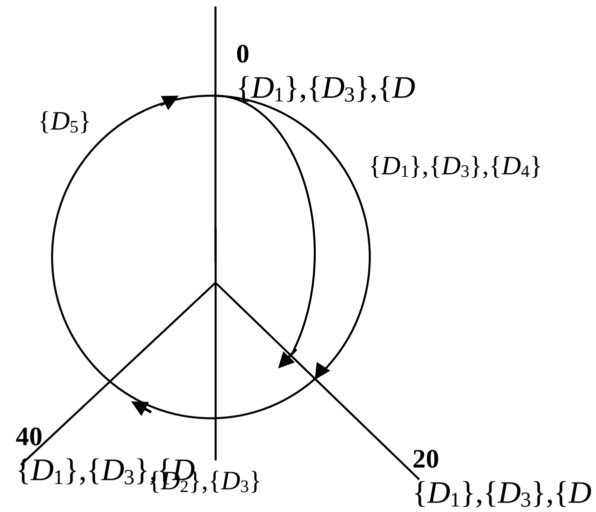

Fig. 4: Phasing of traffic lights. Cycle time 20 seconds.

$u(10)=(1,0,1,1,0)^T$, $u(8)=(0,1,1,0,0)^T$, $u(5)=(0,0,0,0,1)^T$.

The graphical representation by considering the cycle time to be 60 seconds in shown in the Fig. 4 bellow, where each of the set of signal group is allowed to move for 20 seconds.

VI. CONCLUSIONS

In this paper feasible control vectors of traffic control problem is defined and phasing is done by dividing the signal group of the feasible control vectors. Here set of signal groups from the compatibility graph of signal group G_g is used for phasing such that the entire flow of traffic at an intersection can be controlled by dividing the signal group of the graph G_g.

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