

ON HYDROMAGNETIC BOUNDARY LAYER FLOW^o OF NANOFLUIDS OVER A PERMEABLE MOVING SURFACE WITH NEWTONIAN HEATING

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Abstract— The magnetohydrodynamics (MHD) boundary layer flow of nanofluids past a permeable moving flat plate with convective heating at the plate surface has been studied. The nanofluids considered contain water as the base fluid with copper (Cu) or Alumina (Al_2O_3) as the nanoparticles. The model equations are obtained and solved numerically by applying shooting iteration technique together with the fourth order Runge-Kutta-Fehlberg integration scheme. The influence of pertinent parameters on velocity, temperature, skin friction and Nusselt number are investigated. The obtained results are presented graphically and the physical aspects of the problem discussed quantitatively.

Keywords— Magnetohydrodynamics; convective heating, boundary layer; nanofluids; permeable plate.

I. INTRODUCTION

The study of the flow of an electrically conducting fluids past permeable walls not only possesses a theoretical appeal but also model many biological and engineering problems such as MHD generators, nuclear reactors, geothermal energy extraction, drag reduction in aerodynamics, blood flow problems among others (Moreau, 1990). In metallurgy, the quality of the final product depends mainly on the cooling liquid used and the cooling rate. The combined effects of heat transfer and MHD are useful in achieving the desired characteristics of the final product. Experimental and theoretical studies on convectional electrically conducting fluids indicate that magnetic field markedly changes their transport and heat transfer characteristics. In a pioneering work, Sakiadis (1961) investigated the boundary layer flow induced by a moving plate in a quiescent ambient fluid. Suction of a fluid on the boundary surface, can significantly change the flow field and, as a consequence, affect the heat transfer rate at the surface (Chaudhary and Merkin, 1993; Al-Sanea, 2004; Ishak *et al.*, 2007). Various aspects of boundary layer flow problem have been investigated by several authors (Makinde, 2009, 2011; Makinde and Aziz, 2010; Bachok *et al.*, 2012). Convectional heat transfer fluids such as water, mineral oil and ethylene glycol have poor heat transfer properties compared to those of most solids. An innovative way of improving the heat transfer of fluids is to suspend small amounts of nanometer-sized (10–50 nm) particles and

fibers in the fluids. This new kind of fluids named as “nanofluids” was introduced by Choi (1995). The nanoparticles are made of metals such as aluminium, copper, gold, iron, titanium or their oxides and the most commonly used base fluids are water, ethylene glycol, toluene and oil. The choice of base fluid-particle combination depends on the application for which the nanofluid is intended. Nanofluids, with their various potential applications in industrial, engineering and biomedicine have recently attracted intensive studies (Xuan and Li, 2000; Wen and Ding, 2004; Oztop and Abu-Nada, 2008; Bachok *et al.*, 2010; Makinde and Aziz, 2011; Kuznetsov and Nield, 2010; Anjali and Andrews, 2011). The remarkably improved convective heat transfer coefficient makes the nanofluid a superior heat transfer medium for cooling application.

Most convectional fluids used for producing nanofluids are liquids and their electrical conductivity properties are lower than that of the metallic or nonmetallic nanoparticles. The presence of the nanoparticles enhance the electrical conductivity property of the nanofluids, hence are more susceptible to the influence of magnetic field than the convectional base fluids. Recently, several authors (Nourazar *et al.*, 2011; Hamad *et al.*, 2011; Hamad, 2011; Ghasemi *et al.*, 2011) numerically investigated the natural convection of nanofluids under the influence of a magnetic field. Their theoretical studies on magnetic nanofluids assumed that both the nanoparticles and the convectional base fluids have equal electrical conductivity properties. In reality, this is not the case and ignoring the difference in electrical conductivity property of both the nanoparticles and the convectional base fluids may affect the outcome of the investigations carried out. To our knowledge, no attempt has been made in the past to study the effects of the complex interaction between the electrical conductivity of the convectional base fluids and that of the nanoparticles on the hydromagnetic flow with Newtonian heating. The main objective of this paper is to numerically investigate the change in the electrical conductivity of the convectional base fluids as a result of the nanoparticles and the subsequent interaction with the magnetic field at the boundary layer flow over a flat permeable surface with convective heating. The a set of coupled non-linear ordinary differential equations for momentum and energy balance are solved numerically by applying shooting iteration technique together with

fourth order Runge–Kutta integration scheme. Several results showing velocity and temperature profiles, skin friction and Nusselt number are presented graphically and discussed quantitatively.

II. MATHEMATICAL FORMULATION

We consider a steady unidirectional boundary layer flow of an electrically conducting nanofluid (Cu-water and Al_2O_3 -water) past a semi-infinite permeable moving flat plate in the presence of a uniform transverse magnetic field of strength B_0 applied parallel to the y -axis (see Fig. 1). It is assumed that the induced magnetic field and the external electric field are negligible. At the boundary, the permeable plate is moving at a velocity U_0 with a hot convective fluid of temperature T_f flowing below it and a cold nanofluid of temperature $T < T_f$ flowing above the plate. Far away from the plate, $u=0$, $T=T_\infty$.

The x -axis is taken along the direction of plate and y -axis normal to it. The surface temperature is assumed to be maintained by convective heat transfer at a constant temperature T_f . Under the boundary-layer approximation the nanofluid equations for continuity, momentum and energy balance governing the problem under consideration in one dimension are written as

$$\frac{\partial u}{\partial y} = 0, \quad (1)$$

$$-V \frac{du}{dy} = -\frac{1}{\rho_{nf}} \frac{dp}{dx} + \frac{\mu_{nf}}{\rho_{nf}} \frac{d^2 u}{dy^2} - \frac{\sigma_{nf} B_0^2 u}{\rho_{nf}}, \quad (2)$$

$$-V \frac{dT}{dy} = \frac{K_{nf}}{(\rho c_p)_{nf}} \frac{d^2 T}{dy^2} + \frac{\mu_{nf}}{(\rho c_p)_{nf}} \left(\frac{du}{dy} \right)^2 + \frac{\sigma_{nf} B_0^2 u^2}{(\rho c_p)_{nf}}, \quad (3)$$

with boundary conditions,

$$u(x,0) = U_0, \quad -k_f \frac{\partial T}{\partial y}(x,0) = h_f [T_f - T(x,0)], \quad (4)$$

$$u(x,\infty) = 0, \quad T(x,\infty) = T_\infty, \quad (5)$$

where (u,v) are the velocity components of the nanofluid in the (x,y) directions respectively, U_0 is the plate velocity, T is the temperature of the nanofluid, T_f is the temperature of the hot convective fluid, μ_{nf} is the

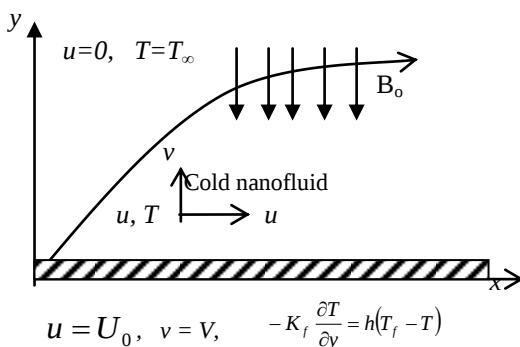


Fig. 1. Flow configuration and coordinate system

Table 1: Thermophysical properties of water, Copper and Alumina (Bachok *et al.*, 2012)

Physical properties	Fluid phase (water)	Cu	Al_2O_3
c_p ($\text{J.kg}^{-1}\text{K}^{-1}$)	4179	385	765
ρ (kg.m^{-3})	997.1	8933	3970
k ($\text{W.m}^{-1}\text{K}^{-1}$)	0.613	400	40
σ (S/m)	5.5×10^{-6}	58×10^6	35×10^6

dynamic viscosity of the nanofluid, ρ_{nf} is the density of the nanofluid, σ_{nf} is the electrical conductivity of the nanofluid, α_{nf} is the thermal diffusivity of the nanofluid, $(\rho c_p)_{nf}$ is the heat capacitance of the nanofluid and k_{nf} is the thermal conductivity of the nanofluid.

Introducing the following dimensionless variables and quantities into the governing conservation Eqs. (1)-(5)

$$\eta = \frac{yU_0}{\nu_f}, W = \frac{u}{U_0}, S = \frac{V}{U_0}, \theta = \frac{T - T_\infty}{T_f - T_\infty}, v_f = \frac{\mu_f}{\rho_f}, \text{Pr} = \frac{\nu_f}{\alpha_f},$$

$$\text{Bi} = \frac{h\nu_f}{U_0 k_f}, \alpha_f = \frac{k_f}{(\rho c_p)_f}, \text{Ha} = \frac{\sigma_f B_0^2 \nu_f}{\rho_f U_0^2}, \text{Br} = \frac{\mu_f U_0^2}{k_f (T_f - T_\infty)},$$

$$\mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}}, \rho_{nf} = (1-\phi)\rho_f + \phi\rho_s,$$

$$(\rho c_p) = (1-\phi)(\rho c_p)_f + \phi(\rho c_p)_s,$$

$$\sigma_{nf} = (1-\phi)\sigma_f + \phi\sigma_s, \frac{k_{nf}}{k_f} = \frac{(k_s + 2k_f) - 2\phi(k_f - k_s)}{(k_s + 2k_f) + \phi(k_f - k_s)},$$

we obtain,

$$\frac{d^2 W}{d\eta^2} + S(1-\phi)^{2.5}(1-\phi + \phi\rho_s/\rho_f) \frac{dW}{d\eta} - \text{Ha}(1-\phi)^{2.5}(1-\phi + \phi\sigma_s/\sigma_f)W = 0, \quad (6)$$

$$\frac{d^2 \theta}{d\eta^2} + \frac{S \text{Pr} k_f}{k_{nf}} (1-\phi + \phi(\rho c_p)_s/(\rho c_p)_f) \frac{d\theta}{d\eta} + \frac{k_f \text{Br}}{k_{nf} (1-\phi)^{2.5}} \left(\frac{dW}{d\eta} \right)^2 + \frac{\text{Ha} \text{Br} k_f}{k_{nf}} (1-\phi + \phi\sigma_s/\sigma_f)W^2 = 0, \quad (7)$$

$$\frac{k_f \text{Br}}{k_{nf} (1-\phi)^{2.5}} \left(\frac{dW}{d\eta} \right)^2 + \frac{\text{Ha} \text{Br} k_f}{k_{nf}} (1-\phi + \phi\sigma_s/\sigma_f)W^2 = 0,$$

$$W = 1, \quad \frac{d\theta}{d\eta} = \text{Bi}(\theta - 1) \quad \text{at} \quad \eta = 0, \quad (8)$$

$$W = 0, \quad \theta = 0, \quad \text{as} \quad \eta \rightarrow \infty, \quad (9)$$

where W and θ are the dimensionless nanofluid velocity and temperature respectively, Pr represents the Prandtl number, Br is the Brinkmann number, S is the suction velocity parameter, Ha is the Hartmann number, Bi is the Biot number, ϕ is the solid volume fraction parameter of the nanofluid, μ_f is the viscosity of the base fluid, ρ_f and ρ_s is the density of the base fluid and nanoparticle respectively, k_f and k_s is the thermal conductivities of the base fluid and nanoparticle respectively, $(\rho c_p)_f$ and $(\rho c_p)_s$ are the heat capacitance of the base fluid and the nanoparticle respectively, σ_f and σ_s is the electrical conductivities of the solid volume fraction, is the electrical conductivity of the base fluid and the nanoparticle respectively. It is important to take note that $\phi = 0$ correspond to a regular fluid scenario with magnetic field effect.

Table 2: Computations showing comparison with Makinde and Aziz (2010) for $Br=0.1$, $Pr=0.72$, $\phi=0$, $B_i=0.1$, $S=1$.

Ha	$\theta(0)$ Makinde and Aziz (2010)	$-\theta'(0)$ Makinde and Aziz (2010)	$W'(0)$ Makinde and Aziz (2010)	$\theta(0)$ Present	$\theta'(0)$ Present	$W'(0)$ Present
0.1	0.1939	0.0806	1.0916	0.1939	0.0806	1.0916
0.5	0.2273	0.0772	1.3660	0.2273	0.0772	1.3660
1	0.2581	0.0742	1.6180	0.2581	0.0742	1.6180

The physical quantities of practical significance in this work are skin friction coefficient and the local Nusselt number Nu , which are defined as

$$C_f = \frac{\tau_w}{\rho_f U_0^2}, \quad Nu = \frac{xq_w}{k_f(T_f - T_\infty)}, \quad (10)$$

where τ_w is the skin friction and q_w is the heat flux from the plate which are given by

$$\tau_w = \mu_{nf} \left. \frac{\partial u}{\partial y} \right|_{y=0}, \quad q_w = -k_{nf} \left. \frac{\partial T}{\partial y} \right|_{y=0} \quad (11)$$

Substituting Eq. (11) into (10), we obtain

$$C_f = \frac{1}{(1-\phi)^{2.5}} W'(0), \quad Re_x^{-1} Nu = -\frac{k_{nf}}{k_f} \theta'(0), \quad (12)$$

where $Re_x = U_\infty x / \nu_f$ is the local Reynolds number and the prime symbol denotes derivatives with respect to η . The set of Eqs.(1)–(2) under the boundary conditions (3)–(4) have been solved numerically using Runge-Kutta-Fehlberg method with shooting technique implemented on Maple 12. From the numerical computations, the local skin-friction coefficient and the local Nusselt number in Eq. (12) are also worked out and their numerical values depicted graphically.

III. RESULTS AND DISCUSSION

The effects of various dimensionless parameters on the velocity, temperature, skin friction and Nusselt's number are illustrated by the set of Figs. 2-15. The Prandtl number is kept constant at 6.2 (Ahmad *et al.*, 2011). The values of the Magnetic parameter Ha considered range from 10^{-10} to 10^{-13} and the nanoparticle volume fraction parameter ϕ is varied from 0 to 0.2. $Ha = 0$ corresponds to absence of magnetic field and $\phi=0$ is regular fluid.

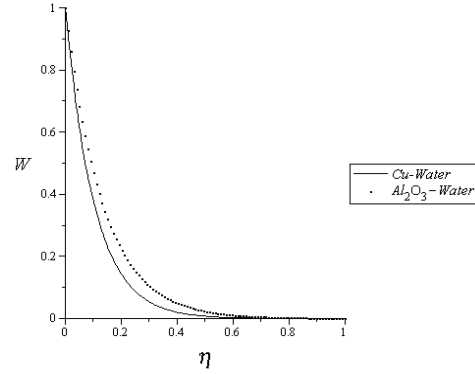
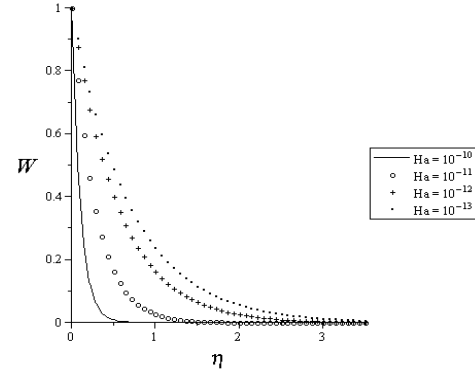
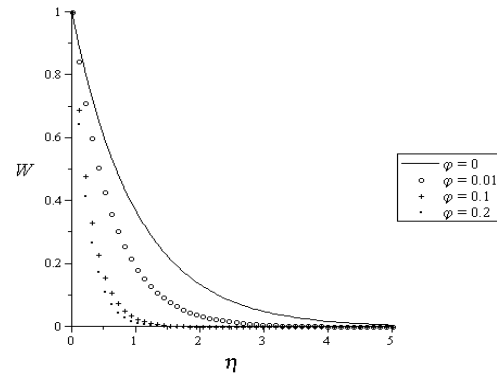
In order to benchmark our numerical results, the special case of heat transfer in MHD flow of convective fluid (i.e. $\phi=0$) over a moving permeable surface is compared with that of Makinde and Aziz (2010) as shown in the Table 2.

It is noteworthy that the numerical results in Table 2 show perfect agreement, hence validate the accuracy of our numerical procedure and the MHD nanofluids results obtained thereafter.

A. Velocity profiles

Figures 2-5 depicts the effects of various physical parameters on the nanofluid velocity profiles. It is noted that for all the pertinent parameters, the velocity is maximum at the moving plate surface but decreases gradually to zero at the free stream far away from the plate surface thus satisfying the boundary conditions. Figure 2 shows that the momentum boundary thickness for Cu-water nanofluid is smaller than that of Al_2O_3 -water nanofluid, consequently, Cu-water nanofluid tends to flow closer to the convectively heated plate surface and

serve as a better coolant than Al_2O_3 -water nanofluid. It is observed in Fig. 3, an increase in the magnetic parameter Ha pushes the fluid towards the plate surface hence decreasing both the momentum boundary layer thickness and the fluid velocity. This is in agreement with the physics of the problem in that, an increase in the magnetic field intensity leads to an increase in the Lorentz force thus producing more resistance to the transport phenomena. A similar trend is observed with increase in the nanoparticle volume fraction ϕ and suction parameter S as ascertained in figs. 4 and 5.


Fig. 2: Velocity profiles for $S = 1$, $Br = Bi = \phi = 0.1$, $Ha = 10^{-10}$

Fig. 3: Effects of Ha on velocity profiles with Cu-water nanofluid for $S = 1$, $Br = Bi = \phi = 0.1$

Fig. 4 : Effects of increasing nanoparticles ϕ on velocity profiles with Cu-water nanofluid for $S = 1$, $Ha = 10^{-11}$, $Br = Bi = 0.1$

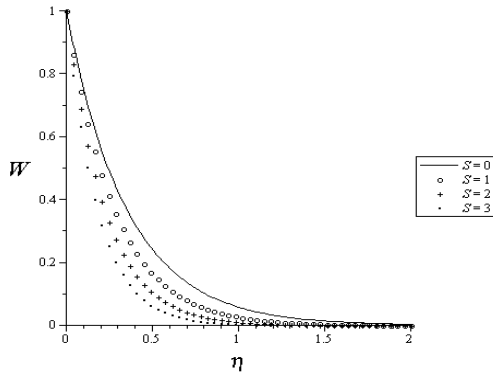


Fig. 5: Effect of S on velocity profiles with Cu-water nanofluid for $Ha=10^{-11}$, $Br=Bi=\phi=0.1$

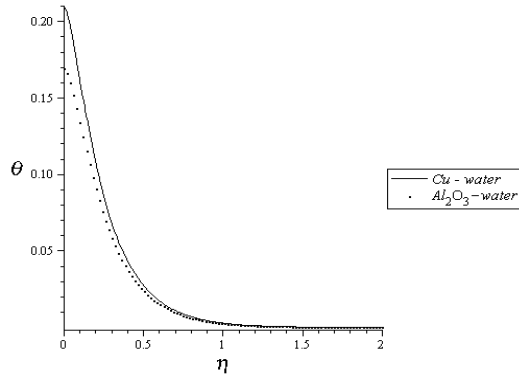


Fig.6: Temperature profiles for $S=1$, $Br=Bi=0.1=\phi=0.1$, $Ha=10^{-10}$

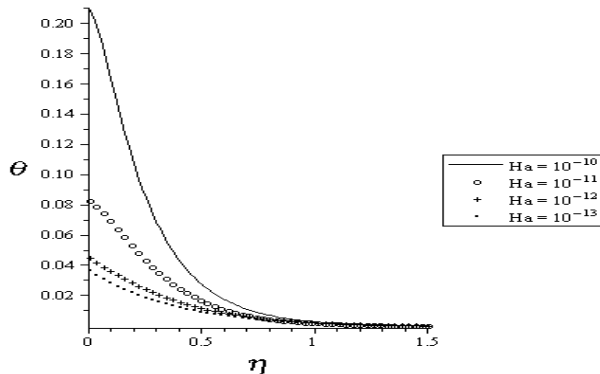


Fig.7: Effect of Ha on temperature profiles with Cu-water nanofluid for $S=1$, $Br=Bi=\phi=0.1$

B. Temperature profiles

Figures 6-11 shows the effects of various parameter on the temperature profile. It is observed that the temperature gradually decreases from a maximum value near the plate surface to zero far away from the plate satisfying the free stream conditions. Figure 6 shows that Cu-water nanofluid thermal boundary layer thickness is greater than that of Al_2O_3 -water nanofluid as expected. This is in accordance with the earlier observation, since the Cu-water nanofluid tends to absorb more heat from the plate surface owing to its close proximity to the hot surface. In Fig. 7, it is noted that an increase in Ha leads to an increase in the temperature, and as a result, the thermal boundary layer thickness increases. Increasing the Bi increases the temperature and the thermal boundary layer thickness, a fact attributed to an increase

in the convective heating as shown in Fig. 8. Similar results are observed with an increase in ϕ and Br as shown in Figs. 9-10. This agrees with the physical behaviour in that when the volume fraction of copper increases the thermal conductivity increases as well, and as a result the thermal boundary layer thickness increases. This observation shows that using nanofluids changes the temperature, thus the use of nanofluids will be of significance in the cooling and heating processes. Figure 11 shows that an increase in $S > 0$ means more nanofluid is sucked out of the porous plate leading to a decrease in the temperature and subsequently, a decrease in the thermal boundary layer thickness. This is expected since increasing S implies that more fluid is sucked out of the permeable plate surface.

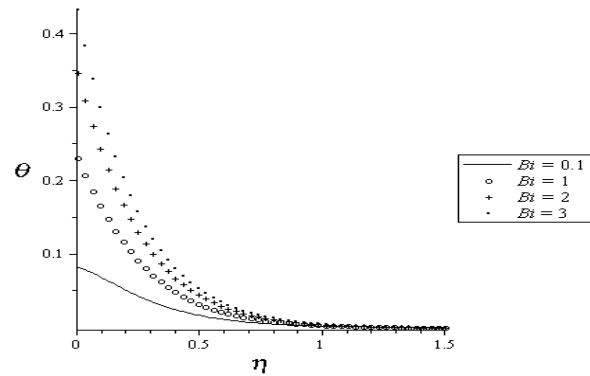


Fig. 8: Effect of Bi on temperature profiles with Cu-water nanofluid for $S=1$, $Ha=10^{-11}$, $Br=\phi=0.1$

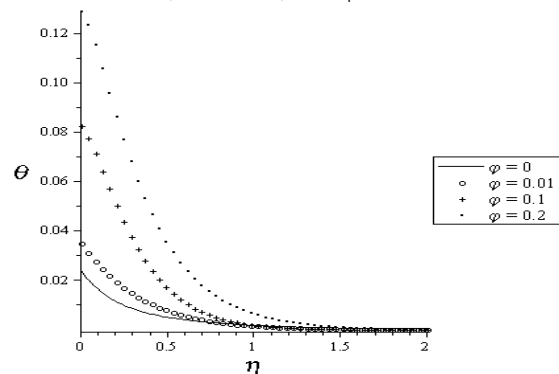


Fig. 9: Effect of ϕ on temperature profiles with Cu-water nanofluid for $S=1$, $Ha=10^{-11}$, $Br=0.1$

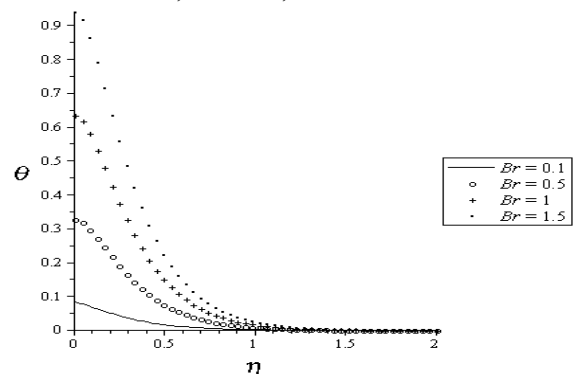


Fig. 10: Effect of Br on temperature profiles with Cu-water nanofluid for $S=1$, $Ha=10^{-11}$, $Bi=\phi=0.1$

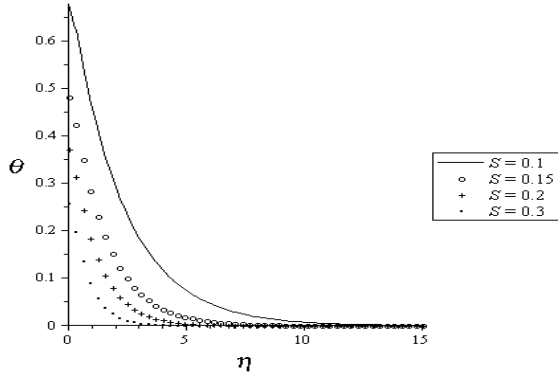


Fig. 11: Effect of S on temperature profiles with Cu-water nanofluid for $Ha= 10^{-11}$, $Br=Bi=\phi= 0.1$

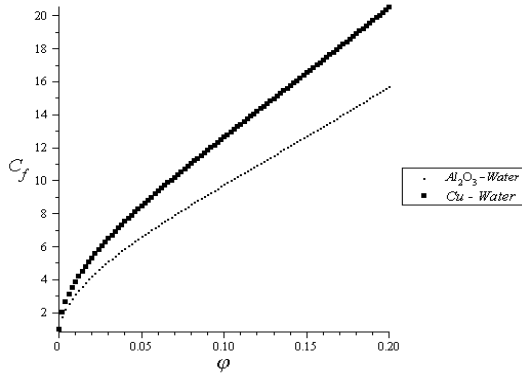


Fig. 12: Skin friction coefficient for $Ha= 10^{-10}$, $S = 1$, $Br = Bi = 0.1$

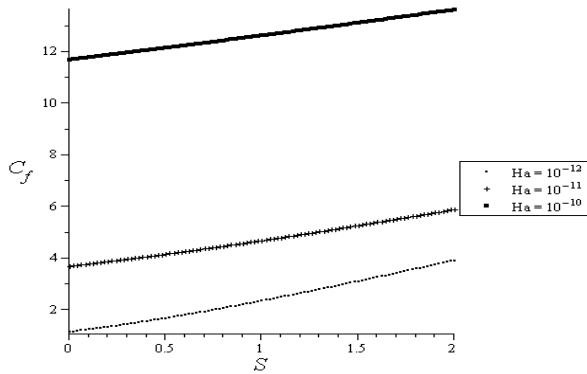


Fig. 13: Skin friction coefficient with Cu-water nanofluid for $Br = Bi = \phi = 0.1$

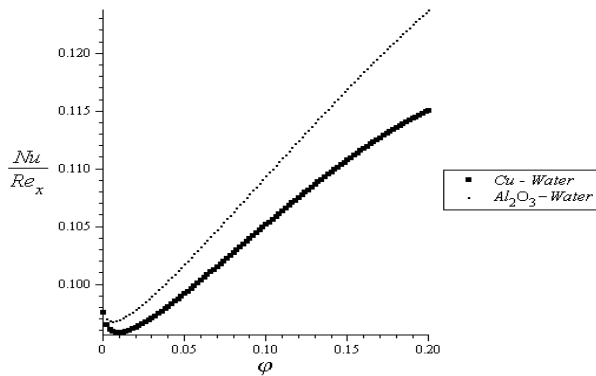


Fig. 14: Nusselt number for $Ha= 10^{-10}$, $S = 1$, $Br = Bi = 0.1$

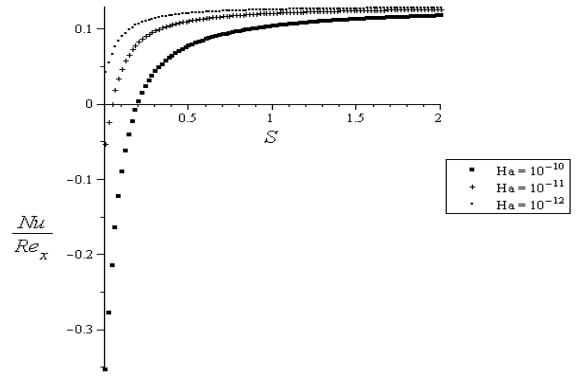


Fig. 15: Nusselt number with Cu-water nanofluid for $Br = Bi = \phi = 0.1$

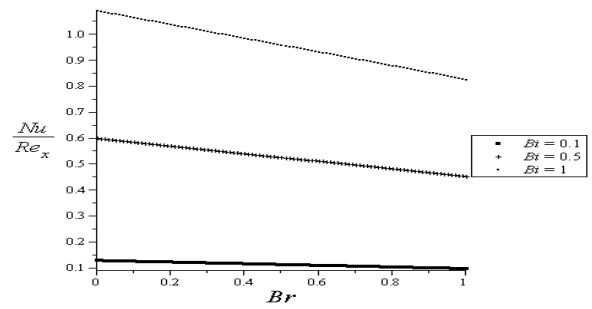


Fig. 16: Nusselt number with Cu-water nanofluid for $S = 1$, $\phi = 0.1$, $Ha = 10^{-12}$

C. Skin Friction

Figures 12-13 shows the variation of the skin friction or the shear stress with ϕ and Ha . It is noted that for high value of ϕ , the skin friction coefficient becomes higher. Cu-water nanofluid has a higher skin friction compared to Al_2O_3 -water nanofluid as shown in Fig.12. This is also expected, since Cu-water moves closer to the plate surface leading to an elevation in the velocity gradient at the plate surface. Figure13 illustrates that decreasing the magnetic field strength decreases the skin friction.

D. Nusselt number

Figures 14-16 show the variation reduced Nusselt number versus different nanofluids, Ha and Bi . It is noted from Fig. 14 that the heat transfer rates increases with the increase in ϕ , although the heat transfer rate at the plate surface for Al_2O_3 -water nanofluid is higher compared to Cu-water nanofluid. A decrease in Ha leads to an increase in the rate of heat transfer at the plate surface as seen in Fig.15. Figure 16 illustrates that increasing the Br , reduces the rate of heat transfer rate, while increasing the Bi increases the rate of heat transfer.

IV. CONCLUSION

In the present study, we have theoretically studied the effects of magnetic field on the boundary layer flow of Cu-water and Al_2O_3 -water nanofluids past a semi-infinite permeable moving flat plate with a convective heat exchange at the surface. The coupled nonlinear governing equations were derived, non-dimensionalised and solved numerically using fourth-order Runge-Kutta-Fehlberg method with shooting technique, putting into consideration the enhanced electrical conductivity of the

convectonal base fluid due to the presence of the nanoparticles. The effects of Ha , ϕ , S and Bi on velocity, temperature, skin friction and local Nusselt number are investigated. It is shown that, increasing the values of Ha , ϕ and S lead to a decrease in the momentum boundary layer thickness and an increase in the thermal boundary layer thickness. From the application point of view, it is obvious that the cooling effect on the convectively heated plate surface is enhanced with increasing values Ha , ϕ and S while an increase in Bi decreases the cooling effect. The skin friction increases with increase in both the magnetic parameter and ϕ , while the Nusselt number increases with increase in ϕ and Bi , but with a decrease in Ha and Br . The results obtained herein justify the physics of the problem, however, experimental data are yet to be found to further validate the formulation of this problem. It is hoped that work such as this will encourage further work in area involving electrically conducting nanofluids and the authors will highlight the interaction between the electrical conductivity of both the convectonal base fluid and the nanoparticle in the presence of a magnetic field.

It is evident from the above findings that the present study has numerous industrial, engineering and biomedical applications such as heat transfer applications: industrial cooling, smart fluids; nanofluid coolant: vehicle cooling, electronics cooling; medical applications: magnetic drug targeting and nanocryosurgery.

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