

ON CORRECTION TO TEMPERATURE FIELD DUE TO VARIABLE THERMAL CONDUCTIVITY IN CASE OF BOUNDARY LAYER FLOW PAST A MOVING CYLINDER WITH SUCTION

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Abstract—□□ The boundary layer flow of viscous incompressible fluid over a moving cylinder with suction has been considered to study heat transfer with variable thermal conductivity. The heat transfer is affected by thermal conductivity for the liquid metals within 0°F to 400°F range. In this case, we observe that the transfer of heat behaves differently in two different regions: $0 \leq \eta \leq 1$ and $\eta > 1$. Hence, we solve the two boundary value problems to draw out recommendations. The results have been discussed graphically.

Keywords— Boundary layer equation, Energy equation, Variable thermal conductivity, Prandtl number.

I. INTRODUCTION

The boundary layer flow with heat transfer due to stretching flat plate or cylinder not only of practical interest in fibre technology, metal extrusion process etc. but also of theoretical interest. The production of polymer sheets and plastic films is based on this technology. There are number of examples which include the cooling of an infinite metallic plate in a cooling bath such as the boundary layer along material handling conveyers, the aerodynamic extrusion of plastic sheets, the boundary layer along a liquid film in condensation processes, paper production, glass blowing, metal spinning and drawing plastic films. The quality of the final product depends on the rate of heat transfer at the stretching surface. Sakiadis (1961) was the first to consider the boundary layer flow on a moving continuous solid surface, whose work was subsequently extended by Rotte and Beek (1969), Bourne and Elliston (1970). Crane (1970) extended this concept to a stretching sheet with linearly varying surface speed and presented an exact solution for steady two-dimensional flow over stretching surface in a quiescent fluid. Since then, many authors have considered various aspects of this problem and obtained similarity solutions. A similarity solution is one in which the number of independent variables is reduced by at least one, usually by a coordinate transformation. The idea is analogous to dimensional analysis, but instead of parameters, like the Reynolds number, the coordinates themselves are collapsed into dimensionless groups that scale the velocities (White, 2006).

On the other hand, Pop *et al.* (1990) have studied the problem of boundary layer flow past a moving longitudinal cylinder in a non-Newtonian power-law fluid at rest. The boundary layer flow due to a stretching ver-

tical surface in a quiescent viscous incompressible fluid when the buoyancy forces are taken into consideration have been considered in the papers by Daskalakis (1993), Chen (1998, 2000), Lin and Chen (1998), Ali (2004), Partha *et al.* (2005) and Ishak *et al.* (2007, 2008). Lin and Shih (1980, 1981) considered the laminar boundary layer and heat transfer along horizontally and vertically moving cylinders with constant velocity and found that the similarity solutions could not be obtained due to the curvature effect of the cylinder. Pop *et al.* (1995) dealt the possibility of mass transfer through the cylinder wall. If the wall of the cylinder is porous or perforated, fluid at a prescribed temperature can be blown into the boundary layer (injection) or fluid at the wall surface can be withdrawn (suction). These mass transfer processes may measurably alter the flow and heat transfer characteristics.

Recently, Rangi and Ahmad (2012) investigated boundary layer flow past a stretching cylinder and heat transfer where they have use Keller-box technique to study flow and temperature fields. In this paper, following Pop *et al.* (1995), we consider heat transfer with variable thermal conductivity in case of moving cylinder with suction to study the heat transfer. We come across the correction to mean temperature field in two different regions.

II. MATHEMATICAL FORMULATION OF THE PROBLEM

Consider a steady, axisymmetric boundary layer flow of a viscous incompressible fluid along a cylinder of radius R , which moves with the constant velocity. It is assumed that the surface of the cylinder is at a uniform temperature T_w , which is greater than the ambient temperature T_∞ . All over the cylinders surface, fluid is sucked in with a constant radial velocity v_w . We use the coordinates x and r , where the x -axis is taken along the cylinder axis and the r -axis is in the radial direction as shown in Fig 1.

Under the assumption of boundary layer theory, the equations which model the problem under consideration are

$$\frac{\partial}{\partial x}(ru) + \frac{\partial}{\partial r}(rv) = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial r} = \frac{\nu}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u}{\partial r} \right) \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial r} = \frac{1}{r} \frac{\partial}{\partial r} \left(\alpha r \frac{\partial T}{\partial r} \right) \quad (3)$$

with the boundary conditions

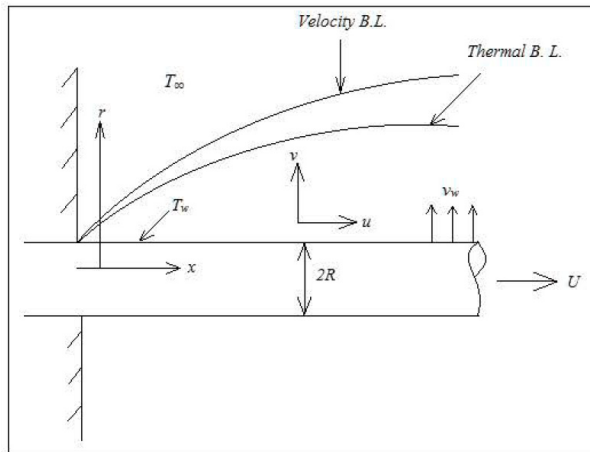


Figure 1. Physical model and coordinate system

$$\begin{aligned} u = U, v = v_w, T = T_w \quad \text{at} \quad r = R, \\ u \rightarrow 0, T \rightarrow T_\infty \quad \text{as} \quad r \rightarrow R, \end{aligned} \quad (4)$$

where u and v are the radial and axial velocity components respectively, T is the fluid temperature, v is kinematic viscosity while density and specific heat are taken constant, α is thermal diffusivity and v_w (<0) is the velocity of suction.

We define the following dimensionless boundary layer variables:

$$\text{The axial coordinate } \xi = \frac{4}{R} \sqrt{\frac{v_x}{U}}, \quad (5a)$$

$$\text{the similarity variable } \eta = \frac{r^2 - R^2}{\xi R^2} \quad \text{and the temperature}$$

$$\text{distribution} \quad \theta(\xi, \eta) = \frac{T - T_\infty}{T_w - T_\infty} \quad (5b)$$

$$\text{The stream function } \psi = R\sqrt{v_x U} f(\xi, \eta), \quad (6a)$$

such that

$$u = \frac{1}{r} \frac{\partial \psi}{\partial r} \quad \text{and} \quad v = -\frac{1}{r} \frac{\partial \psi}{\partial \xi} \quad (6b)$$

The continuity Eq. (1) is satisfied by Eq. (5) and Eq.(6).

Thus Eq. (2) is transformed to

$$(1 + \xi\eta) \frac{\partial^3 f}{\partial \eta^3} + (\xi + f) \frac{\partial^2 f}{\partial \eta^2} = \xi \left(\frac{\partial f}{\partial \eta} \frac{\partial^2 f}{\partial \xi \partial \eta} - \frac{\partial f}{\partial \xi} \frac{\partial^2 f}{\partial \eta^2} \right) \quad (7a)$$

with boundary conditions

$$\text{At } \eta=0 \quad \frac{\partial f}{\partial \eta} = 2 \quad \text{and} \quad f + \xi \frac{\partial f}{\partial \xi} = -2\sigma\xi$$

$$\text{and as } \eta \rightarrow \infty \quad \frac{\partial f}{\partial \eta} = 0 \quad (7b)$$

where $\sigma = \frac{v_w R_0}{4\nu}$ is the fluid suction ($\sigma < 0$).

It has been observed that for liquid metals the thermal conductivity α varies with temperature in an approximately linear manner in the range from 0°F to 400°F. Therefore, we assume α as $\alpha = \alpha_\infty(1 + \varepsilon\theta)$.

Now, the Eq. (3) reduces to

$$\text{Pr} \left\{ \xi \left(\frac{\partial \theta}{\partial \xi} \frac{\partial f}{\partial \eta} - \frac{\partial f}{\partial \xi} \frac{\partial \theta}{\partial \eta} \right) \right\} = \text{Pr} \left(f + \frac{\xi}{\text{Pr}} \right) \frac{\partial \theta}{\partial \eta} + (1 + \xi\eta) \frac{\partial^2 \theta}{\partial \eta^2} + \quad (8a)$$

$$\varepsilon \left[\xi \theta \frac{\partial \theta}{\partial \eta} + (1 + \xi\eta) \left\{ \theta \frac{\partial^2 \theta}{\partial \eta^2} + \left(\frac{\partial \theta}{\partial \eta} \right)^2 \right\} \right]$$

with boundary conditions

$$\text{at } \eta=0 \quad \theta = 1$$

$$\text{and as } \eta \rightarrow \infty \quad \theta = 0 \quad (8b)$$

From the Eq. (8a), we note that the heat transfer takes place in two parts, that is, one part of heat transfer is due to temperature difference and the other part is due to variable thermal conductivity. In Eq. (8a), if $\varepsilon=0$, we get the temperature field θ for the problem where thermal conductivity does not contribute, i.e. thermal conductivity is constant. Our aim is to analyze the temperature field which occurs due to variation in thermal conductivity. Thus the temperature field governed by the following boundary value problem

$$\left\{ \xi \left(\frac{\partial \theta_m}{\partial \xi} \frac{\partial f}{\partial \eta} - \frac{\partial f}{\partial \xi} \frac{\partial \theta_m}{\partial \eta} \right) \right\} = \left(f + \frac{\xi}{\text{Pr}} \right) \frac{\partial \theta_m}{\partial \eta} + (1 + \xi\eta) \frac{\partial^2 \theta_m}{\partial \eta^2} \quad (9a)$$

$$\eta=0 \quad \theta_m = 1$$

$$\theta_m \rightarrow 0 \quad \text{as } \eta \rightarrow \infty \quad (9b)$$

The temperature field which occurs due to variation of thermal conductivity is governed by

$$\left[\xi \theta_v \frac{\partial \theta_v}{\partial \eta} + (1 + \xi\eta) \left\{ \theta_v \frac{\partial^2 \theta_v}{\partial \eta^2} + \left(\frac{\partial \theta_v}{\partial \eta} \right)^2 \right\} \right] = 0 \quad (10a)$$

$$\eta=0 \quad \theta_v = 1$$

$$\theta_v \rightarrow 0 \quad \text{as } \eta \rightarrow \infty \quad (10b)$$

The solution of the problem posed by the Eq. (9a) and Eq.(9b) was discussed by Pop *et al.* (1995). Our aim is to analysis the contribution made by thermal conductivity. So, we write

$$\left[\left\{ \theta_v \frac{d^2 \theta_v}{d\eta^2} + \left(\frac{d\theta_v}{d\eta} \right)^2 \right\} + \xi \left[\theta_v \frac{d\theta_v}{d\eta} + \eta \left\{ \theta_v \frac{d^2 \theta_v}{d\eta^2} + \left(\frac{d\theta_v}{d\eta} \right)^2 \right\} \right] \right] = 0 \quad (11)$$

Applying the regular perturbation method as applied by Ahmad *et al.* (2010), we equate the terms independent of ξ and the dependent of ξ to zero and we get

$$\theta_v \frac{d^2 \theta_v}{d\eta^2} + \left(\frac{d\theta_v}{d\eta} \right)^2 = 0 \quad (12a)$$

$$\theta_v \frac{d\theta_v}{d\eta} + \eta \left\{ \theta_v \frac{d^2 \theta_v}{d\eta^2} + \left(\frac{d\theta_v}{d\eta} \right)^2 \right\} = 0 \quad (12b)$$

The Eq. (12a) is non-linear differential equation. Assuming its solution as

$$\theta_v(\eta) = (1 - \eta)^\alpha$$

we have

$$2\alpha^2 - \alpha = 0$$

The roots of this equation are 0 and $\frac{1}{2}$, therefore

$$\theta_v(\eta) = A + B\sqrt{1 - \eta} \quad (13)$$

The general solution of Eq. (12a) is real only when $0 \leq \eta \leq 1$. Therefore, the heat transfer due to variable thermal conductivity which is free from axial co-ordinate takes

place within the dynamic region $0 \leq \eta \leq 1$. Thus, the boundary conditions (10b) may be replaced by

$$\begin{aligned} \eta=0, \theta_v &= 1 \\ \eta \rightarrow \infty, \theta_v &\rightarrow 0 \end{aligned} \quad (14)$$

Hence, the solution of the problem given by the Eq. (13) and Eq. (14) is

$$\theta_v(\eta) = \sqrt{1-\eta}$$

Looking at the Eq. (12b), we find that

$$\frac{d}{d\eta} \left(\theta_v \eta \frac{d\theta_v}{d\eta} \right) = 0 \quad (15)$$

whose solution comes out to be

$$\theta_v = \sqrt{2C \ln(\eta)} \quad (16)$$

where C is the arbitrary constant. Here, we observe that θ_v is real only for $\eta > 1$. Therefore this part of heat transfer occurs only in $]1, \infty[$. Also, we observe that this part of heat transfer depends on axial co-ordinate ξ . Finally, we have

$$\frac{\theta_v}{\sqrt{2C}} = \sqrt{\ln(\eta)} \quad \text{for } \eta > 1.$$

III. DISCUSSION AND RESULTS

Analyzing the heat transfer with variable thermal conductivity in case of moving cylinder with suction, we come across that the contribution of variable thermal conductivity to heat transfer occurs in two regions: one in $0 \leq \eta \leq 1$ and other in $1 < \eta < \infty$. These two regions may be the outcome of suction. As the fluid from the immediate neighborhood of outer surface of cylinder has been sucked, the fluid from the outside region $1 < \eta < \infty$ rushes to the region $0 \leq \eta \leq 1$. Therefore, a new dynamical region comes into picture in which the heat transfer pattern is different from the heat transfer in $0 \leq \eta \leq 1$. We summarize the results as below:

(i) Within the dynamic region $0 \leq \eta \leq 1$, the temperature field is $\theta = \theta_m + \sqrt{1-\eta}$ which is independent of axial co-ordinate ξ . θ_m is referred as mean temperature and studied by Pop *et al.* (1995).

(ii) Within the region $1 < \eta < \infty$, the temperature field becomes $\theta = \theta_m + \xi \theta_v = \theta_m + \xi (2C \ln(\eta))^{1/2}$ which varies with axial co-ordinate ξ linearly.

(iii) If we put $\varepsilon=0$, the results are agreed with Pop *et al.* (1995).

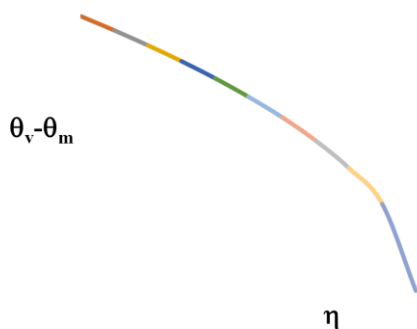


Figure 2. Correction in mean temperature within $0 < \eta < 1$

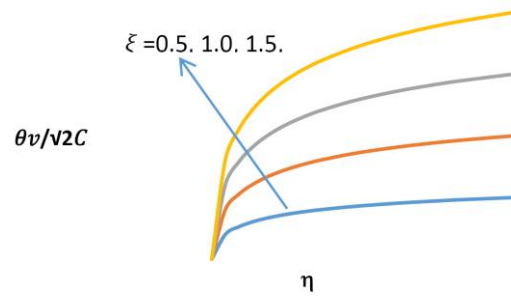


Figure 3. Correction in mean temperature θ_m within $1 < \eta < \infty$ for $\xi=0.5, 1.0, 1.5, 2.0$

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