MHD FLOW, HEAT AND MASS TRANSFER ABOUT A PERMEABLE ROTATING VERTICAL CONE IN PRESENCE OF RADIATION, CHEMICAL REACTION AND HEAT GENERATION OR ABSORPTION EFFECTS

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***Abstract*— The problem of MHD flow, heat and mass transfer about a rotating vertical cone in presence of thermal radiation, chemical reaction and heat generation or absorption effects has been investigated. The governing nonlinear partial differential equations have been transformed into ordinary differential equations (ODE) by using suitable transformations and ODEs are solved by using MATLAB's built in solver bvp4c. Velocity, temperature and concentration profiles of the fluid are presented graphically. A comparison of present results for velocity and temperature is made with the numerical results available in literature and our results are in good agreement.**

***Keywords*— Rotating cone, radiation, chemical reaction, bvp4c.**

I. INTRODUCTION

MHD flow, heat and mass transfer problems about rotating cone-shaped bodies are often encountered in many engineering applications because of their uses in turbines, various propulsion devices for aircrafts, spin stabilized missiles, space vehicles, designing of canisters for nuclear waste disposal, nuclear reactor cooling system and geothermal reservoirs.

The thermal radiation effects on MHD flow are important in many engineering applications, such as in advance power plants for nuclear reactors, high speed flights, re-entry space vehicles and processes involving high temperatures. Previous experimental and theoretical investigations reveal that at high temperature the presence of thermal radiation alters the distribution of temperature in the boundary layer, which in turn affects the flow, heat and mass transfer.

Chemical and metallurgical industries require the study of heat and mass transfer in presence of chemical reaction. There are two types of chemical reactions namely, homogeneous and heterogeneous chemical reactions. A chemical reaction is said to be homogeneous if it takes place in the solution and heterogeneous if it takes place in the interface or in a restricted area. The order of chemical reaction depends on several factors. The first order chemical reaction is the simplest one in which the rate of reaction is directly proportional to the species concentration.

MHD flow, heat and mass transfer about a rotating cone was investigated by many researchers. Hartnett and Deland (1961) examined the influence of Prandtl number on the heat transfer by rotating bodies. The effects of an axial magnetic field on the flow and heat transfer about a rotating disk have been analyzed by Sparrow and Cess (1962). Heiring (1965) discussed the free convection by a rotational non-isothermal cone at low Prandtl number. Tien and Tsuji (1965) presented a theoretical analysis of laminar forced flow and heat transfer about a rotating cone. Kumari and Pop (1998) investigated the free convection over a vertical rotating cone with constant wall heat flux. Takhar *et al.* (2003) investigated unsteady mixed convection flow from a rotating vertical cone with a magnetic field. Anilkumar and Roy (2004) studied numerically the unsteady mixed convection flow on a rotating cone in a rotating fluid. Afify (2004) discussed the effect of radiation on free convective flow and mass transfer past a vertical isothermal cone surface with chemical reaction in the presence of a transverse magnetic field. Chamkha and Al-Mudhaf (2005) investigated the unsteady heat and mass transfer from a rotating vertical cone with a magnetic field and heat generation or absorption effects. Pullepu *et al*. (2008) investigated numerically the laminar free convection flow past a non-isothermal vertical cone by using finite difference method. Sharma and Singh (2010) analyzed the separation of species of a binary fluid mixture confined between two concentric rotating circular cylinders in presence of a strong radial magnetic field. Bég and Ghosh (2010) studied analytically magnetohydrodynamic radiation-convection with surface temperature oscillation and secondary flow effects. Pullepu *et al.* (2014) investigated the effects of chemical reaction on unsteady free convective and mass transfer flow from a vertical cone with heat generation or absorption in presence of variable wall temperature and variable wall concentration. The aim of this paper is to study MHD flow, heat and mass transfer about a vertical rotating cone in presence of thermal radiation, chemical reaction and heat generation or absorption effects. The nonlinear partial differential equations are converted into ordinary differential equations by using suitable transformations and then solved by using MATLAB's built in solver bvp4c.

The following assumptions are made

1. The system is considered as axi-symmetric.

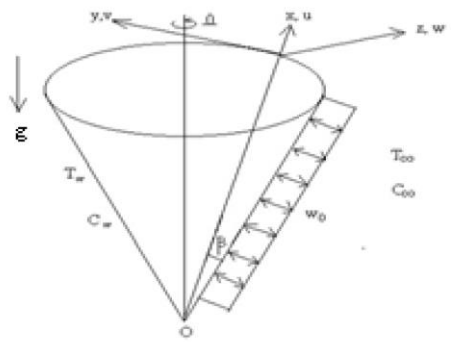

Figure 1. Physical model and coordinate system.

2. The magnetic Reynolds number is small so that the induced magnetic field is neglected.
3. The Joule heating of the fluid is neglected.
4. The Hall Effect of magnetohydrodynamics is neglected.
5. The medium is optically-thin with relatively low density.
6. There exists first order chemical reaction between the fluid and the species concentration and is homogeneous.

II. FORMULATION OF THE PROBLEM

Consider a steady, laminar, hydromagnetic, boundary layer flow of an electrically conducting and chemically reacting incompressible viscous fluid along an axisymmetric, permeable, heated and vertical rotating cone in ambient fluid with uniform angular velocity Ω around the axis of the cone. A uniform magnetic field of strength B_0 is applied in the direction of z-axis. The acceleration due to gravity g is acting downward parallel to the axis of the cone. Uniform suction or injection of the fluid with velocity w_0 is taking place at the surface of the cone.

The physical model and the coordinate system are shown in the Fig. 1. The curvilinear coordinate system (x, y, z) is taken such that x-axis is along the tangential direction, y-axis is along the circumferential direction and z-axis is along the normal direction of the cone respectively. Let u, v, w are the velocity components along the tangential, circumferential and normal directions to the cone respectively. The surface temperature T_w and species concentration at the surface C_w of the fluid are assumed to vary linearly with the distance x. Temperature of the fluid away from the surface T_∞ and species concentration away from the surface C_∞ are assumed to be constant.

Under the above assumptions and Boussinesq's approximation, the dimensional boundary layer equations governing the flow, heat and mass transfer can be expressed as

$$\frac{\partial u}{\partial x}+\frac{u}{x}+\frac{\partial w}{\partial z}=0, \qquad (1)$$

$$u\frac{\partial u}{\partial x}+w\frac{\partial u}{\partial z}-\frac{v^2}{x}=\nu\frac{\partial^2 u}{\partial z^2}+g\beta_T\left(T-T_\infty\right)\cos\beta + g\beta_C\left(C-C_\infty\right)\cos\beta-\frac{\sigma B_0^2 u}{\rho}, \qquad (2)$$

$$u\frac{\partial v}{\partial x}+w\frac{\partial v}{\partial z}-\frac{uv}{x}=\nu\frac{\partial^2 v}{\partial z^2}-\frac{\sigma B_0^2 v}{\rho}, \qquad (3)$$

$$u\frac{\partial T}{\partial x}+w\frac{\partial T}{\partial z}=\frac{1}{\rho C_p}\left\{k\frac{\partial^2 T}{\partial z^2}+Q_0\left(T-T_\infty\right)-\frac{\partial q_r}{\partial z}\right\} \qquad (4)$$

and

$$u\frac{\partial C}{\partial x}+w\frac{\partial C}{\partial z}=D_m\frac{\partial^2 C}{\partial z^2}-k_C\left(C-C_\infty\right) \qquad (5)$$

together with the boundary conditions

$$u=0, v=\Omega x\sin\beta, w=w_0, T=T_w(x), \quad C=C_w(x) \text{ when } z=0 \qquad (6)$$

and

$$u\to 0, v\to 0, T\to T_\infty, C\to C_\infty \text{ when } z\to\infty \qquad (7)$$

where T is the temperature of the fluid, C is the species concentration, k is the thermal conductivity, C_p is the specific heat at constant pressure, g is the acceleration due to gravity, ν is the kinematic viscosity, β is the semi-vertical angle of the cone, σ is the electrical conductivity, ρ is the density of the fluid, q_r is the radiation heat flux, β_T and β_C are coefficients of thermal expansion and mass expansion, Ω is the angular velocity of the cone around its axis, D_m is the mass diffusivity, Q_0 is the heat generation or absorption coefficient, k_c is the dimensional chemical reaction parameter respectively.

Dimensional boundary layer Eqs. (1) to (5) can be made dimensionless by using the following transformations:

$$\eta=\left(\frac{\Omega\sin\beta}{\nu}\right)^{1/2}z,\quad u(x,z)=-\frac{1}{2}\Omega x\sin\beta f'(\eta),$$
$$v(x,z)=\Omega x\sin\beta h(\eta),\quad w(x,z)=\left(\nu\Omega\sin\beta\right)^{1/2}f(\eta),$$
$$T(x,z)-T_\infty=\left(T_w-T_\infty\right)\theta(\eta)\frac{x}{L},$$

and

$$C(x,z)-C_\infty=\left(C_w-C_\infty\right)\phi(\eta)\frac{x}{L}, \qquad (8)$$

Prime represents derivative with respect to η. L is the characteristic length measured along the surface of the cone.

For optically-thin gray medium the radiation heat flux gradient is given (Bég and Ghosh, 2010) by the equation

$$\frac{\partial q_r}{\partial z}=-4a\sigma^*\left(T_\infty^4-T^4\right) \qquad (9)$$

where a and σ^* are absorption coefficient and the Stefan- Boltzmann constant respectively. It is assumed that temperature differences within the flow are sufficiently small such that T^4 can be expanded in a Taylor's series about T_∞ and after rejecting higher order terms it becomes

$$T^4=4T_\infty^3 T-3T_\infty^4. \qquad (10)$$

With these transformations Eq. (1) of continuity is identically satisfied and Eqs. (2) to (5) reduce to the following set of ordinary differential equations:

$$f'''-ff''+\frac{1}{2}\left(f'\right)-2h^2-2Ri\left(\theta+N\phi\right)-Mf'=0, \qquad (11)$$

$$h''-\left(fh'-f'h\right)-Mh=0, \qquad (12)$$

$$\theta'' - \Pr\left(f\theta' - \frac{1}{2} f'\theta \right) + \delta \Pr\theta - \Pr R_d \theta = 0 \qquad (13)$$

and

$$\phi'' - Sc\left(f\phi' - \frac{1}{2} f'\phi \right) - Sc\gamma\phi = 0 \qquad (14)$$

together with the boundary conditions

$$f' = 0,\ f = f_w, h = 1, \theta = 1, \phi = 1 \text{ when } \eta \to 0 \qquad (15)$$

and

$$f' = 0, h = 0, \theta = 0, \phi = 0 \text{ when } \eta \to \infty \qquad (16)$$

where

$Ri = \dfrac{Gr}{\mathrm{Re}^2}$, $Gr = \dfrac{g\beta_T \cos\beta (T_w - T_\infty) L^3}{v^2}$, $\mathrm{Re} = \dfrac{\Omega L^2 \sin\beta}{v}$,

$\Pr = \dfrac{\mu C_p}{k}$, $N = \dfrac{\beta_C (C_w - C_\infty)}{\beta_T (T_w - T_\infty)}$, $M = \dfrac{\sigma B_0^2}{\rho \Omega \sin\beta}$, $Sc = \dfrac{v}{D_m}$,

$\delta = \dfrac{Q_0}{\rho C_p \Omega \sin\beta}$, $R_d = \dfrac{16 v a \sigma^* T_\infty^3}{k \Omega \sin\beta}$ and $\gamma = \dfrac{k_c}{\Omega \sin\beta}$.

represent Richardson number, Grashof number, Reynolds number, Prandtl number, buoyancy parameter, magnetic parameter, Schmidt number, heat generation / absorption parameter, radiation parameter and chemical reaction parameter respectively. Here the suction / injection velocity is given by

$$f_w = \frac{w_0}{(v\Omega \sin\beta)^{1/2}},$$

where f_w=0 corresponds to the case of an impermeable rotating cone, f_w<0 is for suction, f_w>0 is for injection, δ<0 for heat absorption and δ>0 for heat generation.

It may be noted that if we omit the term Pr $f'\theta/2$ from Eq. (13) and take N=0, δ=0, R_d=0, f_w=0, Ri=0 then Eqs. (11) to (13) are identical to those of Sparrow and Cess (1962).

II. METHOD OF SOLUTION

Equations (11) to (14) are nonlinear ODEs and exhibit no closed-form solution. Therefore, they must be solved numerically subject to the boundary conditions Eq. (15) and Eq. (16). To solve our BVP we have employed MATLAB's built in solver bvp4c.

IV. RESULTS AND DISCUSSIONS

The velocity, temperature and concentration profiles are presented graphically to gain some insight into the effects of various parameters involved in the problem. In order to assess the accuracy of the numerical method, the present results of the physically important parameters i.e the wall shear stresses in the tangential and circumferential directions, ambient velocity and local rate of heat transfer, under the similar conditions, are compared with results obtained by Sparrow and Cess (1962). Comparisons are shown in Table 1 and Table 2. Results are found to be in very good agreement. Local rate of mass transfer for the present problem is obtained and presented in Table 3.

Figures 2 to 6 represent the velocity components, temperature and concentration profiles for values of R_d=[0.5, 1, 1.5], δ = 0.5, Pr=0.72, Ri=1, Sc=0.6, M=1, N=1; γ=1 and f_w=0.1.

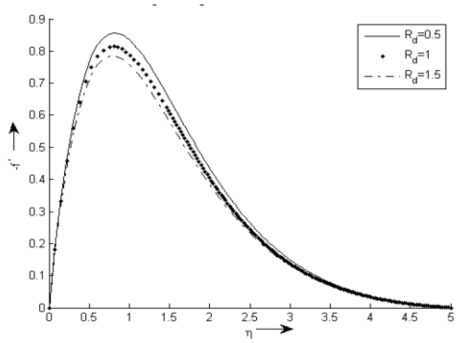

Figure 2. Tangential velocity profiles against η.

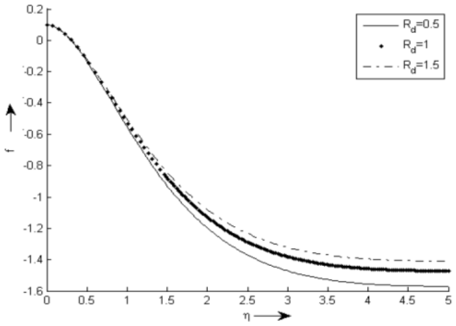

Figure 3. Normal velocity profiles against η.

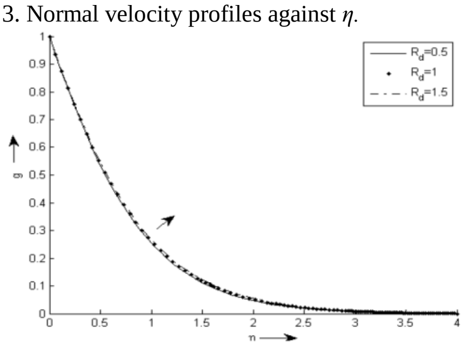

Figure 4. Circumferential velocity profiles against η

Figures 2 to 6 reveals that the effect of the increase in the values of R_d is to decrease the temperature and the tangential velocity component of the fluid but to increase the normal, the circumferential velocity components of the fluid and concentration of species at any point in the boundary layer. This means larger the values of the radiation parameter thinner the thermal boundary layer thickness. Physically, large values of radiation parameter imply decrease the rate of energy transport to the fluid. Thus, due to the reduction in buoyancy forces associated with the decreasing temperature enhances the concentration of species.

Figures 7 to 11 represent the velocity components, temperature and concentration profiles for values of γ=[0, 0.5, 1], δ=0.5, Pr=0.72; R_d=0.5, Ri=1, Sc=0.6, M=1, N=1 and f_w=0.1.

Figures 7 to 11 reveal that the effect of increase in the values of γ is to increase the normal, circumferential velocity components and temperature of the fluid but to decrease the tangential velocity component and concentration of species at any point in the boundary layer. It implies that as the value of γ increases the thermal boundary layer thickness enhances and the concentration boundary layer thickness reduces.

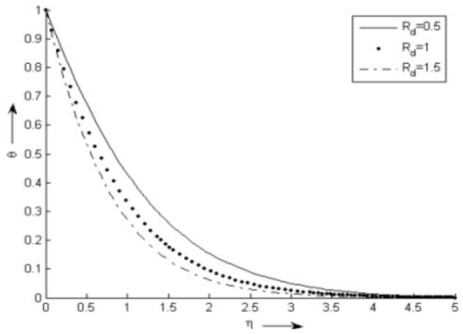

Figure 5. Temperature profiles against η.

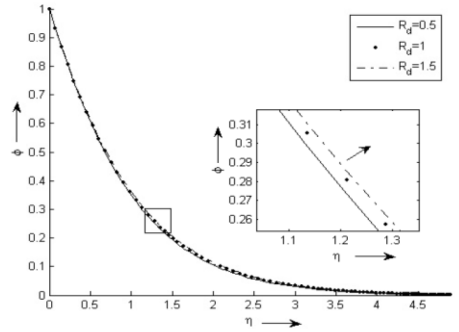

Figure 6. Concentration profiles against η.

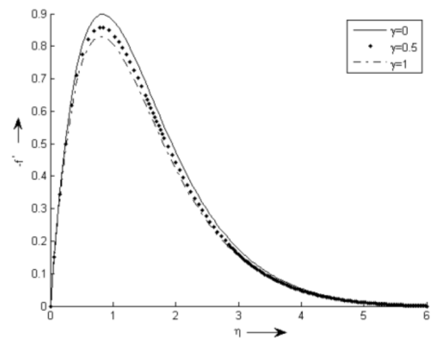

Figure 7. Tangential velocity profiles against η.

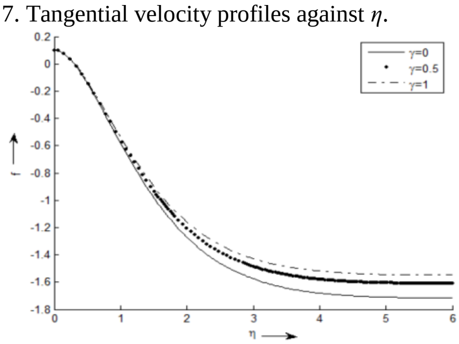

Figure 8. Normal velocity profiles against η.

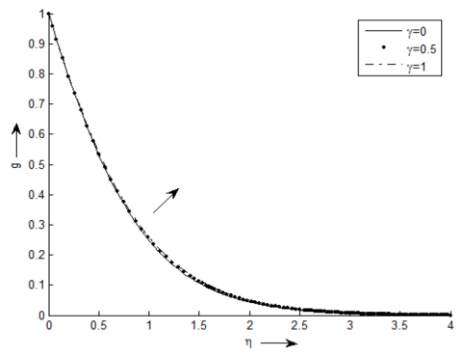

Figure 9. Circumferential velocity profiles against η.

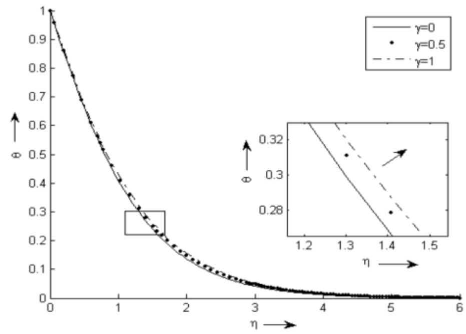

Figure 10. Temperature profiles against η.

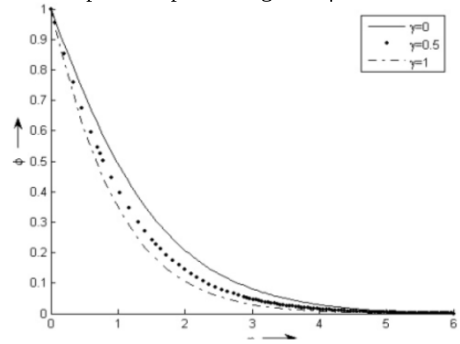

Figure 11. Concentration profiles against η.

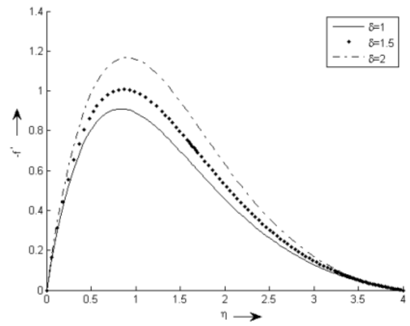

Figure 12. Tangential velocity profiles against η.

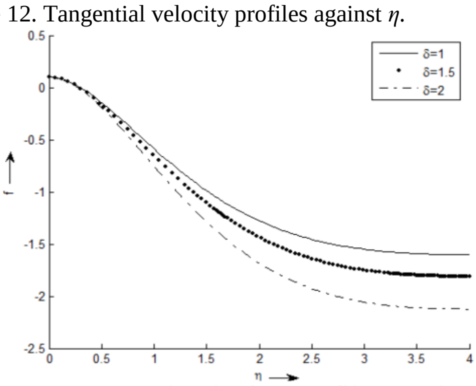

Figure 13. Normal velocity profiles against η.

Figures 12 to 16 represent the velocity components, temperature and concentration profiles for values of δ= [1, 1.5, 2], Pr=0.72; R_d=0.5, Ri = 1; Sc = 0.6; N=1; M=1; γ=1 and f_w=0.1. The effect of increase in the values of the heat generation/absorption parameter δ is to increase the tangential velocity component and temperature of the fluid but to decrease the normal velocity component, the circumferential velocity component and concentration of species of the fluid at any point in the boundary layer. Physically, heat generation implies rise in temperature and as a result thermal boundary layer thickness increases. It is also observed from the Fig. 15 that there is a drastic increase in temperature of the fluid near the surface of the cone due to thermal buoyancy forces caused by heat generation.

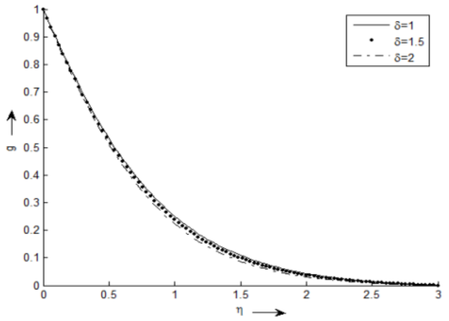

Figure 14. Circumferential velocity profiles against η.

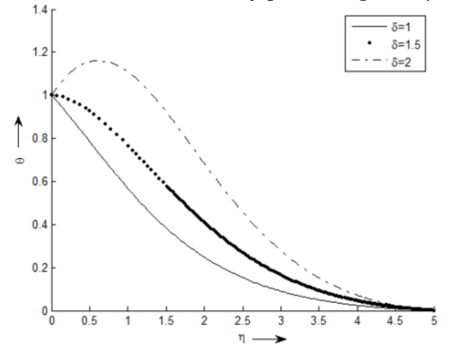

Figure 15. Temperature profiles against η.

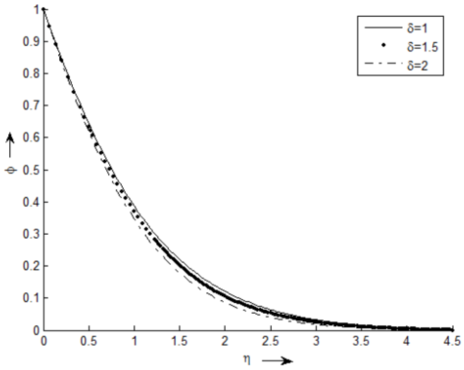

Figure 16. Concentration profiles against η.

Table1. Comparison of local surface shear stresses and ambient velocity by taking δ=0; Ri=0; N=0; R_d=0; f_w=0

Present result			
M	$-f''(0)$	$-g'(0)$	$-f(\infty)$
0	1.0205	0.6159	0.8845
0.5	0.7703	0.8487	0.4589
1	0.6185	1.0691	0.2533
2	0.4611	1.4421	0.1086
3	0.3810	1.7477	0.0618
4	0.3314	2.0103	0.0408
Previous result			
M	$-f''(0)$	$-g'(0)$	$-f(\infty)$
0	1.021	0.616	0.885
0.5	0.770	0.849	0.459
1	0.619	1.069	0.253
2	0.461	1.442	0.109
3	0.381	1.748	0.0618
4	0.331	2.010	0.0408

Table2. Comparison of local rate of surface heat transfer.

Previous results for $-\theta'(0)$ (Sparrow and Cess)			
M	Pr=0.1	Pr=1	Pr=10
0	0.0766	0.3960	1.1340
0.5	0.0428	0.2820	0.9560
1	0.0244	0.1940	0.8010
2	0.0108	0.0982	0.5710
3	0.00614	0.0588	0.4220
4	0.00407	0.0395	0.3180
Present results for $-\theta'(0)$			
M	Pr=0.1	Pr=1	Pr=10
0	0.0757	0.3964	1.1340
0.5	0.0429	0.2827	0.9515
1	0.0245	0.1938	0.7985
2	0.0108	0.0981	0.5733
3	0.0062	0.0588	0.4213
4	0.0042	0.0399	0.3183

Table3. Local rate of mass transfer in the present case is given below.

δ	M	R_d	γ	f_w	$-\phi'(0)$
1	1	0.5	1	0.5	0.8136
1.5	1	0.5	1	0.5	0.8310
2	1	0.5	1	0.5	0.8574
0.5	1	1	0	0	0.5670
0.5	1	1	0.5	0	0.7657
0.5	1	1	1	0	0.9289
0.5	1.5	1	1	0	0.9138
0.5	2	1	1	0	0.9014
0.5	0.5	1.5	0.5	-1	1.1256
1	0.5	1.5	0.5	-1	1.1363

Table 3 shows the local rate of mass transfer $-\phi'(0)$ for Pr=0.72, N=1, Sc=0.6,Ri=1 and different values of δ, M, R_d, γ, f_w. The behaviour of this number is self evident.

V. CONCLUSION

From the above discussions it can be concluded that

1. The tangential velocity component increases for the increasing values of δ and decreases for the increasing values of R_d and γ.
2. The normal and the circumferential velocity components increase for the increasing values of R_d and γ but decreases for the increasing values of δ.
3. Temperature of the fluid increases with the increase in the values of γ and δ but decreases with the increase in the values of R_d.
4. Concentration of species of the fluid increases with the increase in the values of R_d but decreases with the increase in the values of γ and δ.

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NOMENCLATURE

a absorption coefficient
B_0 magnetic field strength
C species concentration (kg.m^{-3})
C_w species concentration at the surface (kg.m^{-3})
C_∞ species concentration away from the surface (kg.m^{-3})
c_p specific heat at constant pressure(J.kg^{-1}.K)
D_m mass diffusivity (m^{2}s^{-1})
f, h similarity velocity functions
f_w dimensionless suction /injection velocity (m.s^{-1})
g acceleration due to gravity (m.s^{-2})
Gr Grashof number
k thermal conductivity (W.m^{-1}.K^{-1})
k_c dimensional chemical reaction parameter
L characteristic length (m)
M magnetic parameter
N buoyancy parameter
Pr Prandtl number
Q dimensional heat generation parameter
q_r radiation heat flux
R_d dimensionless radiation parameter
Re Reynolds number
Ri Richardson number
Sc Schmidt number
T fluid temperature (K)
T_w surface temperature (K)
T_∞ fluid temperature away from surface (K)
u tangential velocity component (m.s^{-1})
v circumferential velocity component (m.s^{-1})
w normal velocity component (m.s^{-1})
w_0 dimensional suction/injection velocity (m.s^{-1})
(x, y, z) curvilinear coordinates

Greek symbols

Ω angular velocity(rad.s^{-1})
ρ fluid density (kg.m^{-3})
ν kinematic viscosity (m2s^{-1})
β semi-vertical angle (radian)
β_T, β_c coefficients of thermal and mass expansions (K^{-1})
η similarity variables
θ dimensionless temperature
ϕ dimensionless species concentration
δ dimensionless heat generation parameter
γ dimensionless chemical reaction parameter
σ electrical conductivity(siemens.m^{-1})
σ^* Stefan-Boltzmann constant

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